Precision Metrology 16 -Roundness Measurement

Roundness: Deviation from ideal reference circle

-True circle? True round?

-About 70% of all engineering components have axis of rotation

-No round parts having truly round profile

Causes of Non-roundness or out of roundness or roundness error

Machining(Milling/Turning/Grinding): Spindle error, Tool wear, Chatter, Defects in bearing, Elastic deformation of workpiece, Chucking

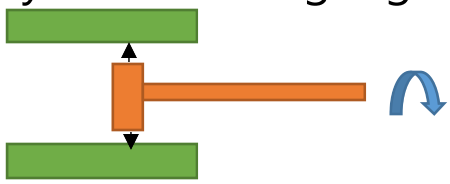
Centreless grinding: Lobed circle pattern in ball grinding

Drawing/Extrusion: Wear in Die/Mold, Defects in Surface

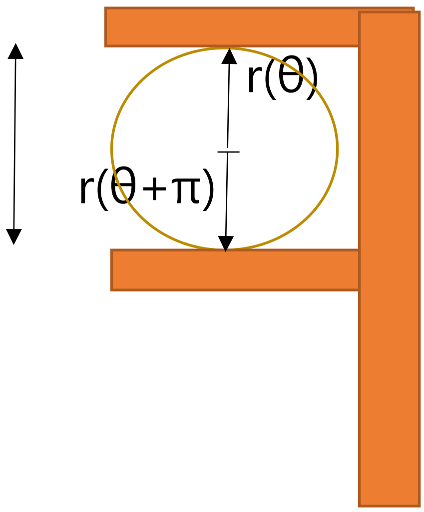
Roundness Measurement

(1) Diametre measurement (two points method)

cylinder/bore gauge

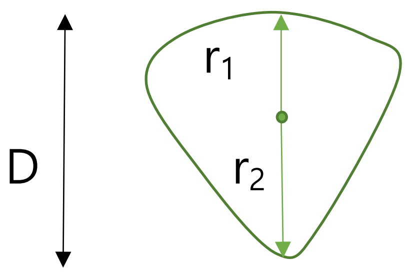


$D(\theta)=r(\theta)+r(\theta+\pi)$



<u>Diametre measurement vs Radius measurement</u>

Ex) Lobed circle



Inner Dia.$=2r_1$

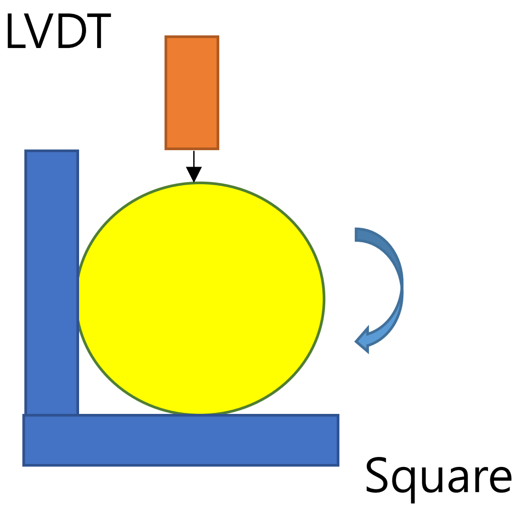
Outer Dia.$=2r_2$

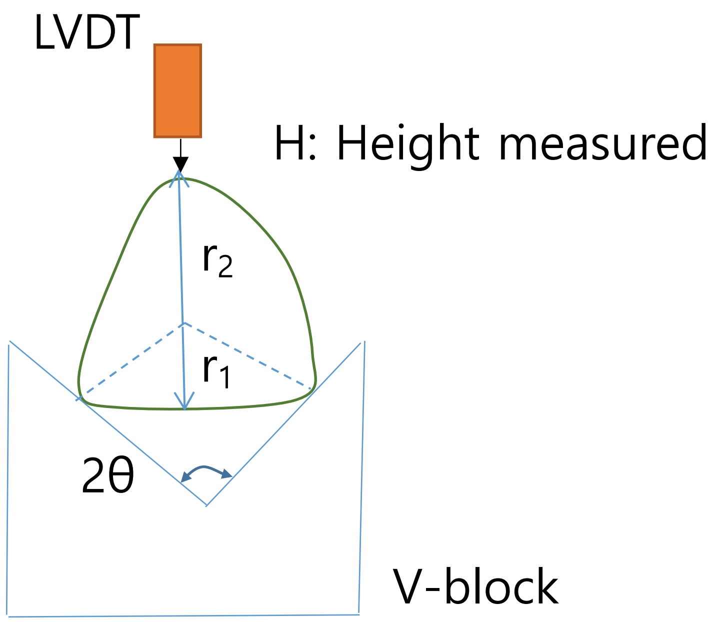
D=Diametre$=r_1+r_2=$const.

Causes: Jaws chucking, or centerless grinding

∴Lobed circle (especially odd-numbered) cannot be measured

(2) 3 Points method





$H_2 = r_2 + r_2/\sin\theta$: measured at 2 position

$H_1 = r_1 + r_1/\sin\theta$: measured at 1 position

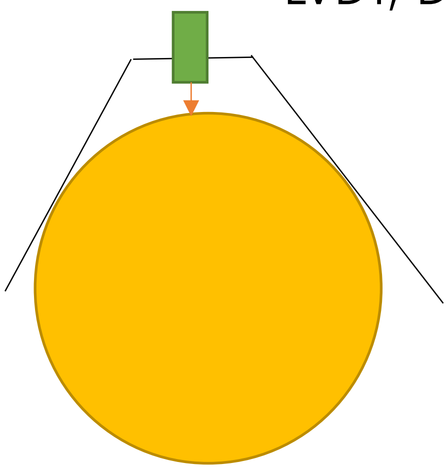
Height difference $= H_2 - H_1 = (r_2 - r_1) + (r_2 - r_1)/\sin\theta$

$= (r_2 - r_1)[1 + 1/\sin\theta] > (r_2 - r_1)$

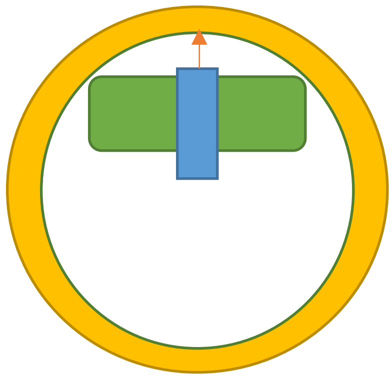
∴Data distortion or magnification for lobed circle

Cylinder Gauge

LVDT, Dial Gauge, Cap gauge



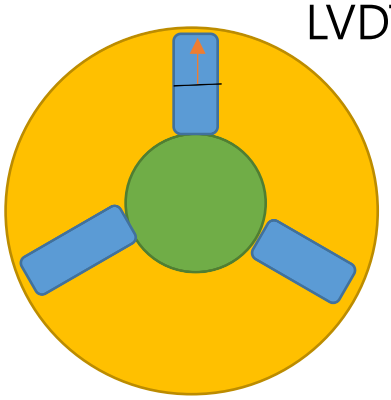
Outer Cylinder



Inner Cylinder

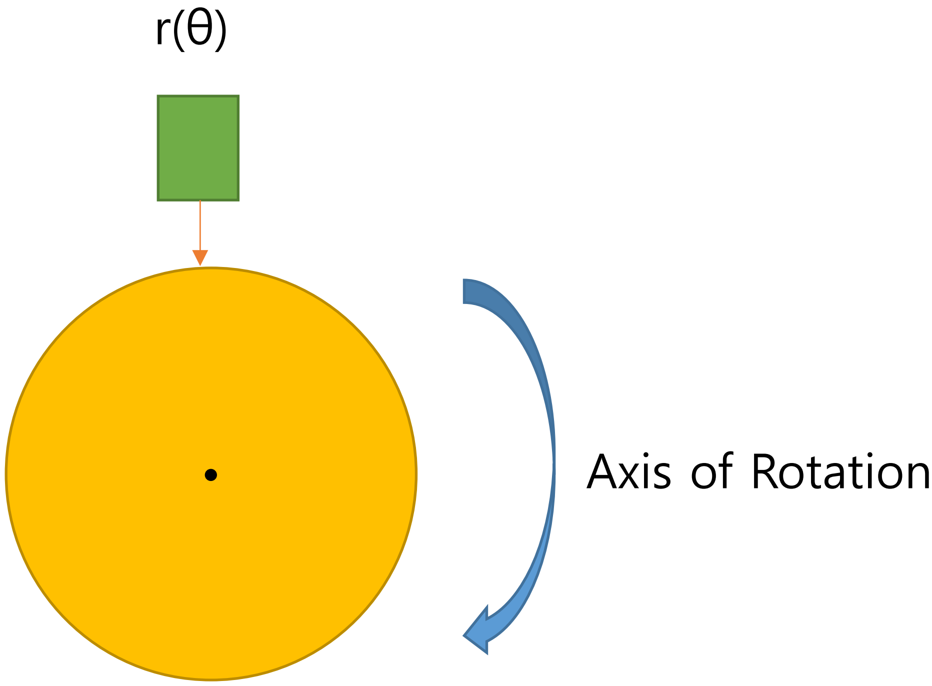
Bore Gauge

LVDT(Dial Gauge)



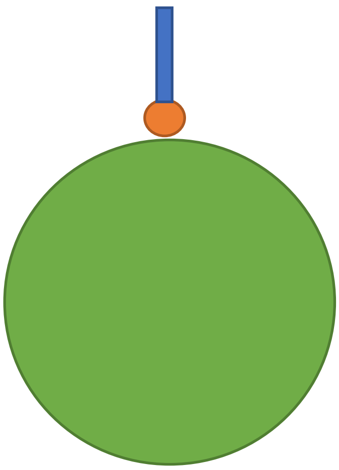
(3) Radius measurement method

LVDT, Dial Gauge, Cap Sensor



(4) CMM (Coordinate Measuring Machine)

To: Measure (Xi, Yi) along the Circle



Roundness Calculation

:Deviation from the ideal reference circle

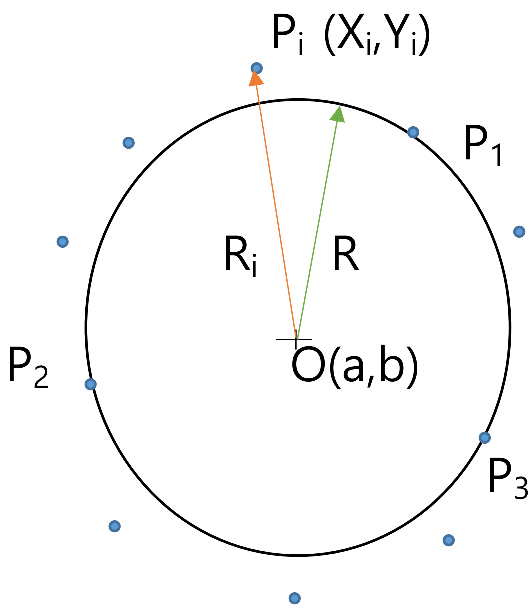
Four Reference Circles:

They are Minimum Circumscribed Circle or Centre (MCC); Maximum Inscribed Circle or Centre (MIC); Least Squares Circle or Centre (LSC); Minimum Zone Circle or Centre (MZC);

(1) Maximum Inscribed Circle or Centre(MIC)

:Largest possible inscribing circle, clue to the Shaft diameter to fit in Hole, or Plug Gauge Centre

Equation of Circle: $(X-a)^2+(Y-b)^2=R^2$



(a,b): Centre, R : Radius

a, b, R can be calculated by finding P_1, P_2, P_3;

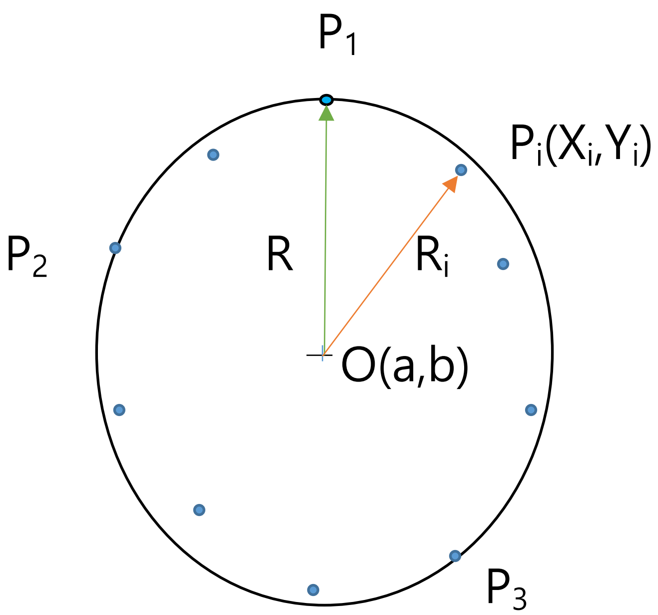
Roundness deviation $= R_i - R = \sqrt{[(X_i-a)^2+(Y_i-b)^2]} - R$

Roundness error $= \max R_i - \min R_i$

P_1, P_2, P_3 can be found by an iterative procedure.

(2)Minimum Circumscribed Circle or Centre(MCC)

:Smallest possible circumscribing circle, clue to Hole diametre to fit in Shaft, or Ring Gauge Centre



(a,b): Centre, R: Radius

a,b,R can be calculated by finding P_1,P_2,P_3;

Roundness deviation

$\delta R_i = R_i - R = \sqrt{[(X_i-a)^2+(Y_i-b)^2]} - R$

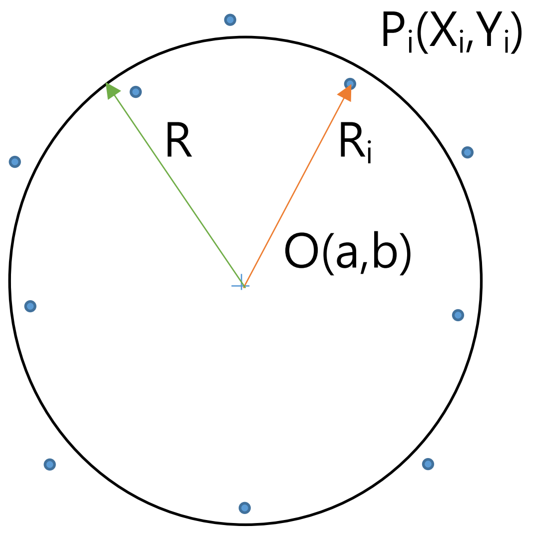
Roundness error

$= \max \delta R_i - \min \delta R_i = \max R_i - \min R_i$

P_1,P_2,P_3 can be found by an iterative procedure.

(3)Least Squares Circle or Centre (LSC)

:Least squares based best fit circle or centre



(a,b): Centre, R: Radius

$I=\sum(R_i-R)^2=\sum[\sqrt{(X_i-a)^2+(Y_i-b)^2}-R]^2$ be minimum

But this is nonlinear formulation!

Thus $(R_i^2-R^2)^2$ can be used instead of $(R_i-R)^2$

$J=\sum(R_i^2-R^2)^2=\sum[(X_i-a)^2+(Y_i-b)^2 - R^2]^2$ minimum

$J=\sum(X_i^2+Y_i^2-2aX_i-2bY_i+a^2+b^2-R^2)^2$

Let $A=-2a$, $B=-2b$, $C=a^2+b^2-R^2$

$J=\sum(X_i^2+Y_i^2+AX_i+BY_i+C)^2$ minimum

$\partial J/\partial A=2\sum(X_i^2+Y_i^2+AX_i+BY_i+C)X_i=0$

$\partial J/\partial B=2\sum(X_i^2+Y_i^2+AX_i+BY_i+C)Y_i=0$

$\partial J/\partial C=2\sum(X_i^2+Y_i^2+AX_i+BY_i+C)=0$

$\therefore$ A,B,C can be solved; a, b, R can be solved.

Roundness deviation,

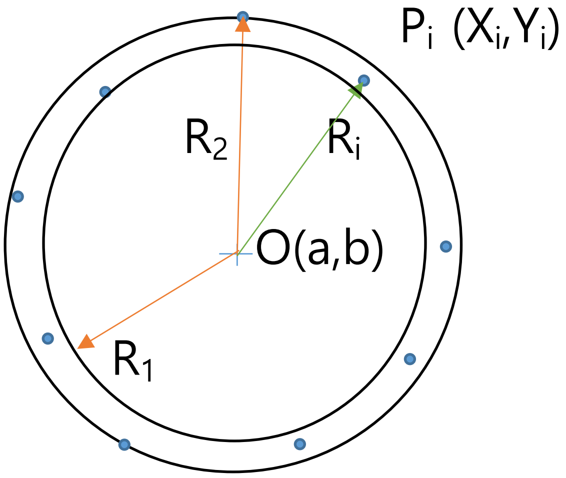
$\delta R_i=R_i-R=\sqrt{[(X_i-a)^2+(Y_i-b)^2]}-R$

Roundness error

$=\max \delta R_i - \min \delta R_i = \max R_i - \min R_i$

(4)Minimum Zone Circle or Centre (MZC)

:Two concentric circles that give minimum radial separation, or MRS circle/centre



(a,b): Centre;

R_2:Maximum radius, R_1:Minimum radius

R_2-R_1: Radial separation

Min $R_2^2 - R_1^2$

s.t. $R_1^2 \le (X_i - a)^2 + (Y_i - b)^2 \le R_2^2$

Let $A=2a$, $B=2b$,

$C_2=R_2^2-(a^2+b^2)$, $C_1=R_1^2-(a^2+b^2)$, then becomes

Min C_2-C_1

s.t.

$AX_i+BY_i+C_2 \geq X_i^2+Y_i^2$

$AX_i+BY_i+C_1 \leq X_i^2+Y_i^2$

Linear Programming with Simplex Search

Let $\mathbf{C}=[0,0,1,-1]$; $\mathbf{X}=[A,B,C_2,C_1]^T$

Min $\mathbf{CX}$

St $\mathbf{A_1X} \geq \mathbf{B}$, $\mathbf{A_2X} \leq \mathbf{B}$

Where

$$\mathbf{A}_1=\begin{bmatrix} X_1 & Y_1 & 1 & 0 \\ X_2 & Y_2 & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ X_N & Y_N & 1 & 0 \end{bmatrix} \quad \mathbf{B}=\begin{bmatrix} X_1^2+Y_1^2 \\ X_2^2+Y_2^2 \\ \cdots \\ X_N^2+Y_N^2 \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} X_1 & Y_1 & 0 & 1 \\ X_2 & Y_2 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots \\ X_N & Y_N & 0 & 1 \end{bmatrix}$$

Thus A, B, C_2, C_1 can be solved; a, b, R_2, R_1 solved

Roundness deviation, δRi

$\delta R_i = R_i - R_1 = \sqrt{[(X_i - a)^2 + (Y_i - b)^2]} - R1$

Roundness error=max δR_i- min δR_i

=max R_i- min R_i