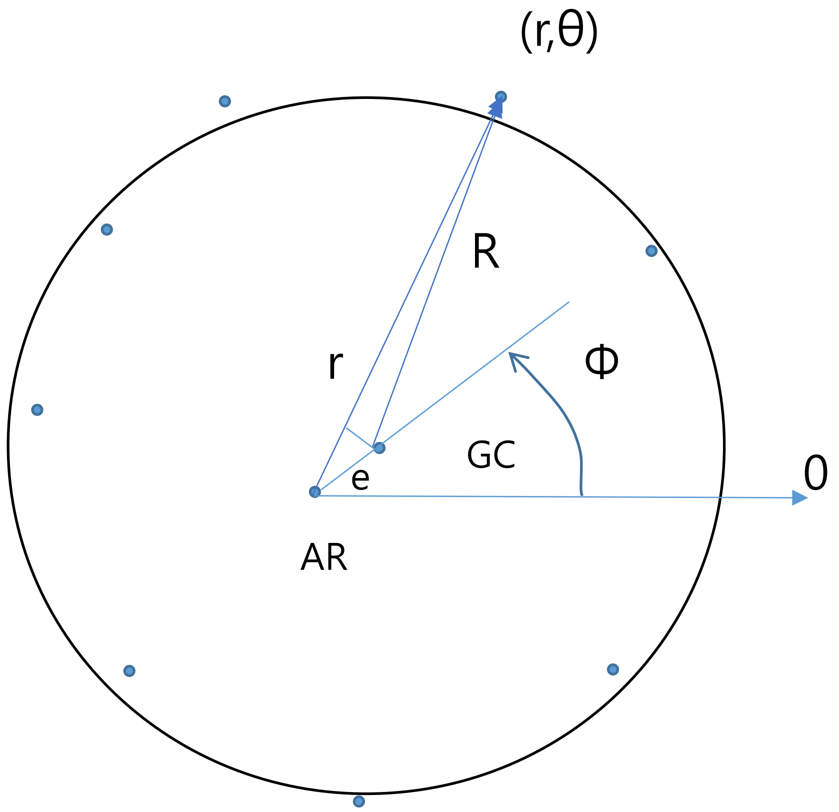
Precision Metrology 17 ; Roundness evaluation

Roundness Evaluation in Polar Coordinate System



$r(\theta)=e\cos(\theta-\Phi)+\sqrt{[R^2-e^2\sin^2(\theta-\Phi)]}$

$=e\cos(\theta-\Phi)+R\sqrt{[(1-e^2/R^2\sin^2(\theta-\Phi)]}$

$\fallingdotseq e[\cos\theta\cos\Phi+\sin\theta\sin\Phi]+R=a\cos\theta+b\sin\theta+R$;

where $a=e\cos\Phi$, $b=e\sin\Phi$ are eccentricity

In order to fit into the least squares partial arc, ranging from θ_1 to θ_2

$J=\int[r(\theta)-(a\cos\theta+b\sin\theta+R)]^2 d\theta$ be minimum,

and a,b,R should be found; where $\int$ is the integration over θ_1 to θ_2

$\partial J/\partial R=2\int[r(\theta)-a\cos\theta-b\sin\theta-R](-1)=0$

$\therefore\ a\int\cos\theta d\theta+b\int\sin\theta d\theta+R(\theta_2-\theta_1)=\int r(\theta)d\theta$ (1)

$\partial J/\partial a=2\int[r(\theta)-a\cos\theta-b\sin\theta-R](-\cos\theta)d\theta=0$

$\therefore a\int\cos^2\theta d\theta+b\int\sin\theta\cos\theta d\theta+R\int\cos\theta d\theta=\int r(\theta)\cos\theta d\theta$ (2)

$\partial J/\partial b=2\int[r(\theta)-a\cos\theta-b\sin\theta-R](-\sin\theta)d\theta=0$

$\therefore a\int\cos\theta\sin\theta d\theta+b\int\sin^2\theta d\theta+R\int\sin\theta d\theta=\int r(\theta)\sin\theta d\theta$ (3)

<u>From (1),(2),(3)</u>

$a=[A\{\int r(\theta)\cos\theta d\theta-B\int r(\theta)d\theta\}$

$+C\{\int r(\theta)\sin\theta d\theta-D\int r(\theta)d\theta\}]/E$

$b=[F\{\int r(\theta)\sin\theta d\theta-D\int r(\theta)d\theta\}+C\{\int r(\theta)\cos\theta d\theta-B\int r(\theta)d\theta\}/E$

$R=[\int r(\theta)d\theta-a\int\cos\theta d\theta-b\int\sin\theta d\theta]/(\theta_2-\theta_1)$

where

$A=\int \sin^2\theta d\theta-\{\int \sin\theta d\theta\}^2/(\theta_2-\theta_1)$

$B=\{\sin\theta_2-\sin\theta_1\}/(\theta_2-\theta_1)$

$C=\int \cos\theta d\theta \int \sin\theta d\theta/(\theta_2-\theta_1)-\int \sin\theta\cos\theta d\theta$

$D=\{\cos\theta_1-\cos\theta_2\}/(\theta_2-\theta_1)$

$E=AF-C^2$ where $F=\int \cos^2\theta d\theta-\{\int \cos\theta d\theta\}^2/(\theta_2-\theta_1)$

If $\theta_1=0$, $\theta_2=2\pi$;

$A=\pi$, $B=0$, $C=0$, $D=0$, $F=\pi$, $E=\pi^2$

Thus $R=\int r d\theta /2\pi \fallingdotseq \Sigma r_i/N$, where N= number of points

$a=\int r\cos\theta d\theta/\pi \fallingdotseq 2\Sigma X_i/N$, and $b=\int r\sin\theta d\theta/\pi \fallingdotseq 2\Sigma Y_i/N$

This is a remarkable result for the least squares circle.

(Discuss any necessary assumption?)

Roundness deviation, $\delta r_i=r_i-(a\cos\theta_i+b\sin\theta_i+R)$

Thus roundness error= max (δr_i) – min (δr_i)

In (X, Y) coordinate system;

Roundness Deviation, $\delta r_i= \sqrt{(X_i-a)^2+(Y_i-b)^2} - R$

Thus roundness error= max (δr_i) – min (δr_i)

Lobing Coefficients for Roundness Profile, $r(\theta)$:

As $r(\theta)$ is of 2π period, thus can be expanded with the Fourier Series Expansion

$r(\theta)=R+\Sigma(A_n\cos n\theta+B_n\sin n\theta)$,

where Σ is the summation from 1 to ∞

$R=\int r(\theta)d\theta/2\pi \fallingdotseq \Sigma r_i/N$

$A_n=\int r(\theta)\cos n\theta d\theta/\pi \fallingdotseq 2\Sigma r_i\cos n\theta_i/N$

$B_n=\int r(\theta)\sin n\theta d\theta/\pi \fallingdotseq 2\Sigma r_i\sin n\theta_i/N$

Signal Topology for Roundness (by D.Whitehouse)

Coeff.	Cause	Effect
1	Setup	Eccentricity, once per rev
2	Part Ovality	Ellipse
	Tilt (setup)	
3	Clamping	Tri-lobe
4-5	Genuine	Unequal angle lobe
	Setup	Equal angle lobe
5-20	Stiffness of Machine	Roundness error
20-50	Stiffness	Roundness error
	Chatter	Vibration
50-1000	Manufact. Process	Roundness error Noise

Relevant Terminology in Roundness measurement

Run-out :

Maximum deviation of measurement data during the radial measurement over the 360 deg revolution.

Radial Run-out :

Maximum deviation along a circle of the part

Physically, radial runout=Roundness error + Eccentricity

Total Run-out :

Maximum deviation along a cylinder of the part

Physically, total run-out

=Roundness error+Eccentricity+Straightness+Tilt

Concentricity

:Deviation of centres between the concentric circles

Coaxialty

:Deviation of centres between the coaxial shafts

Cylindricity

:Departure from a true (ideal) cylinder, combination of roundness and straightness

Measurement methods:

(1) Radial section measurement

(2) Generatix method

(3) Helical line method

(4) Points method

Cylindricity calculation

(1) Maximum Inscribed Cylinder

(2) Minimum Circumsribed Cylinder

(3) Least Squares Cylinder

(4) Minimum Zone Cylinder

For the LSC, given measured data (r_i , θ_i , Z_i);

$r_i=(A_0+A_1Z_i)\cos\theta_i+(B_0+B_1Z_i)\sin\theta_i+R$

where $A_0+A_1Z_i$ is the eccentricity in X direction,

$B_0+B_1Z_i$ is the eccentricity in Y direction.

$J=\Sigma[ri-\{(A_0+A_1Z_i)\cos\theta_i+(B_0+B_1Z_i)\sin\theta_i+R\}]^2$ be minimum

A_0,A_1,B_0,B_1,R : unknowns to determine

Once the unknowns are solved,

Cylindricity deviation

$\delta r_i=r_i-\{(A_0+A_1Z_i)\cos\theta_i+(B_0+B_1Z_i)\sin\theta_i+R\}$

Cylindricity error= max δr_i – min δr_i

HW) Formulate the least squares cylinder, and derive the equations to solve the unknows.

Evaluate the cylindricity with the measurement data provided.

Also, discuss the Conicity evaluation.