Precision Metrology 19-Volumetric error calculation

:To build up the volumetric error, based on the geometric error components, and to propagate the geometric errors into the full 3D volume of 3 axis machine

Translational Errors;

positional, horizontal and vertical straightness errors

$\delta x(x)$, $\delta y(x)$, $\delta z(x)$;

$\delta y(y)$, $\delta x(y)$, $\delta z(y)$;

$\delta z(z)$, $\delta x(z)$, $\delta y(z)$;

Rotational Errors; Roll, Pitch, Yaw

$Ex(x)$, $Ey(x)$, $Ez(x)$;

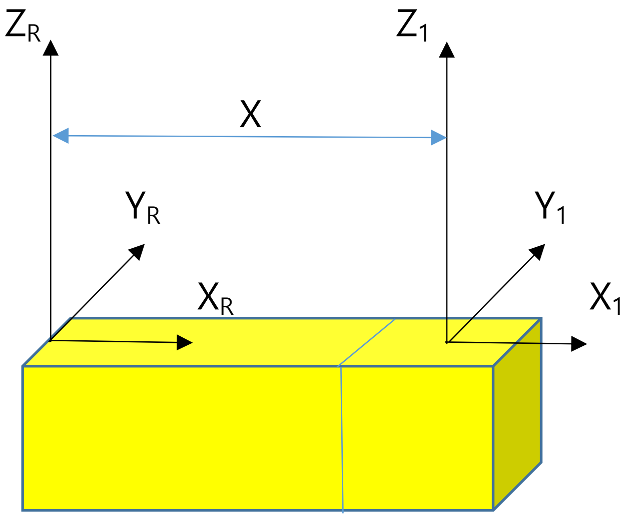
$Ey(y)$, $Ex(y)$, $Ez(y)$;

$Ez(z)$, $Ex(z)$, $Ey(z)$;

Squareness Errors;

α ; XY plane

$\beta 1$; YZ plane, $\beta 2$; XZ plane

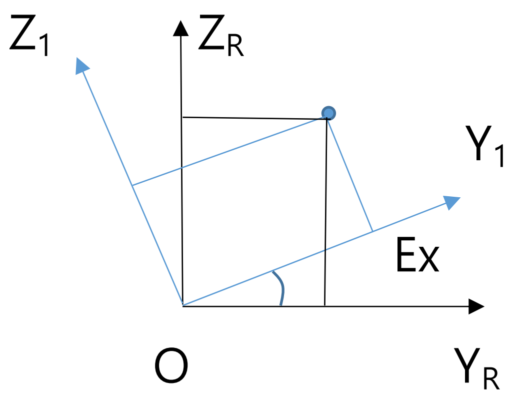


[$O_R X_R Y_R Z_R$]: Reference Coordinate System

[$O_1 X_1 Y_1 Z_1$]: Moving Coordinate System fixed on X-Slide

Two coordinates are initially aligned as the same.

<u>Roll Motion of Slide, Ex</u>



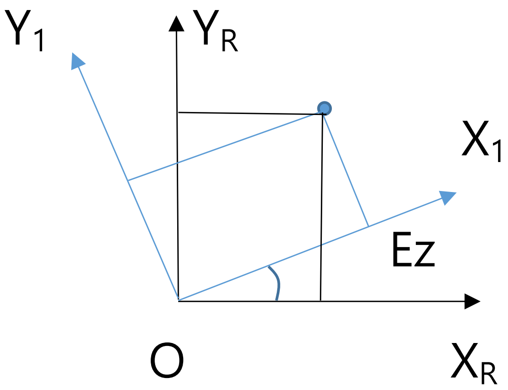
$Y_R = Y_1 \cos Ex - Z_1 \sin Ex \fallingdotseq Y_1 - Z_1 Ex$; if $Ex \ll 1$

$Z_R = Y_1 \sin Ex + Z_1 \cos Ex \fallingdotseq Y_1 Ex + Z_1$; if $Ex \ll 1$

In 3D Transformation matrix form, $\mathbf{X}_R=\mathbf{T}_R\mathbf{X}_1$; that is

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -Ex \\ 0 & Ex & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

<u>Yaw motion of Slide, Ez</u>



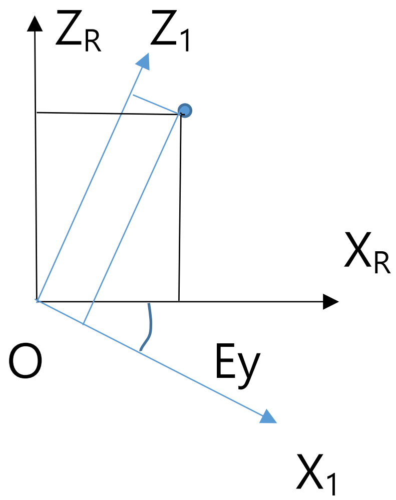
$$X_R=X_1\cos Ez-Y_1\sin Ez \fallingdotseq X_1-Y_1 Ez \ ; \text{ if } Ez \ll 1$$

$$Y_R=X_1\sin Ez+Y_1\cos Ez \fallingdotseq X_1 Ez+Y_1 \ ; \text{ if } Ez \ll 1$$

In 3D Transformation matrix form, $\mathbf{X}_R=\mathbf{T}_Y\mathbf{X}_1$; that is

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} 1 & -Ez & 0 \\ Ez & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

<u>Pitch motion of Slide, Ey</u>



$X_R=X_1\cos Ey+Z_1\sin Ey \fallingdotseq X_1+Z_1Ey$; if $Ey \ll 1$

$Z_R=-X_1\sin Ey+Z_1\cos Ey \fallingdotseq -X_1Ey+Z_1$; if $Ey \ll 1$

In 3D Transformation matrix form, $\mathbf{X}_R=\mathbf{T}_p\mathbf{X}_1$; that is

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} 1 & 0 & Ey \\ 0 & 1 & 0 \\ -Ey & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

Combining all angular motions by multiplying the 3 matrices to give the transformation matrix for rotational motion of X-slide, $\mathbf{T}x$; where the order of multiplication is arbitrary due to the asymmetric matrices.

$\mathbf{T}x=\mathbf{T}_R\mathbf{T}_Y\mathbf{T}_P$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -Ex \\ 0 & Ex & 1 \end{bmatrix} \begin{bmatrix} 1 & -Ez & 0 \\ Ez & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & Ey \\ 0 & 1 & 0 \\ -Ey & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -Ez(x) & Ey(x) \\ Ez(x) & 1 & -Ex(x) \\ -Ey(x) & Ex(x) & 1 \end{bmatrix}$$

And, it is an asymmetric matrix, too.

Now for translating motion of X slide in introduced;

Translation in X direction=$X+\delta x(x)$

; nominal position +positional error

Translation in Y direction=$\delta y(x)-\alpha X$

;Y straightness error of X axis + squareness error

Translation in Z direction=$\delta z(x)$

;Z straightness error of X axis

Thus, the translating motion of X-slide is expressed as the column vector, **L**x, that is,

$$\mathbf{L}x=\begin{bmatrix} X+\delta x(x) \\ \delta y(x)-\alpha X \\ \delta z(x) \end{bmatrix}$$

Thus a point $P(X_1,Y_1,Z_1)$ on the X slide can be expressed in the reference coordinate system $[X_R,Y_R,Z_R]$;

In the 3D transformation matrix,

$\mathbf{X}_R=\mathbf{T}x\mathbf{X}_1+\mathbf{L}x$; eq(1) and that is,

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} 1 & -Ez & Ey \\ Ez & 1 & -Ex \\ -Ey & Ex & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} X+\delta x(x) \\ \delta y(x)-\alpha X \\ \delta z(x) \end{bmatrix}$$

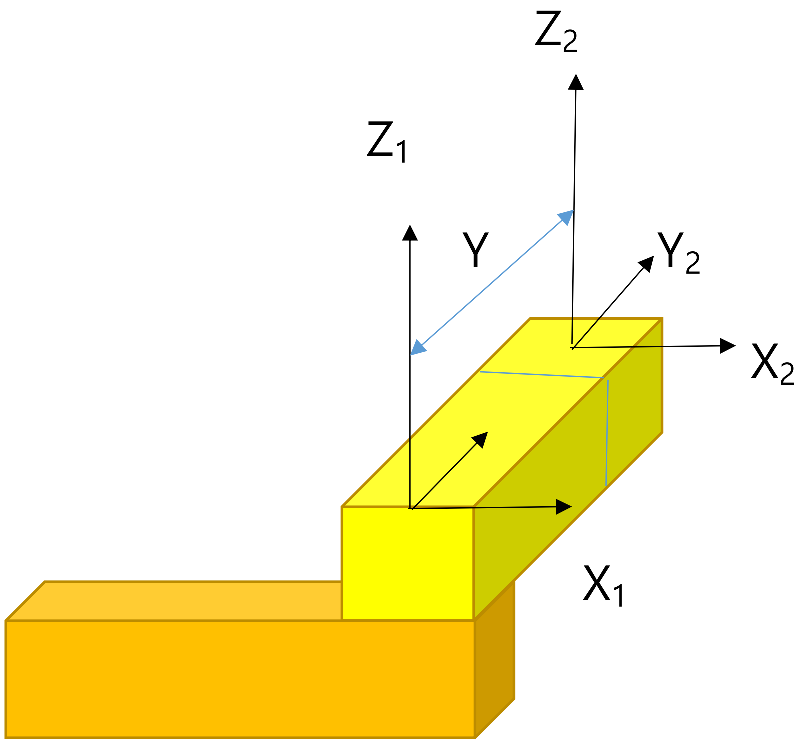
That is,

$$X_R=X+\delta x(x)+X_1-Y_1Ez(x)+Z_1Ey(x)$$

$$Y_R=\delta y(x)-\alpha X+X_1Ez(x)+Y_1-Z_1Ex(x)$$

$$Z_R=\delta z(x)-X_1Ey(x)+Y_1Ex(x)+Z_1$$

Now, introduce the Y-slide motion on the top of the X-slide;



$[O_1X_1Y_1Z_1]$: Moving Coordinates fixed on X slide

$[O_2X_2Y_2Z_2]$: Moving Coordinates fixed on Y slide

Two coordinates are initially aligned as the same.

Angular motion of the Y slide is similarly expressed as the transformation matrix, **T**y, that is

$$\mathbf{T}y = \begin{bmatrix} 1 & -Ez(y) & Ey(y) \\ Ez(y) & 1 & -Ex(y) \\ -Ey(y) & Ex(y) & 1 \end{bmatrix}$$

For translating motion of Y slide,

Translation in X direction=$\delta x(y)$

; X straightness error of Y axis

Translation in Y direction=$Y+\delta y(y)$

; nominal position +positional error

Translation in Z direction=$\delta z(y)$

; Z straightness error of Y axis

Thus the translating motion of Y slide can be expressed as the column vector, **L**y; that is

$$\mathbf{L}y = \begin{bmatrix} \delta x(y) \\ Y+\delta y(y) \\ \delta z(y) \end{bmatrix}$$

Thus a point $P(X_2,Y_2,Z_2)$ on the X slide can be expressed in the $[X_1,Y_1,Z_1]$ coordinate system;

$\mathbf{X}_1=\mathbf{T}y\mathbf{X}_2+\mathbf{L}y$; eq(2)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} 1 & -Ez & Ey \\ Ez & 1 & -Ex \\ -Ey & Ex & 1 \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} + \begin{bmatrix} \delta x(y) \\ Y+\delta y(y) \\ \delta z(y) \end{bmatrix}$$

That is,

$X_1=\delta x(y)+X_2-Y_2Ez(y)+Z_2Ey(y)$

$Y_1=Y+\delta y(x)+X_2Ez(y)+Y_2-Z_2Ex(z)$

$Z_1=\delta z(y)-X_2Ey(y)+Y_2Ex(y)+Z_2$

Applying eq(2) to eq(1), the point $P(X_2,Y_2,Z_2)$ can be expressed in the reference $[X_RY_RZ_R]$ coordinate system.

Thus, $\mathbf{X}_R=\mathbf{T}x\{\mathbf{T}y\mathbf{X}_2+\mathbf{L}y\}+\mathbf{L}x$, that is, ignoring terms over 2nd order;

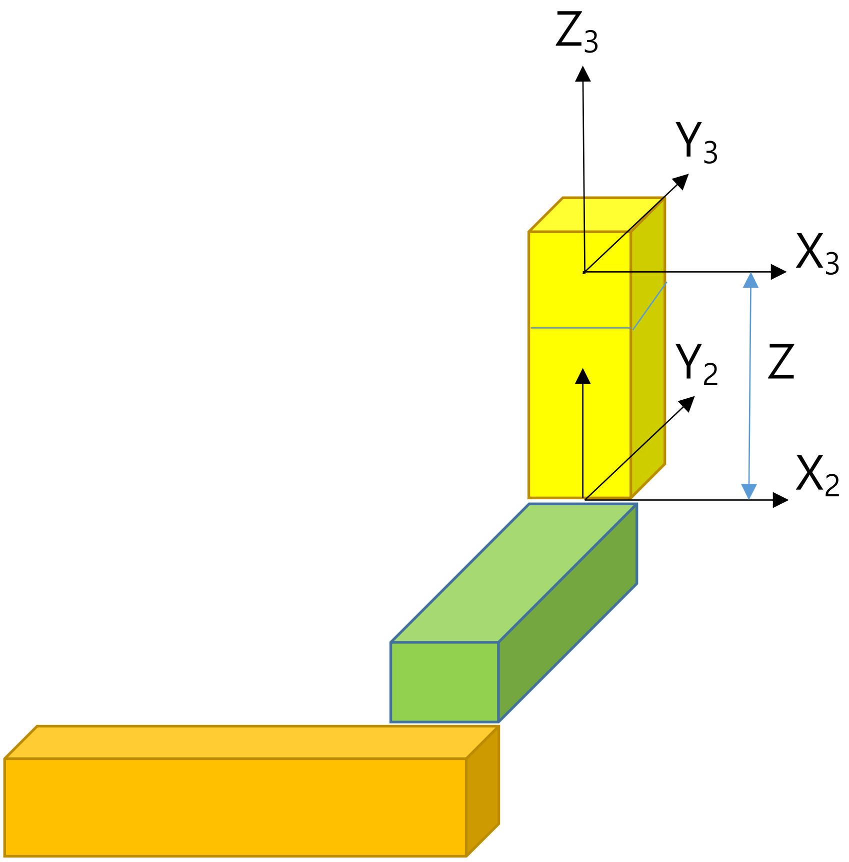
$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} 1 & -Ez & Ey \\ Ez & 1 & -Ex \\ -Ey & Ex & 1 \end{bmatrix} \begin{bmatrix} \delta x(y)+X_2-Y_2Ez(y)+Z_2Ey(y) \\ Y+\delta y(y)+X_2Ez(y)+Y_2-Z_2Ex(y) \\ \delta z(y)-X_2Ey(y)+Y_2Ex(y)+Z_2 \end{bmatrix} + \begin{bmatrix} X+\delta x(x) \\ \delta y(x)-\alpha X \\ \delta z(x) \end{bmatrix}$$

$X_R = X + \delta x(x) + \delta x(y) + X_2 - Y_2 Ez(y) + Z_2 Ey(y) - (Y + Y_2) Ez(x) + Z_2 Ey(x)$

$Y_R = Y + \delta y(y) + \delta y(x) - \alpha X + X_2 Ez(y) + Y_2 - Z_2 Ex(y) + X_2 Ez(x) - Z_2 Ex(x)$

$Z_R = \delta z(x) + \delta z(y) - X_2 Ey(y) + Y_2 Ex(y) + Z_2 - X_2 Ey(x) + (Y + Y_2) Ex(x)$

Now Z slide motion is introduced of the top of the Y slide



[$O_2X_2Y_2Z_2$]: Moving Coordinates fixed on the Y slide

[$O_3X_3Y_3Z_3$]: Moving coordinate fixed on the Z slide

Two coordinates are initially aligned as the same.

The angular motion of Z slide can be expressed as the rotational matrix, **T**z;

$$\mathbf{T}z=\begin{bmatrix} 1 & -Ez(z) & Ey(z) \\ Ez(z) & 1 & -Ex(z) \\ -Ey(z) & Ex(z) & 1 \end{bmatrix}$$

For translating motion of Z slide,

Translation in X direction=$\delta x(z)-\beta_1 Z$

; X straightness error of X axis+ XZ squareness error

Translation in Y direction=$\delta y(z)- \beta_2 Z$

; Y straightness error+ YZ squareness error

Translation in Z direction=$Z+\delta z(z)$

; nominal position + positional error

Thus the translating motion of Z slide can be expressed as the column vector, **L**z; that is

$$\mathbf{L}z = \begin{bmatrix} \delta x(z)-\beta_1 Z \\ \delta y(z)-\beta_2 Z \\ Z+\delta z(y) \end{bmatrix}$$

Thus a point $P(X_3,Y_3,Z_3)$ on the X slide can be expressed in the $[X_2,Y_2,Z_2]$ coordinate system;

$\mathbf{X}_2=\mathbf{T}z\mathbf{X}_3+\mathbf{L}z$; eq(3)

Kinematic Chain: Sequence of slide motion from reference coordinates to the workpiece coordinates; in this case Reference->X slide->Y slide->Z slide.

Applying eq(3),eq(2) to eq(1);

$\mathbf{X}_R=\mathbf{T}x\mathbf{X}_1+\mathbf{L}x=\mathbf{T}x\{\mathbf{T}y\mathbf{X}_2+\mathbf{L}y\}+\mathbf{L}x$

$=\mathbf{T}x[\mathbf{T}y\{\mathbf{T}z\mathbf{X}_3+\mathbf{L}z\}+\mathbf{L}y]+\mathbf{L}x$; eq(4)

This is the total relationship between the coordinates according to the kinematic chain.

When $\mathbf{X}p(Xp,Yp,Zp)$ is the coordinates of probe in the

$\mathbf{X}_3$ coordinate system; eq(4) becomes

$\mathbf{X}_R=\mathbf{T}x[\mathbf{T}y\{\mathbf{T}z\mathbf{X}p+\mathbf{L}z\}+\mathbf{L}y]+\mathbf{L}x$; eq(5)

$\mathbf{T}z\mathbf{X}p+\mathbf{L}z$ =

$$\begin{bmatrix} 1 & -Ez(z) & Ey(z) \\ Ez(z) & 1 & -Ex(z) \\ -Ey(z) & Ex(z) & 1 \end{bmatrix} \begin{bmatrix} Xp \\ Yp \\ Zp \end{bmatrix} + \begin{bmatrix} \delta x(z)-\beta_1 Z \\ \delta y(z)-\beta_2 Z \\ Z+\delta z(z) \end{bmatrix}$$

$$= \begin{bmatrix} \delta x(z)-\beta_1 Z+Xp-YpEz(z)+ZpEy(z) \\ \delta y(z)-\beta_2 Z+XpEz(z)+Yp-ZpEx(z) \\ Z+\delta z(z)-XpEy(z)+YpEx(z)+Zp \end{bmatrix}$$

After ignoring terms over 2'nd order**,**

$\mathbf{T}y\{\mathbf{T}z\mathbf{X}p+\mathbf{L}z\}+\mathbf{L}y=$

$$\begin{bmatrix} 1 & -Ez(y) & Ey(y) \\ Ez(y) & 1 & -Ex(y) \\ -Ey(y) & Ex(y) & 1 \end{bmatrix} \begin{bmatrix} \delta x(z)-\beta_1 Z+Xp-YpEz(z)+ZpEy(z) \\ \delta y(z)-\beta_2 Z+XpEz(z)+Yp-ZpEx(z) \\ Z+\delta z(z)-XpEy(z)+YpEx(z)+Zp \end{bmatrix} + \begin{bmatrix} \delta x(y) \\ Y+\delta y(y) \\ \delta z(y) \end{bmatrix} =$$

$$\begin{bmatrix} \delta x(y)+\delta x(z)-\beta_1 Z+Xp-YpEz(z)+ZpEy(z)-YpEz(y)+(Z+Zp)Ey(y) \\ Y+\delta y(y)+\delta y(z)-\beta_2 Z+XpEz(z)+Yp-ZpEx(z)+XpEz(y)-(Z+Zp)Ex(y) \\ Z+\delta z(y)+\delta z(z)-XpEy(z)+YpEx(z)+Zp-XpEy(y)+YpEx(y) \end{bmatrix} = A$$

$$\mathbf{T}x[\mathbf{T}y\{\mathbf{T}z\mathbf{X}p+\mathbf{L}z\}+\mathbf{L}y]+\mathbf{L}x=$$

$$\begin{bmatrix} 1 & -Ez(x) & Ey(x) \\ Ez(x) & 1 & -Ex(x) \\ -Ey(x) & Ex(x) & 1 \end{bmatrix} \begin{bmatrix} \\ A \\ \\ \end{bmatrix} + \begin{bmatrix} X+\delta x(x) \\ \delta y(x)-\alpha X \\ \delta z(x) \end{bmatrix} = \begin{bmatrix} Xa \\ Ya \\ Za \end{bmatrix}$$

where Xa, Ya, Za are the actual position of the probe in the reference or workpiece coordinate system. Thus

$$Xa=X+\delta x(x)+\delta x(y)+\delta x(z)-\beta_1 Z+Xp-YpEz(z)+ZpEy(z)-YpEz(y)+(Z+Zp)Ey(y)$$

$$-Ez(x)(Y+Yp)+Ey(x)(Z+Zp)$$

$$=X+Xp+\delta x(x)+\delta x(y)+\delta x(z)-\beta_1 Z+ZEy(y)-YEz(x)+ZEy(x)$$

$$-Yp(Ez(x)+Ez(y)+Ez(z))+Zp(Ey(x)+Ey(y)+Ey(z))$$

$$Ya=Y+\delta y(y)+\delta y(z)+\delta y(x)-\alpha X-\beta_2 Z+XpEz(z)+Yp-ZpEx(z)+XpEz(y)-$$
$$(Z+Zp)Ex(y)+Ez(x)Xp-Ex(x)(Z+Zp)$$

$$=Y+Yp+\delta y(y)+\delta y(x)+\delta y(z)-\alpha X-\beta_2 Z+ZEx(y)-ZEx(x)$$

$$+Xp(Ez(x)+Ez(y)+Ez(x))-Zp(Ex(x)+Ex(y)+Ex(z))$$

$$Za=Z+\delta z(y)+\delta z(x)+\delta z(z)-XpEy(z)+YpEx(z)+Zp-XpEy(y)+YpEx(y)$$

$$-Ey(x)Xp+Ex(x)(Y+Yp)$$

$$=Z+Zp+\delta z(z)+\delta z(x)+\delta z(y)+YEx(x)$$

$$-Xp(Ey(x)+Ey(y)+Ey(z))+Yp(Ex(x)+Ex(y)+Ex(z))$$

The volumetric error $\Delta X, \Delta Y, \Delta Z$ are the actual position minus the nominal position;

$$\Delta X=Xa-(X+Xp)= \delta x(x)+\delta x(y)+\delta x(z)-\beta_1 Z+ZEy(y)-YEz(x)+ZEy(x)$$

$$-Yp(Ez(x)+Ez(y)+Ez(z))+Zp(Ey(x)+Ey(y)+Ey(z))$$

$$\Delta Y=Ya-(Y+Yp)= \delta y(y)+\delta y(x)+\delta y(z)-\alpha X-\beta_2 Z-ZEx(y)-ZEx(x)$$

$$+Xp(Ez(x)+Ez(y)+Ez(x))-Zp(Ex(x)+Ex(y)+Ex(z))$$

$$\Delta Z=Za-(Z+Zp)= \delta z(z)+\delta z(x)+\delta z(y)+YEx(x)$$

$$-Xp(Ey(x)+Ey(y)+Ey(z))+Yp(Ex(x)+Ex(y)+Ex(z))$$

This is the 3D volumetric error equations or volumetric error map, considering all the geometric error components for the 3 axis machines for the current configuration of machine. Note that the 3D volumetric error equations are changed if the kinematic chains are changed.