

## Precision Metrology 20- Metrology on the Axis of Rotation

Axis of rotation: a line about which rotation occurs

Spindle: a device that provides an axis of rotation

Perfect spindle : a spindle having no motion of axis of rotation relative to the reference coordinate axes

Error motion: changes in position of (the surface of perfect workpiece having the centre line coincident with) the axis of rotation w.r.t a reference coordinate axis;

Possible causes are bearing error due to nonround bearing components; structural error motion due to excitation, mass, compliance, damping, etc.

Sensitive direction: the perpendicular direction to the ideally generated workpiece surface

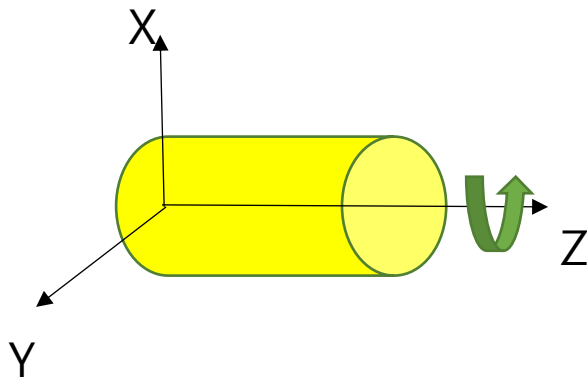
fixed sensitive direction: in which the workpiece is rotated by the spindle and the point of machining(gaging) is fixed. Ex) Lathe

rotating sensitive direction: in which the workpiece

is fixed, and the point of machining (gaging) rotates with the spindle. Ex) Jig borer

Nonsensitive direction: any direction perpendicular to the sensitive direction

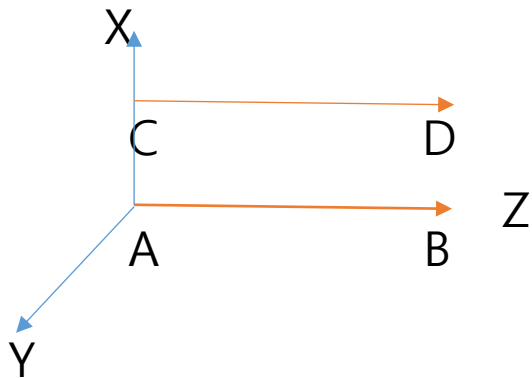
### 6 DOFs' Error motions



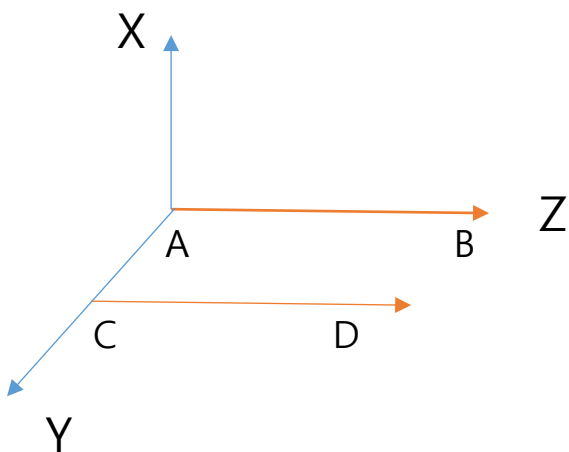
AB: Z reference axis;

CD: Axis of rotation

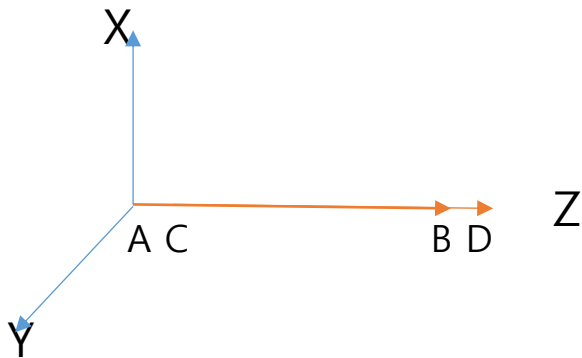
Pure radial motion in  $X$ ,  $X_0(\theta)$ ; error motion in the perpendicular direction to the  $Z$  reference axis



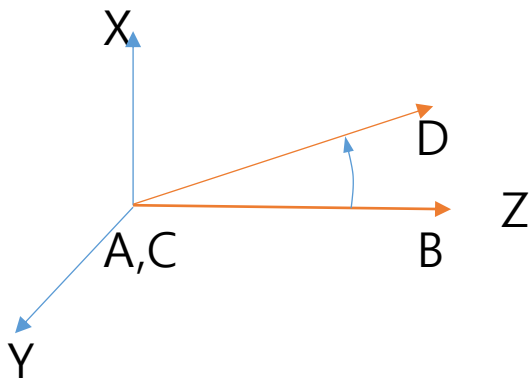
Pure radial motion in  $Y$ ,  $Y_0(\theta)$ ; error motion in the perpendicular direction to the  $Z$  reference axis



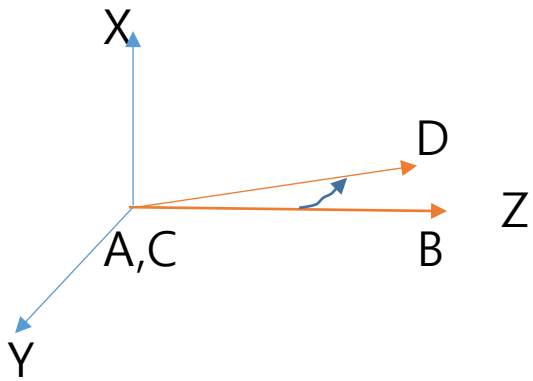
Axial motion in  $Z$ ,  $z(\theta)$ ; error motion collinear with the  $Z$  reference axis, "axial slip", "end-camming", "drunkenness"



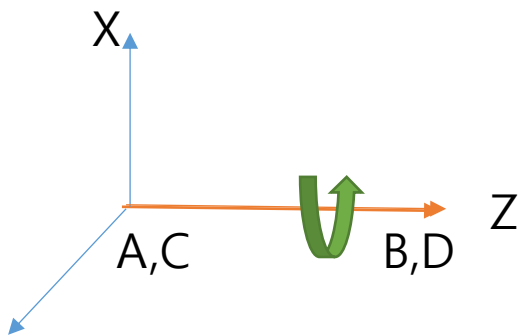
Tilt Motion in  $Y$ ,  $\beta(\theta)$ ; "coning", "wobble", "swash"



Tilt motion in X,  $\alpha(\theta)$ ; "coning", "wobble", "swash"



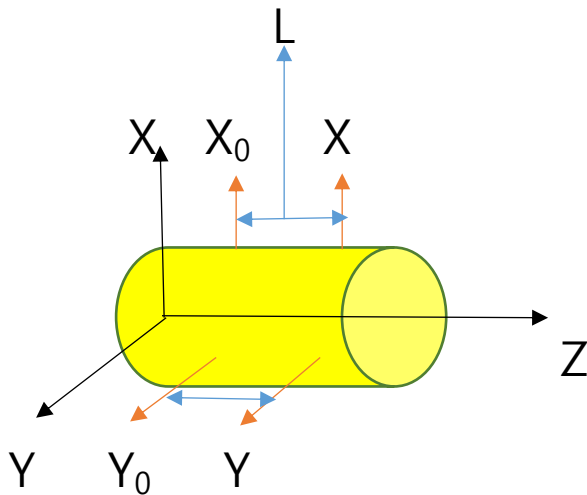
Indexing motion in Z ;  $\gamma(\theta)$



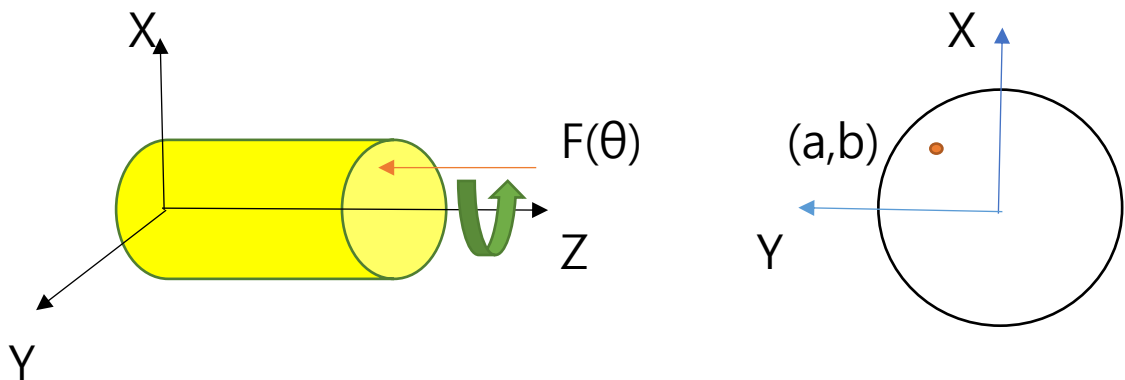
General radial motions;  $X(\theta)$ ,  $Y(\theta)$  at  $L$  axial location

$$X(\theta) = X_0(\theta) + L\beta(\theta)$$

$$Y(\theta) = Y_0(\theta) - L\alpha(\theta)$$

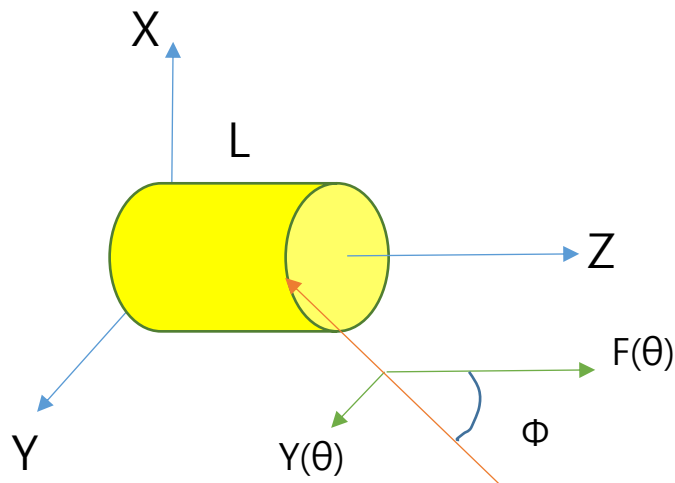


Face motion,  $F(\theta)$ : Error motion parallel to the  $Z$  axis at  $(a,b)$  radial location



$$F(\theta) = Z(\theta) + b\alpha(\theta) - a\beta(\theta)$$

General case of error motion,  $e(\theta)$  at any sensitive direction



$e(\theta)$ : tool-direction, or sensitive direction at  $(L, b, 0)$  offset

$$e(\theta) = F(\theta)\cos\Phi + Y(\theta)\sin\Phi,$$

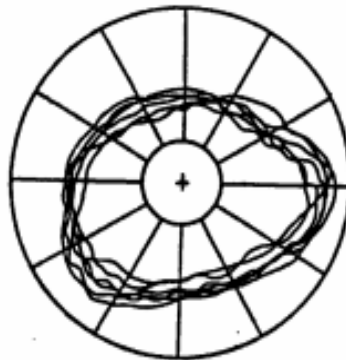
where  $F(\theta)$ : face motion,  $Y(\theta)$ : radial motion

$$F(\theta) = Z(\theta) + b\alpha(\theta), \quad Y(\theta) = Y_0(\theta) - L\alpha(\theta)$$

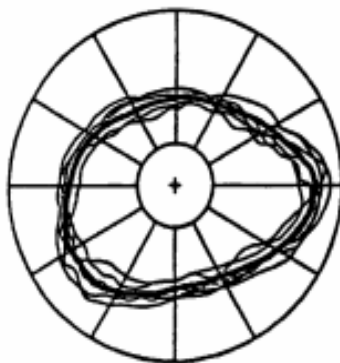
$$\therefore e(\theta) = [Z(\theta) + b\alpha(\theta)]\cos\Phi + [Y_0(\theta) - L\alpha(\theta)]\sin\Phi$$

$$= Z(\theta)\cos\Phi + Y_0(\theta)\sin\Phi + [b\cos\Phi - L\sin\Phi]\alpha(\theta)$$

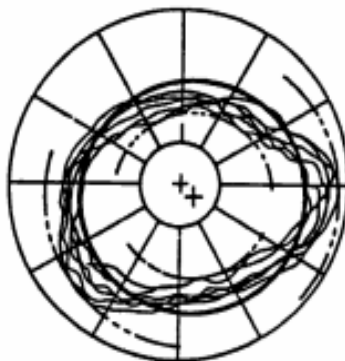
Once  $Z(\theta)$ ,  $Y_0(\theta)$ ,  $\alpha(\theta)$  measured, then  $e(\theta)$ , error motion for any sensitive direction can be predicted.



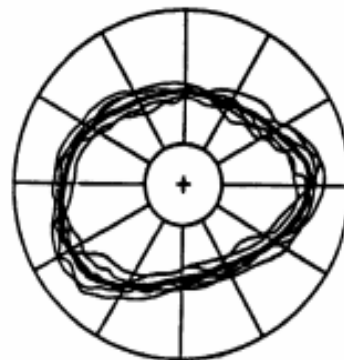
(a) Total Error Motion



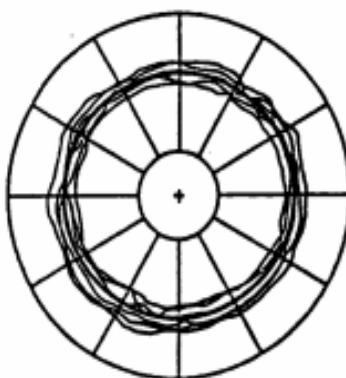
(b) Average Error Motion



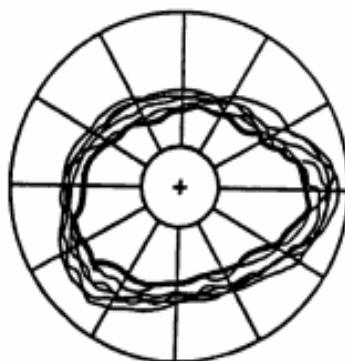
(c) Fundamental Error Motion



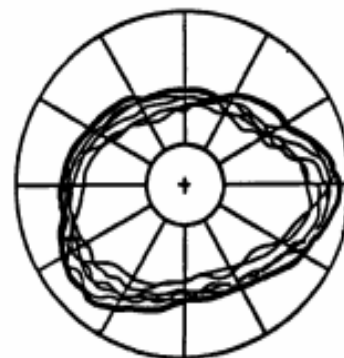
(d) Residual Error Motion



(e) Asynchronous Error Motion



(f) Inner Error Motion



(g) Outer Error Motion

Polar Plots of Error Motion, Source: ANSI/ASME



## <Polar Plot of Error Motion>

Total error motion = Average(or Synchronous) error motion + Asynchronous error motion

Average error motion = Fundamental error motion + Residual error motion, and average error motion is related to the part roundness profile

Asynchronous error motion is mainly due to the structural error motion, and related to the surface roughness. The structural error motion is due to the internal/external source of excitation by mass, compliance, and damping, etc.

Fundamental error motion: once per rev component; due to the eccentricity(radial motion), or due to the squareness error of thrust bearing (axial motion). The sinusoidal fundamental error motion can be eliminated by the best fit circle technique

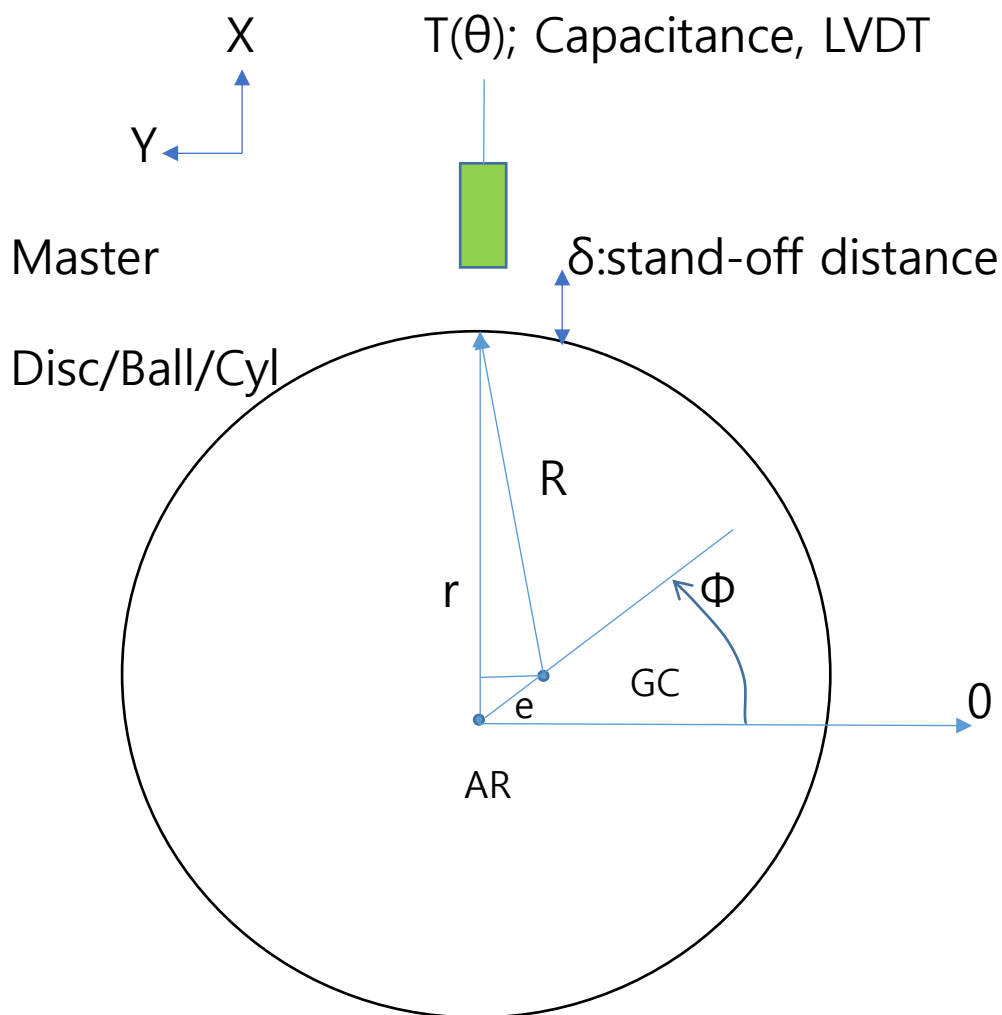
Residual error motion: Average error motion minus Fundamental error motion.

Inner error motion: Inner most data from the total error motion

Outer error motion: Outer most data from the total error motion

### Radial error motion measurement for $X(\theta), Y(\theta)$

(1) Roundness error of master cylinder is ignored



$$r(\theta) = e \cos(\theta - \Phi) + \sqrt{[R^2 - e^2 \sin^2(\theta - \Phi)]}$$

$$\cong e \cos \Phi \cos \theta + e \sin \Phi \sin \theta + R$$

$$= a \cos \theta + b \sin \theta + R, \text{ where } a = e \cos \Phi, b = e \sin \Phi$$

$T(\theta)$ : measured at sensor, + sign for closer

$$T(\theta) = X \text{ radial error motion} + r(\theta) + \delta$$

$$= X(\theta) + a \cos \theta + b \sin \theta + R + \delta$$

$$\cong T_0 + \sum [A_k \cos k\theta + B_k \sin k\theta], \text{ for } k=1 \text{ to } \infty$$

$$\text{Where } T_0 = \int T(\theta) d\theta / 2\pi \cong \sum T_i / N; \text{ for } i=1 \text{ to } N$$

$$A_k = \int T(\theta) \cos k\theta d\theta / \pi \cong 2 \sum T_i \cos k\theta_i / N; \text{ for } i=1 \text{ to } N$$

$$B_k = \int T(\theta) \sin k\theta d\theta / \pi \cong 2 \sum T_i \sin k\theta_i / N; \text{ for } i=1 \text{ to } N$$

Therefore,  $T_0 = R + \delta$ ,  $A_1 = a$ ,  $B_1 = b$ , and

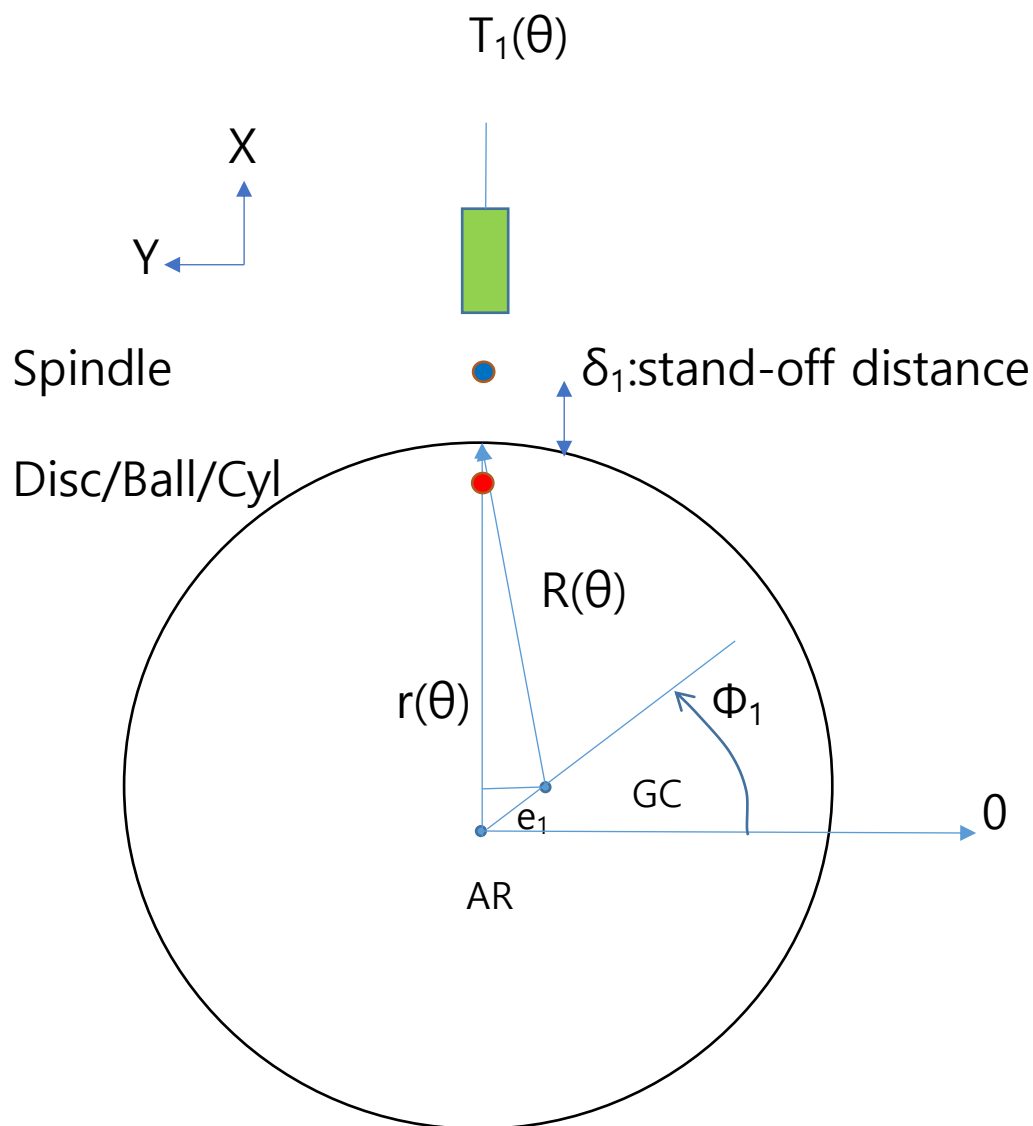
$$X(\theta) = T(\theta) - [a \cos \theta + b \sin \theta + R + \delta]$$

$$= T(\theta) - [T_0 + A_1 \cos \theta + B_1 \sin \theta]: X \text{ radial error motion}$$

The Y radial error motion can be similarly measured when the sensor is moved to  $\theta=90^\circ$  location.

(2) Roundness error of master cylinder is not ignored.

### **T<sub>1</sub> measurement (Forward)**



$$T_1(\theta) = X \text{ radial error motion} + r(\theta) + \delta_1$$

$$= X(\theta) + a \cos \theta + b \sin \theta + R(\theta) + \delta_1$$

$$\equiv T_0 + \sum [A_k \cos k\theta + B_k \sin k\theta], \text{ for } k=1 \text{ to } \infty$$

$$\text{Where } T_0 = \int T_1(\theta) d\theta / 2\pi \equiv \sum T_i / N; \text{ for } i=1 \text{ to } N$$

$$A_k = \int T_1(\theta) \cos k\theta d\theta / \pi \equiv 2 \sum T_i \cos k\theta_i / N; \text{ for } i=1 \text{ to } N$$

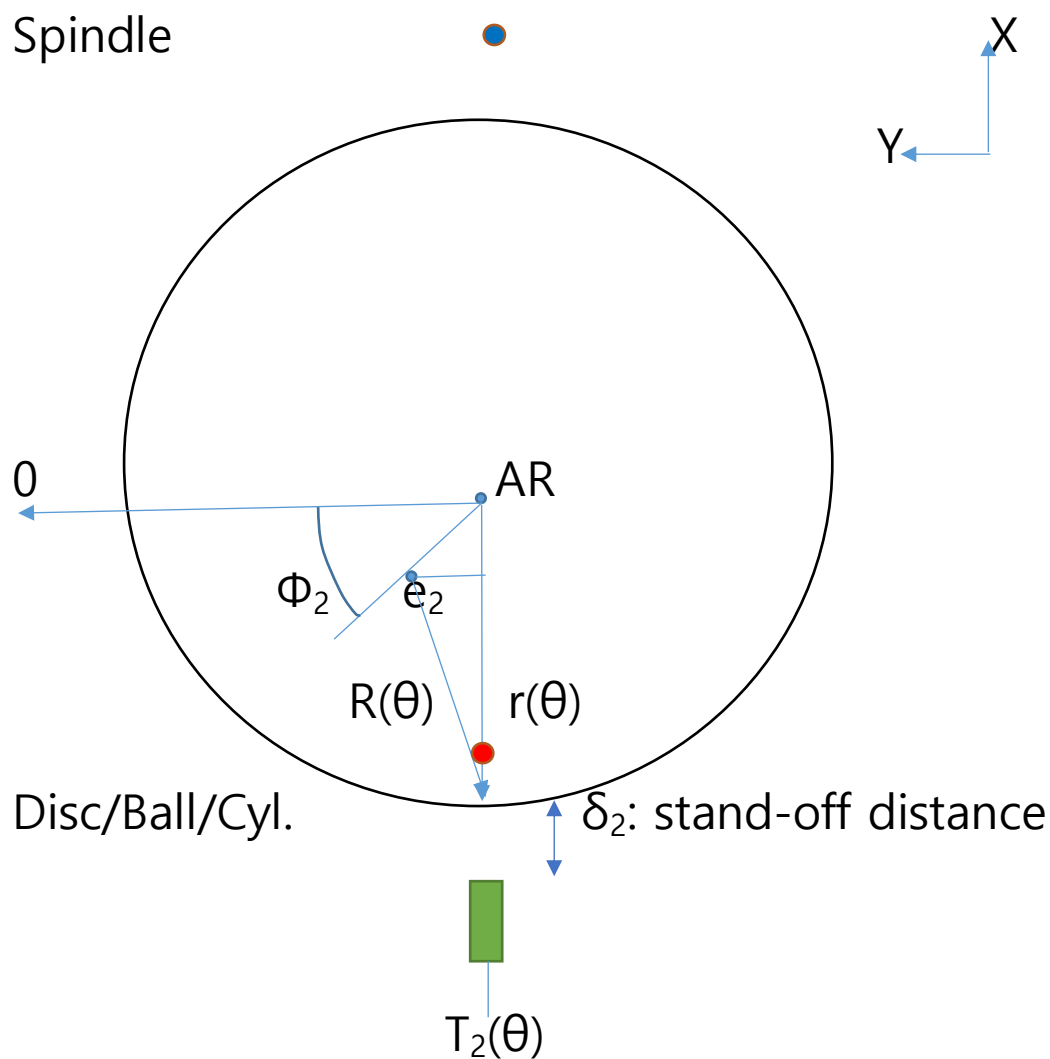
$$B_k = \int T_1(\theta) \sin k\theta d\theta / \pi \equiv 2 \sum T_i \sin k\theta_i / N; \text{ for } i=1 \text{ to } N$$

$$\text{Therefore } a = A_1, b = B_1, \delta_1 = T_0$$

$$\underline{T}_1(\theta) = T_1(\theta) - [a \cos \theta + b \sin \theta + \delta_1] = X(\theta) + R(\theta); \text{ eq(1)}$$

## T<sub>2</sub> measurement (Reverse)

:Master-disc and sensor are rotated at 180 deg, and measured, while the spindle rotor is stationary.



$$T_2(\theta) = -X \text{ radial motion} + r(\theta) + \delta_2$$

$$= -X(\theta) + R(\theta) + a' \cos \theta + b' \sin \theta + \delta_2$$

$$\equiv T'_0 + \sum [A'_k \cos k\theta + B'_k \sin k\theta], \text{ for } k=1 \text{ to } \infty$$

$$\text{Where } T'_0 = \int T_2(\theta) d\theta / 2\pi \equiv \sum T_{2i} / N; \text{ for } i=1 \text{ to } N$$

$$A'_k = \int T_2(\theta) \cos k\theta d\theta / \pi \equiv 2 \sum T_{2i} \cos k\theta_i / N; \text{ for } i=1 \text{ to } N$$

$$B'_k = \int T_2(\theta) \sin k\theta d\theta / \pi \equiv 2 \sum T_{2i} \sin k\theta_i / N; \text{ for } i=1 \text{ to } N$$

$$\text{Therefore } a' = A'_1, b' = B'_1, \delta_2 = T'_0$$

$$\underline{T}_2(\theta) = T_2(\theta) - [a' \cos \theta + b' \sin \theta + \delta_2] = -X(\theta) + R(\theta); \text{ eq(2)}$$

From eq(1),(2)

$$X(\theta) = \{\underline{T}_1(\theta) - \underline{T}_2(\theta)\} / 2$$

$$\text{and } R(\theta) = \{\underline{T}_1(\theta) + \underline{T}_2(\theta)\} / 2$$

Similarly, the Y radial error motion can be measured.

∴ This is the Reversal technique for the Radial error motion measurement

[Q: Two set-ups for each component?

X(θ), Y(θ), R(θ): 3 unknowns to measure?]