

## Precision Metrology 4

### Error classification for Machines

CLASSIFICATION	ERROR & SOURCES
GEOMETRIC & KINEMATIC ERROR	POSITIONAL & ANGULAR ERROR STRAIGHTNESS ERROR ORTHOGONALITY ERROR FUNCTIONAL MOVEMENT ERROR DUE TO MECHANICAL WEAR MISALIGNMENT & PLAY MANUFACTURING DEFECT SCALE ERRORS
LOAD INDUCED ERROR	WEIGHT OF COMPONENTS (STRUCTURAL DEFORMATION) WEIGHT OF WORKPIECE MACHINING(PROBING) FORCES
THERMAL INDUCED ERROR	DRIVE MOTOR & CHANGE IN AIR INLET TEMPERATURE THERMAL EXPANSION & STRUCTURAL BENDING
ENVIRONMENTAL ERROR	CHANGE IN AIR PRESSURE EXTERNAL VIBRATION/NOISE
SOFT WARE ERROR	ALGORITHM ERROR

TABLE 2.1 ERROR CLASSIFICATION

\*Source: Ph.D Thesis of Dr. H.J.Pahk, University of Manchester

## I. Geometric Errors

:Errors due to Geometric Inaccuracy of Machine Elements

6 DOFs(Degree of Freedoms) errors introduced for the linear motion of a machine axis on the guideway, or slider.

(DOFs=Degree of Freedoms=Number of Independent)

How many DOFs for "a free body in 3D space"?

3 translational errors;  $\delta x$ ,  $\delta y$ ,  $\delta z$

$x,y,z$  indicate the direction of translational errors

3 rotational errors;  $E_x$ ,  $E_y$ ,  $E_z$

$x,y,z$  indicate the direction of rotational errors, or axis of rotation;

For the Y axis of slide; y is added to the notation in the bracket such as (y)

$\delta x(y), \delta y(y), \delta z(y); E_x(y), E_y(y), E_z(y)$

Positional error,  $\delta y(y)$

Horizontal straightness error ,  $\delta x(y)$

Vertical straightness error,  $\delta z(y)$

Pitch angular error,  $E_x(y)$

Yaw angular error,  $E_z(y)$

Roll angular error,  $E_y(y)$

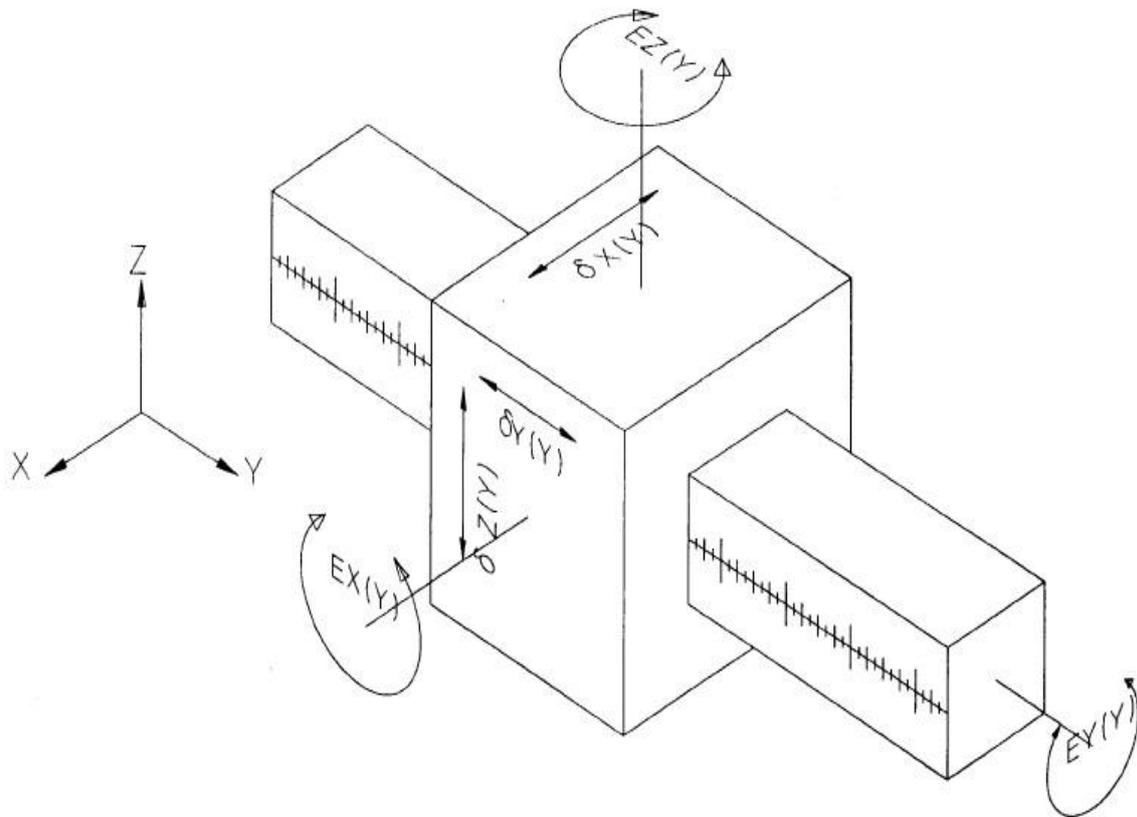


FIG.3.1 6 DEGREE OF FREEDOMS OF ONE GUIDE WAY

\*Source: Ph.D Thesis of Dr. H.J.Pahk, University of Manchester

### (1) Positional error, $\delta y(y)$

: Errors observed along the direction of travel, due to thermal expansion, lead screw error, scale error, wavelength error, interpolation error, thermal expansion, etc, and typically few 10 $\mu$ m.

(a) Thermal expansion in optical scale, or in leadscrew:

$\Delta L = \alpha L T$ , and  $\alpha$  = thermal expansion coefficient, 12 ppm/K, for steel, 24 ppm/K for aluminum,

1 ppm/K = 1 part per million/K = 1  $\mu\text{m}/\text{m}/\text{K}$

T = temperature increase in Kelvin (K)

(b) Lead screw error:

$L = n p$ , and  $n$  is no. of turn,  $p$  is the screw pitch.

$\Delta L = n \Delta p$ , and  $\Delta p$  is the screw pitch error.

Scale grade:  $C_0 < C_1 < C_2 < C_3$

(c) Wavelength error;

$L = m \lambda$ , and  $m$  is the no. of wavelength.

$\Delta L = m \Delta \lambda$ , and  $\Delta \lambda$  is the wavelength error, typically

1 ppm/K for He-Ne laser source

(d) Interpolation and control error

4096 divisions for one 20  $\mu\text{m}$  scale interval by the software or electronics-gates

Sinusoidal, truncational, gates signaling errors can be engaged during the interpolation process

Error due to the control system such as transient /steady state error, following/overshooting error, and compensation error, etc.

(2) Straightness error;  $\delta x(y)$ ,  $\delta z(y)$

:Errors observed in the transverse direction that perpendicular to the direction of travel, and is mainly due to the non-straight of the guideway from manufacturing defects, thermal deformation, or wear. Typically, few  $\mu\text{m}$  for 1 m travel. Horizontal straightness error observed in the xy plane, and vertical straightness error in the yz plane.

### (3) Angular error; $E_x(y)$ , $E_y(y)$ , $E_z(y)$

Errors of axis of rotation observed along the guideway.

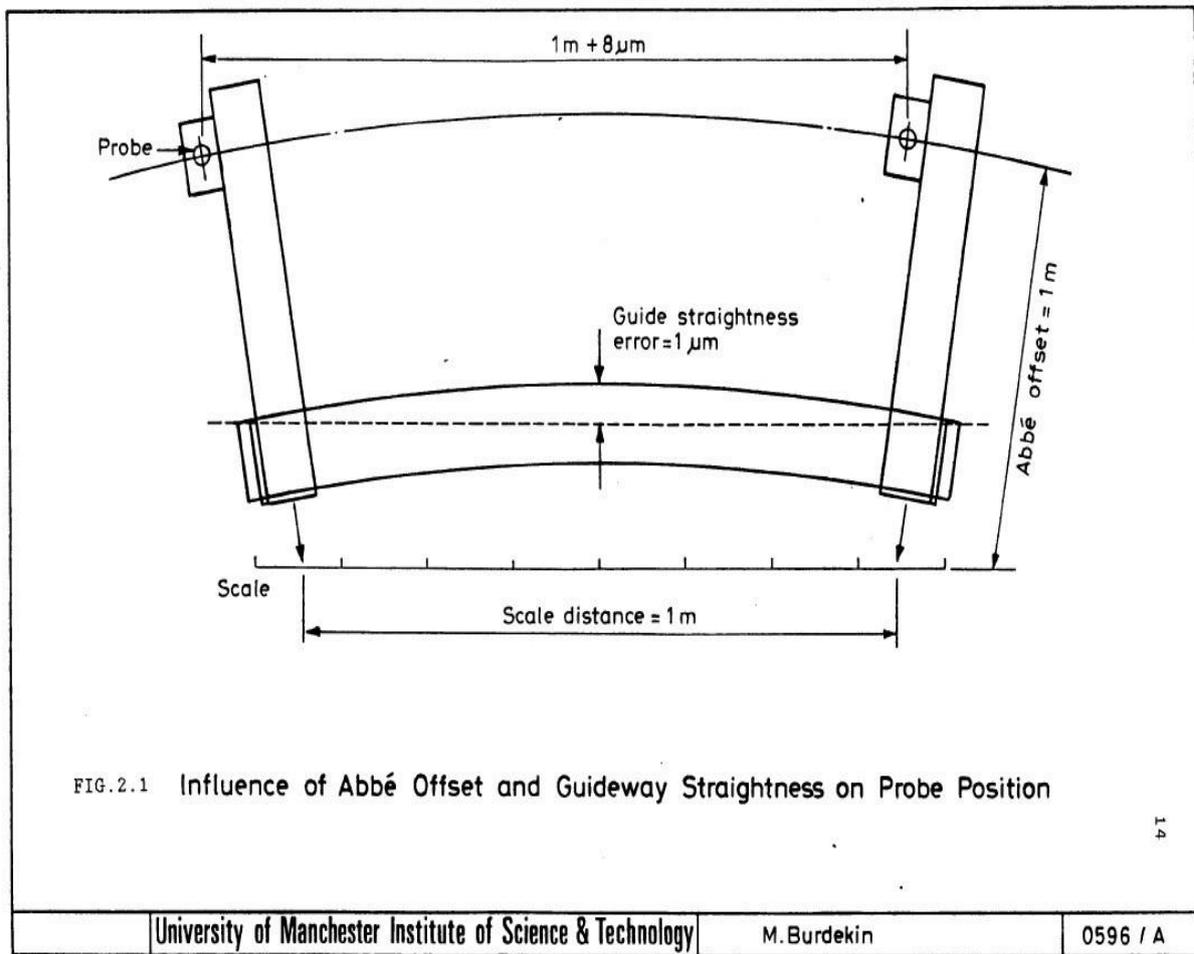
$E_x(y)$ : Pitch angular error observed in the horizontal transverse direction(X) along the guideway. It can be due to vertical straightness error, or thermal deformation(thermal bending) of guideway.

$E_z(y)$ : Yaw angular error observed in the vertical transverse direction(Z) along the guideway. It can be due to the horizontal straightness error, or thermal deformation (thermal bending) of guideway.

$E_y(y)$ : Roll angular error observed in the direction of travel(Y) along the guideway. It can be due to the flatness, warping of the guideway.

Once the angular errors are observed, the Abbe errors are always induced. The Abbe error is defined as the product of Abbe offset and angular error, where the

Abbe offset is defined as the distance between the line of measurement(scale) and the line of dimension being measured.



\*Source: Lecture Note of Dr. M. Burdekin, University of Manchester

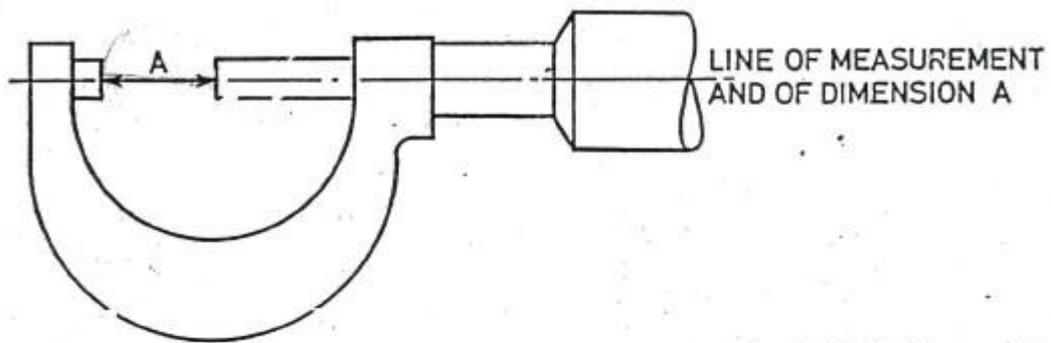


Fig. 3.2. A hand micrometer conforming to the principle of alignment.

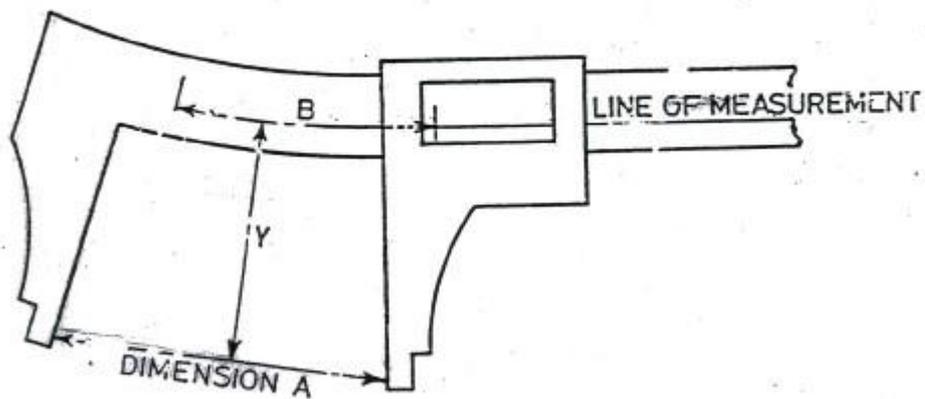


Fig. 3.3. A vernier caliper does not conform to the principle of alignment.

\*Source: 'Metrology for Engineers', 5<sup>th</sup> Ed. published by Cassel, and authored by J.F.W. Galyer and C.R. Shotbolt

## The Abbe Principle or 'Principle of Alignment'

:The line of measurement(scale), and the line of the dimension being measured should be coincident. The Abbe offset is always to be removed or minimized for precision alignment, or sometimes called as 'principle of alignment'. This principle is very fundamental to precision metrology and design.

∴ Abbe Error induced = Angular Error X Abbe Offset

For the general 3D case;

The Abbe Error in 3D =  $[\Delta X, \Delta Y, \Delta Z] = \mathbf{E} \times \mathbf{A} =$

$$\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ E_x & E_y & E_z \\ A_x & A_y & A_z \end{bmatrix} = \mathbf{i}(E_y \cdot A_z - E_z \cdot A_y) - \mathbf{j}(E_x \cdot A_z - E_z \cdot A_x) + \mathbf{k}(E_x \cdot A_y - E_y \cdot A_x)$$

where  $\mathbf{E}$  ( $E_x, E_y, E_z$ ) is the angular,

and  $\mathbf{A}$  ( $A_x, A_y, A_z$ ) is the angular error

Ex) For the fig.2.1

The vertical straightness profile of 1um at the center of guideway can be mathematically modeled as the parabolic function,

$$s(x)=4(x/L)(1-x/L) =4 \{(x/L)-(x/L)^2\} [\text{um}]$$

where  $x$  and  $L(=1)$  in m,  $x$  is in  $[0,L]$

The angle observed at the probe can be obtained from the  $ds/dx$ , (*why? assumption?*)

$$ds/dx=4(1/L-2x/L^2) [\text{um/m, urad}]$$

Thus the pitch angular error,  $E_y$ , observed when the probe is travelling from  $x=0$  to  $x=L$  position

$$E_y=ds/dx|_{x=0} - ds/dx|_{x=L} = 4/L-(-4/L)=8/L=8 \text{ (urad)}$$

$\therefore$  Abbe error induced with 1m Abbe offset

$$= E \cdot A = 8 \cdot 1 = 8 [\text{um}]$$

## Observation:

-True value =  $1\text{m} + 8\mu\text{m}$ , while Nominal value =  $1\text{m}$

$\therefore$  Positional Error = True – Nominal =  $8\mu\text{m}$

- $1\mu\text{m}$  straightness error is very precise case for  $1\text{m}$  guideway

-Error magnification of angular error via Abbe offset

-How to overcome?

*To avoid/reduce the Abbe offset by design, or*

*To compensate the error by numerical error correction*

HW4) For the fig. 2.1,

- (1) Make another mathematical modeling for the vertical straightness profile along the guideway.
- (2) Derive the angular error observed during the travel along the guideway
- (3) Calculate the Abbe error experienced at the probe