

## Precision Metrology 5

### II. Kinematic Errors

Relative deviation in movement of several moving machine elements, or error in functional movement

#### 1. Translational-Translational movement

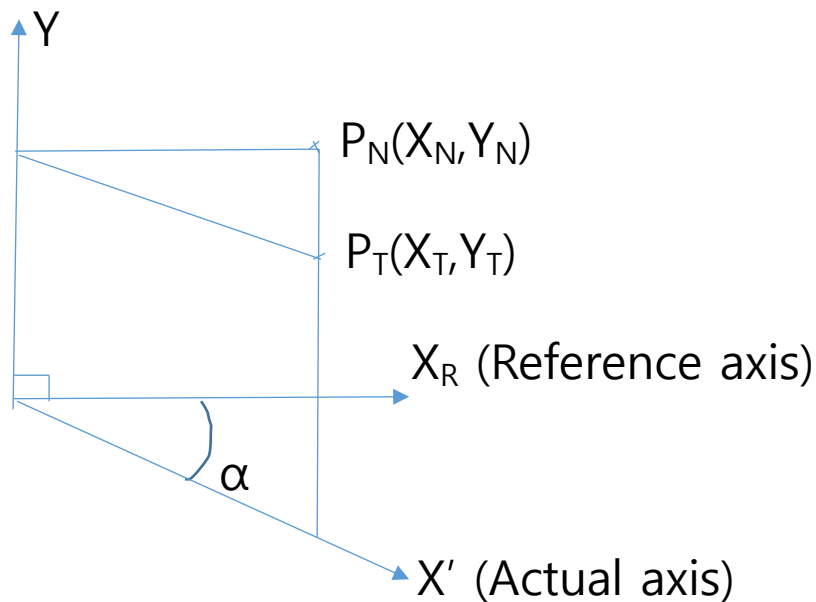
##### 1) Squareness error, out of squareness [urad]

:Due to the non-squareness of the nominal perpendicular axis, or orthogonality.

Q) Is it really 90deg between X and Y axis?

No exact 90 deg, squareness, perpendicularity, or orthogonality exist between the motions of axis during the multi-axis movement. This is mainly due to the mis-assembly or mis-alignment between the machine axis. Typically, few 10 urad exists between precision machine axis movements.

-Squareness error,  $\alpha$ , between X and Y axis



When machine moves nominally to  $P_N(X_N, Y_N)$ , true position would be  $P_T(X_T, Y_T)$  in reference axis.

$$X_T = X_N \cdot \cos \alpha \approx X_N$$

$$Y_T = Y_N - X_N \cdot \sin \alpha \approx Y_N - X_N \cdot \alpha$$

where  $\alpha$  = squareness error = out of squareness [urad]

and  $\alpha$  is typically  $10 \text{ urad} = 10^{-5} \text{ rad}$

$$\cos \alpha = 1 - \frac{\alpha^2}{2!} + \dots \approx 1 \quad (\because \alpha^2 \approx (10^{-5})^2 = 10^{-10})$$

$$\sin \alpha = \alpha - \frac{\alpha^3}{3!} + \dots \approx \alpha$$

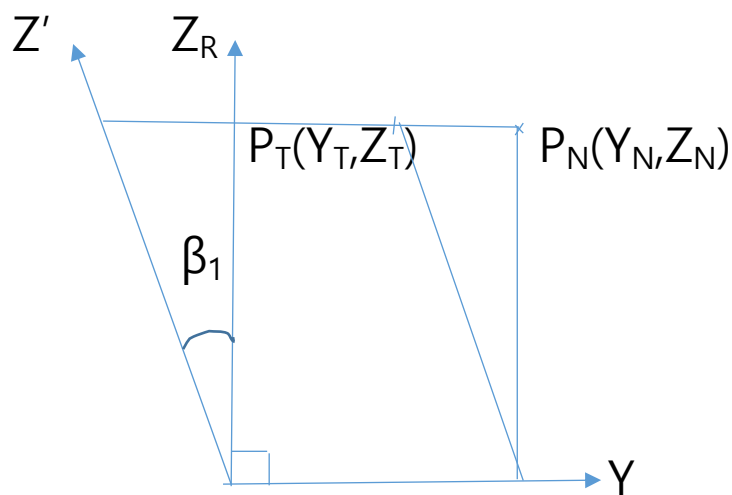
Thus, the squareness error contributes to the errors;

Error in  $X$ ,  $\Delta X = X_T - X_N = 0$

Error in  $Y$ ,  $\Delta Y = Y_T - Y_N = -\alpha \cdot X_N$

Similarly,

Squareness error,  $\beta_1$ , between  $Y$  and  $Z$  axis



$$Z_T = Z_N \cdot \cos \beta_1 \approx Z_N \quad (\because \beta_1 \approx 0)$$

$$Y_T = Y_N - Z_N \cdot \sin \beta_1 \approx Y_N - Z_N \cdot \beta_1 \quad (\because \beta_1 \approx 0)$$

where  $\beta_1$  is the out of squareness, and typically few 10 urad

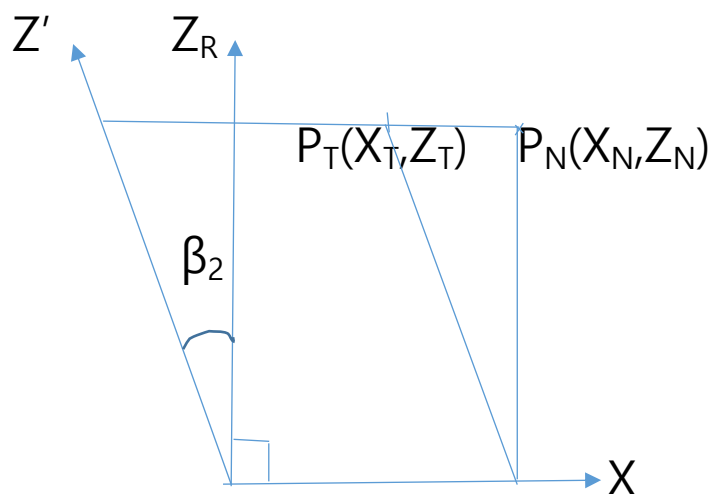
Thus, the squareness error contributes to the errors;

$$\text{Error in } Y, \Delta Y = Y_T - Y_N = -\beta_1 \cdot Z_N$$

Error in Z,  $\Delta Z = Z_T - Z_N = 0$

Similarly,

Squareness error,  $\beta_2$ , between X and Z axis



$$Z_T = Z_N \cdot \cos \beta_2 \approx Z_N \quad (\because \beta_2 \approx 0)$$

$$X_T = X_N - Z_N \cdot \sin \beta_2 \approx X_N - Z_N \cdot \beta_2 \quad (\because \beta_2 \approx 0)$$

where  $\beta_2$  is the out of squareness, and typically few 10 urad

Thus, the squareness error contributes to the errors;

$$\text{Error in X, } \Delta X = X_T - X_N = -\beta_1 \cdot Z_N$$

$$\text{Error in Z, } \Delta Z = Z_T - Z_N = 0$$

Three squareness errors,  $\alpha$ ,  $\beta_1$ ,  $\beta_2$  are defined in the 3D space, and they are all independent.

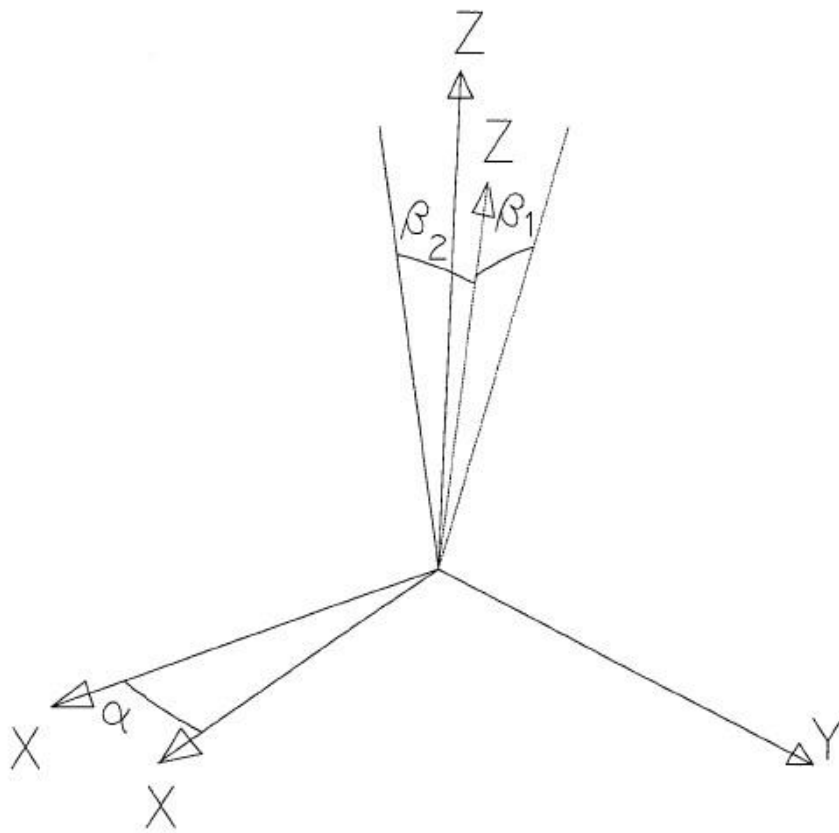


FIG . 3 . 2 ORTHOGONALITY ERRORS

(Source: PhD Thesis, Dr.H.J.Pahk, Univ. of Manchester)

For a 3 axis machine, having 3 slide axis,

Geometric errors for each axis

X axis (or X slide):

$\delta x(x), \delta y(x), \delta z(x); Ex(x), Ey(x), Ez(x)$  : 6 components

Y axis (or Y slide):

$\delta x(y), \delta y(y), \delta z(y); Ex(y), Ey(y), Ez(y)$  : 6 components

Z axis (or Z slide):

$\delta x(z), \delta y(z), \delta z(z); Ex(z), Ey(z), Ez(z)$  : 6 components

Kinematic Errors between each axis

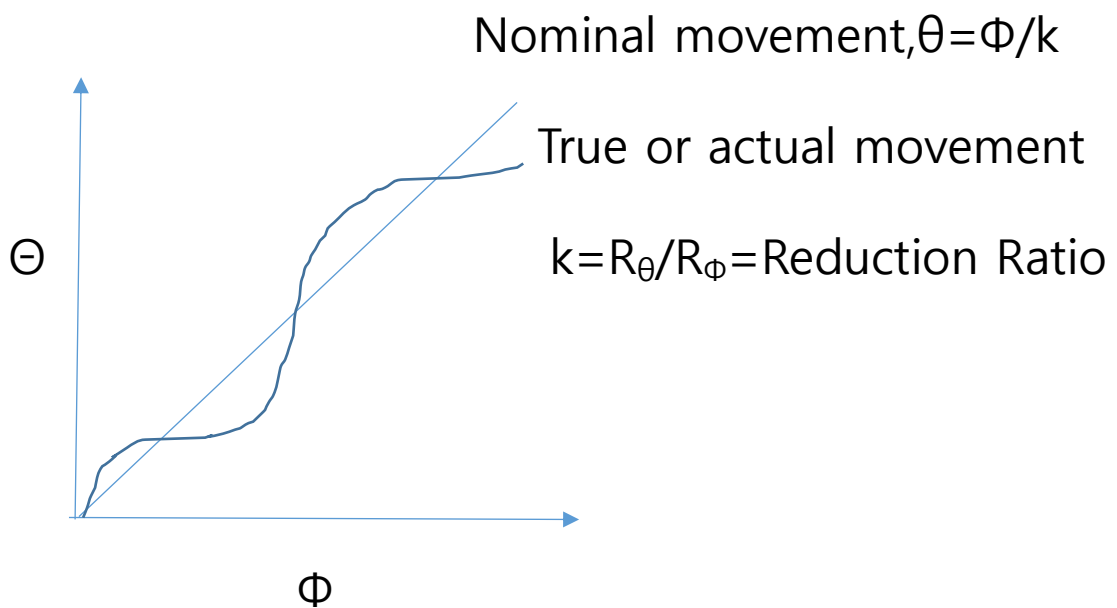
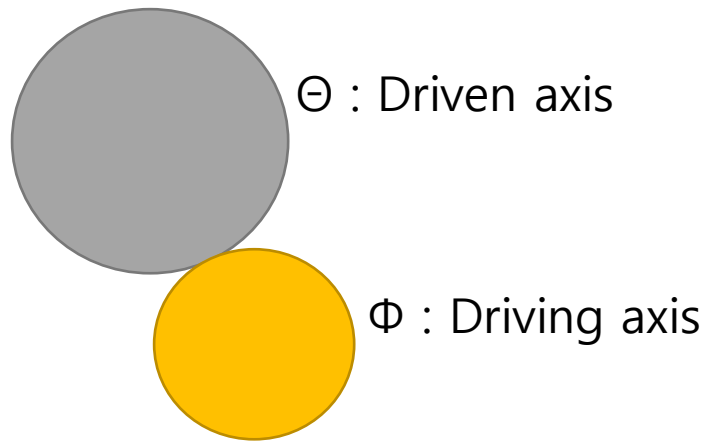
$\alpha$  (or  $\alpha_{xy}$ ) in XY plane

$\beta_1$  (or  $\alpha_{yz}$ ) in YZ plane

$\beta_2$  (or  $\alpha_{xz}$ ) in XZ plane

In total, 21 geometric/kinematic error components are independently existing, and they contribute to the accuracy and repeatability of the 3 axis machine in the 3D space.

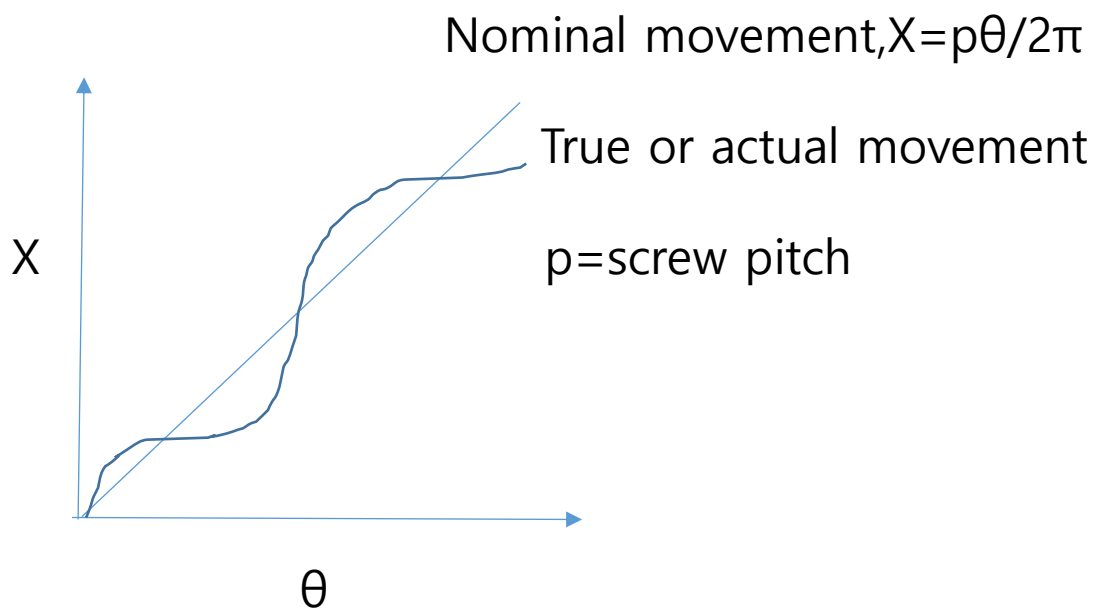
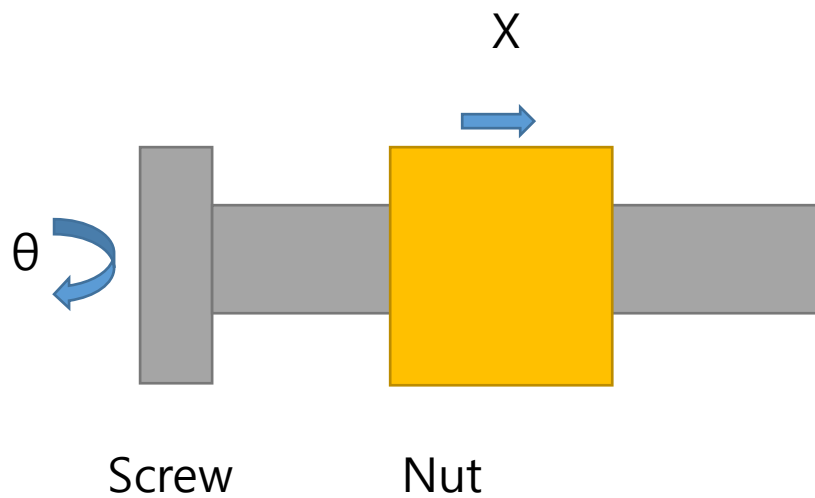
## 2. Rotational-Rotational movement



Error = True - Nominal; Error components due to eccentricity in axis, tooth profile error in gear, sliding/friction in belt/chain, torque, friction in friction drive, etc. Highly periodical component

Ex) Gears, Couplings, Belt, Chain, Friction Drive,

### 3. Rotational-Translational movement



Error = True - Nominal; Error components due to the lead screw pitch error, thermal expansion, backlash, carrying load, friction, encoder resolution, control scheme (closed or open loop, gains)

Ex) Most wide application in machine drive



### III. Load Induced Errors

:Machine deformation due to Load

->Relative motion between the tool/probe and work piece, degrading machine's accuracy

#### Possible Loads

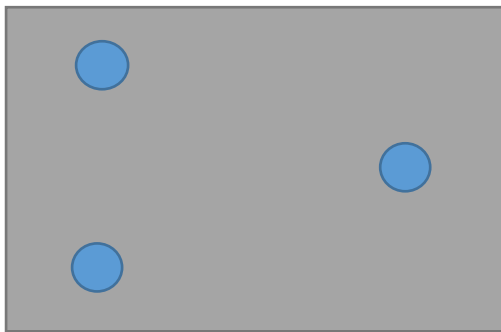
1) Loads due to the machine mounting

- Non-kinematic or redundant mount/design with inappropriate tolerance can give excessive load, resulting load induced error
- kinematic mount/design principle: to constraint a body by a minimum number of physical constraints
- Redundant constraint may cause excessive load, giving large machine deformation

Ex1) 3 supports for precision machine mounting

- 3 supports for precision mounting of small/medium sized machines, in order to avoid excessive load, even under floor movement.

-For the case over 3 supports, the size of redundant supports should be within the tolerance of plane determined by the 3 supports.



3 mounts

Kinematic mount

$C=3$ ; minimum const.

Stable

Small/Medium m/c

Accom. floor warp.



4 or more mounts

Non-kinematic mount

$C \geq 4$ ; redundant const.

Stable with tolerance

Large m/c

No accom. floor warp.

(Also, 3 legs tripod can accommodate any geometry of the resting surface.)

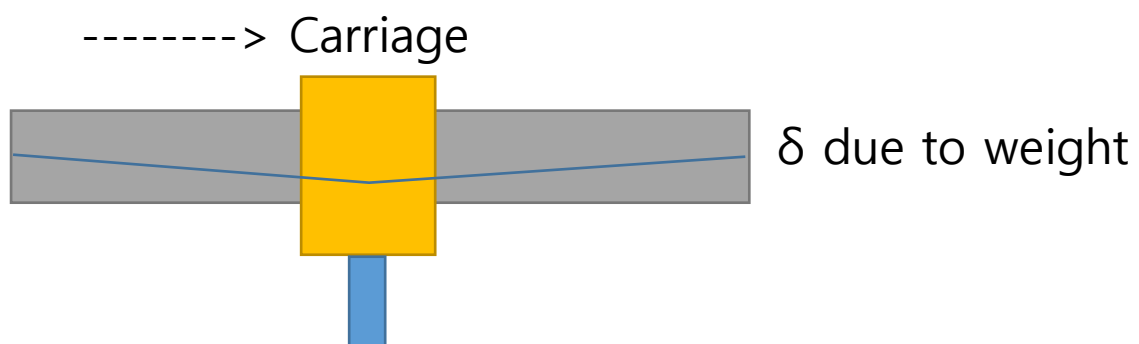
To overcome this, Kinematic Mount/Design is required

2) Static loads due to weight of components, or workpiece

-Weight of components or workpiece->stiffness of load-bearing elements (e.g. bearings, pads)->machine deformation.

-Cross coupling effect of geometric/kinematic errors

Cross coupling effect: motion of one machine element influencing the motion of others

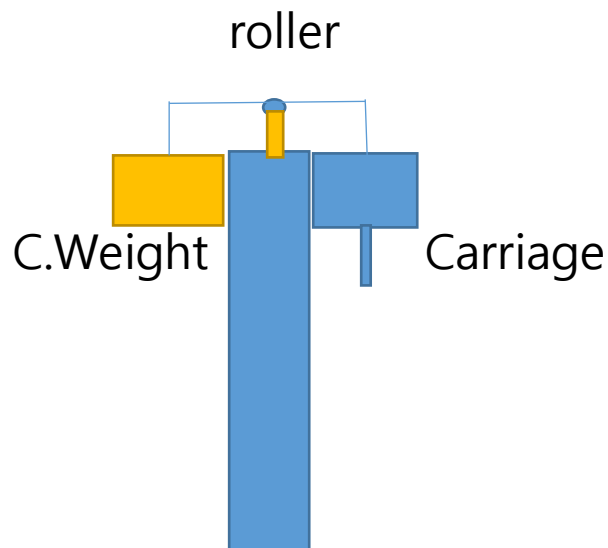


Methods for overcoming the deformation due to the dead weight of components:

① Convex shape of guides and guideway

Ex) Head design for portal m/c, Jib design for drilling m/c, Saddle design for milling m/c

② Load removal by counter weight, Hydraulic force



③ Separate frame or 'Metrology frame'

:to separate the frame from the load bearing machine structure

④ Numerical error correction

:to compensate numerically after prediction

### 3) Dynamic or Vibrational loads

#### Forced vibration vs. Self-excited vibration

Vibration with excited Freq.      One/Multi Natural Freq.

-Disturb. Via. Foundation

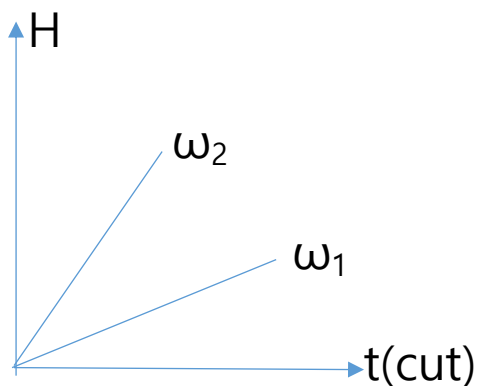
-Without disturbance

-Unbalance by eccent.

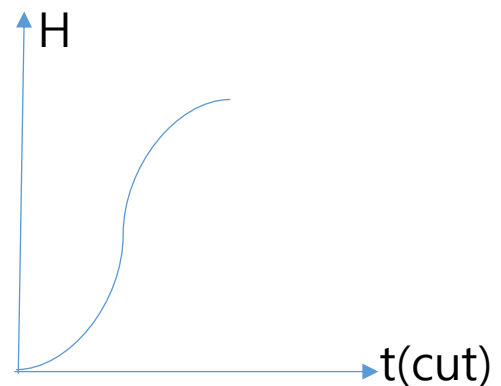
-Process and mechanism

tooth impact, bearing flaw

-Chatter(e.g.)



Slope=cons. for each  $\omega$



Slope=changing,

or coupled

Where  $H$ =Mag. of Vibration,  $t$ =Depth of cut

$\therefore$ High Stiffness Structure of  $m/c$  is to overcome this