

Precision Metrology 10: Sources of Errors for positional error measurement

Sources of Errors in positional error measurement

1. Wavelength compensation

:Mainly due to no compensation or incorrect compensation

- (1) Error in Temperature sensors, Pressure sensors, Humidity sensors due to malfunctioning of sensors. It needs verification or calibration prior to measurement.
- (2) Inappropriate locations of sensors; thus it is necessary to locate the sensors as close as possible to the beam path.
- (3) Manual compensation sometimes help, as the wavelength change can be precisely calculated manually.

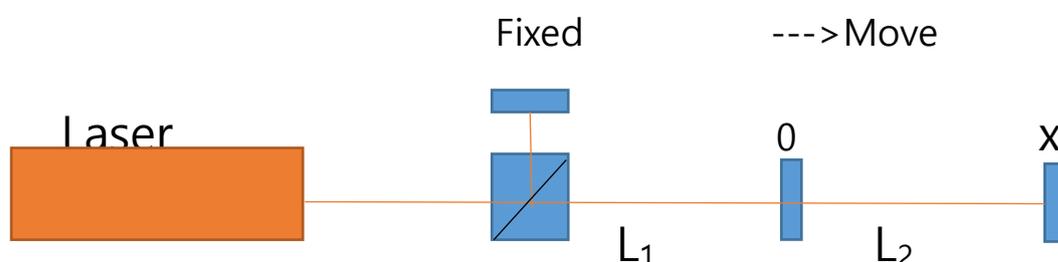
2. Temperature change on the machine parts during measurement, especially due to the thermal

deformation in lead screw, or scale, guideway, bearing parts. All measurements must be performed under the best approximation of operating conditions for machine. Several sets of complete warm-up sequences are essential to the positional error measurement.

3. Dead path error

: Due to the uncompensated wavelength in the dead path, while the wavelength is already influenced for the whole beam path, during the measurement procedures by the environmental change

Dead path: Distance (L_1) from the interferometer optics to the moving reflector at the RESET or Start position.



The L_1+L_2 length is influenced by the wavelength change. But only L_2 length is corrected by the wavelength compensation.

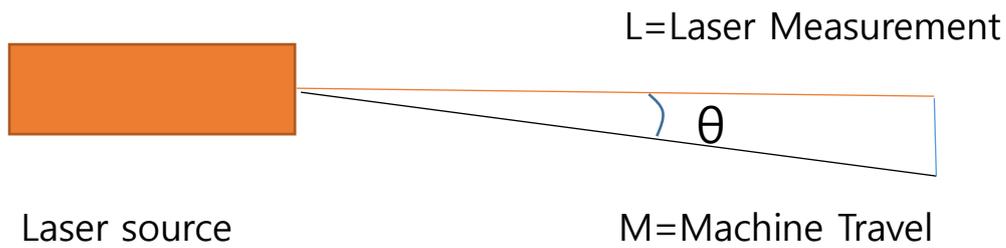
When the wavelength change is $\Delta\lambda$ due to the environmental change,

$$\text{Dead path error} = (L_1 + L_2) \cdot \Delta\lambda / \lambda - L_2 \cdot \Delta\lambda / \lambda$$

$= L_1 \cdot \Delta\lambda / \lambda$; under-evaluation by this amount compared to the true value. But, L_1 is difficult to measure precisely, therefore the dead path (L_1 length) should be minimized.

4. Cosine error (or error of misalignment)

: Error due to the misalignment of the beam path to the motion axis of machine. It is the error due to the misalignment of the measuring direction and scale direction



When θ is the angle of misalignment,

Cosine Error, CE,

$$= M - L = L/\cos\theta - L$$

$$= L(\sec\theta - 1)$$

where $\sec\theta = 1/\cos\theta = 1/(1 - \theta^2/2 + \dots)$

$\approx 1 + \theta^2/2$; with θ in [rad]

Thus $CE = L\theta^2/2$, and $CE/L = \theta^2/2$ (per unit length)

θ [rad]	$\theta^2/2$ [rad]
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0.01	50E-6 (50ppm=50um per 1m)
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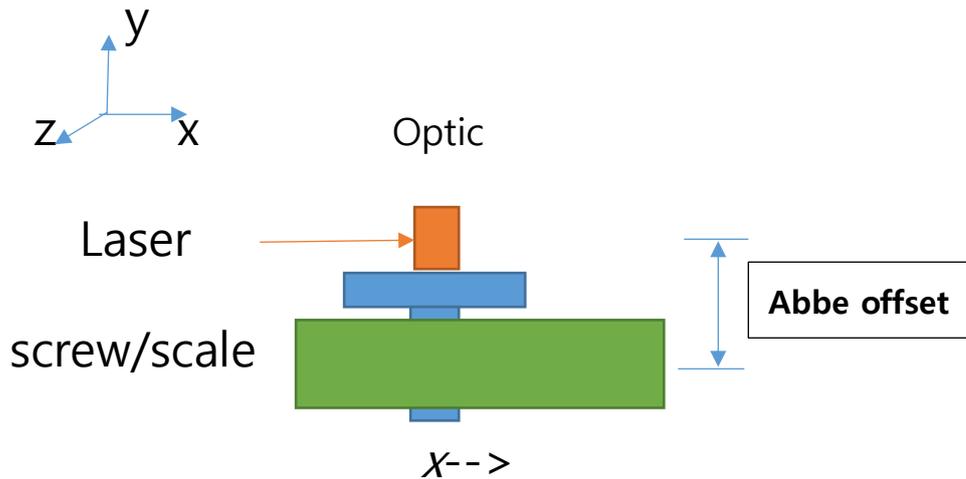
0.005	12.5E-6 (12.5 ppm= 12.5 um per 1m)
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0.001	0.5E-6 (0.5ppm=0.5um per 1m)
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Thus, few um per 1m possibly occurs for this case, and thus careful beam alignment is very important.

5. Abbe error

:Error due to the Abbe offset engaged during the measurement setups



The Abbe offset is due to the optics setup, then adding the Abbe error to the laser measurement;

Abbe error = $E_z(x) \cdot \text{Abbe offset}$ (2D space)

Abbe error = $\mathbf{E} \times \mathbf{A}$ (3D space)

Where $E_z(x)$ is the angular pitch at x position, **E is**
 $\mathbf{E}(E_x, E_y, E_z)$ at x position

Strategy

- (1) To measure the screw or scale precisely, the laser should be close to the screw/scale, that is to reduce the Abbe offset
- (2) When the Abbe offset is unavoidable, measure the Abbe offset and the angular pitch, $E_z(x)$, then compensate the positional error;

$$\Delta X_c(x) = \Delta X_m(x) - \text{Abbe Offset} \cdot E_z(x)$$

where $\Delta X_c(x)$ is the compensated positional error, while $\Delta X_m(x)$ is the measured positional error at x position.

Ex) When angular pitch error=10 arcsec, Abbe Offset=100mm, at x position; Abbe error induced?

\therefore Abbe error = Angular pitch \cdot Abbe offset

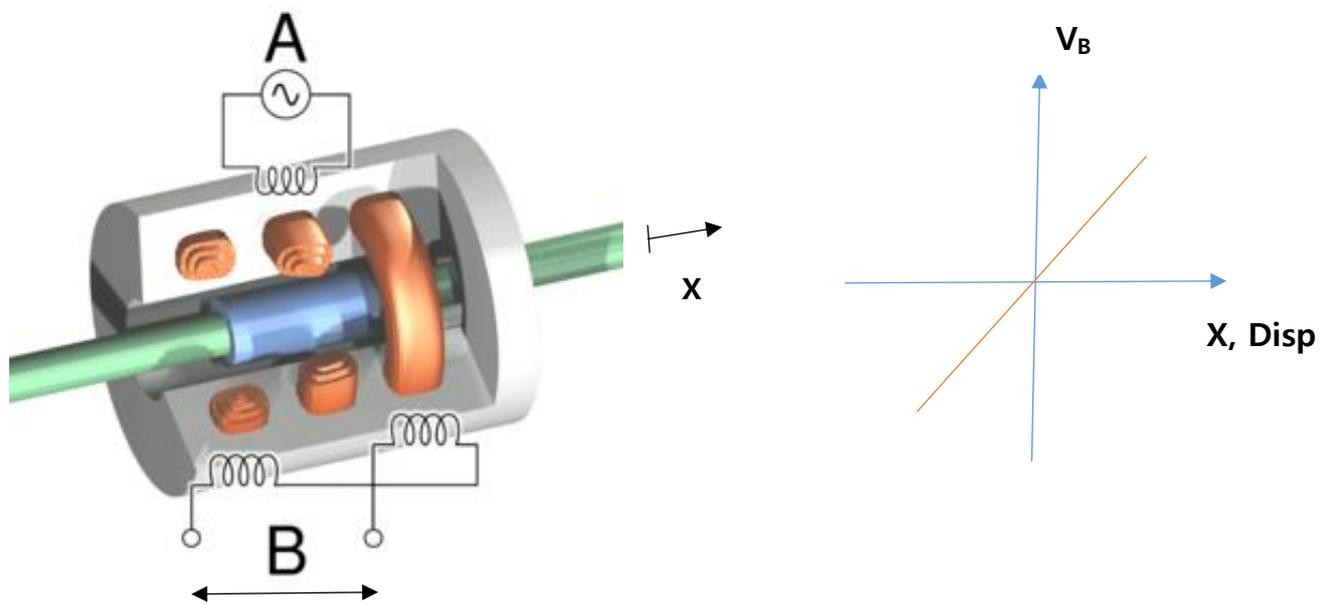
= (48urad)(0.1m) = 4.8um, and it should be subtracted from the laser measurement data, in order to get the correct positional error of screw/scale at x position.

Small displacement measurement

: To measure the small distance up to 10 mm with fine resolution

Commercially available sensors for small distance

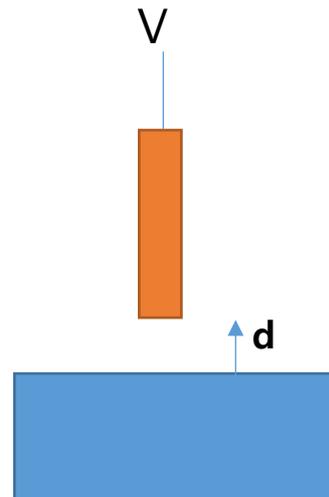
<u>Type</u>	<u>Range</u>	<u>Resolution</u>
LVDT	~10mm	0.01um
Capacitance Gauge	~1mm	0.001um
Optical sensor	~10mm	0.1um



LVDT (Source:Wikipedia)

$$V = L \cdot di/dt = j\omega Li, \text{ if } i = Ie^{j\omega t} \therefore |V| \propto L \propto X$$

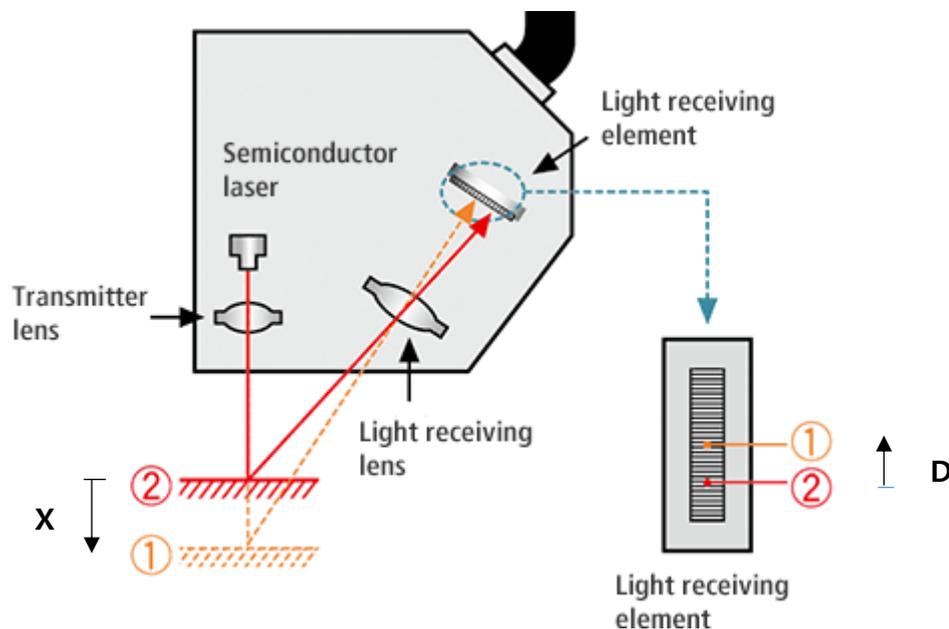
Where L: Inductance, X: Displacement



$$C = \epsilon A / d, \quad V = \int i dt / C = d / \epsilon A \cdot \int i dt$$

$$\therefore V \propto d$$

Capacitance Sensor (Source:Wikipedia)

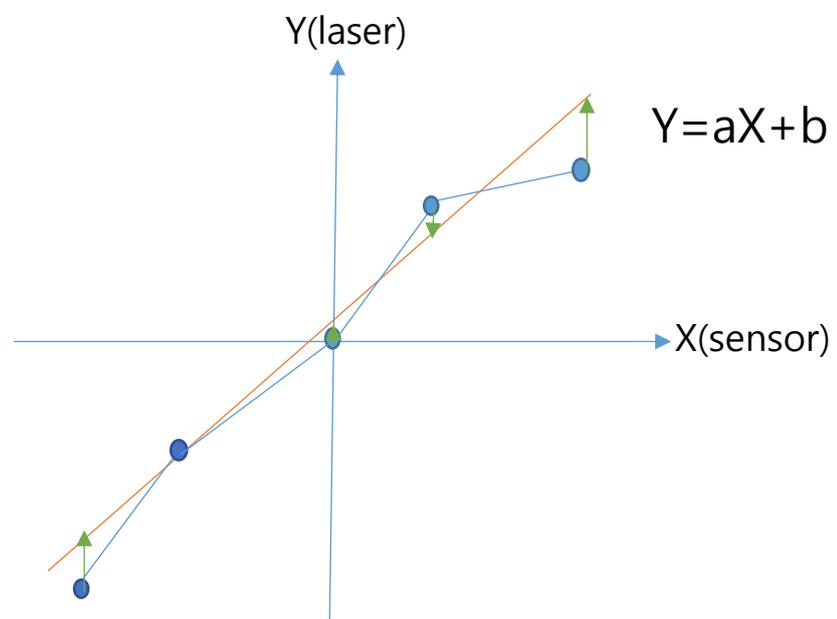
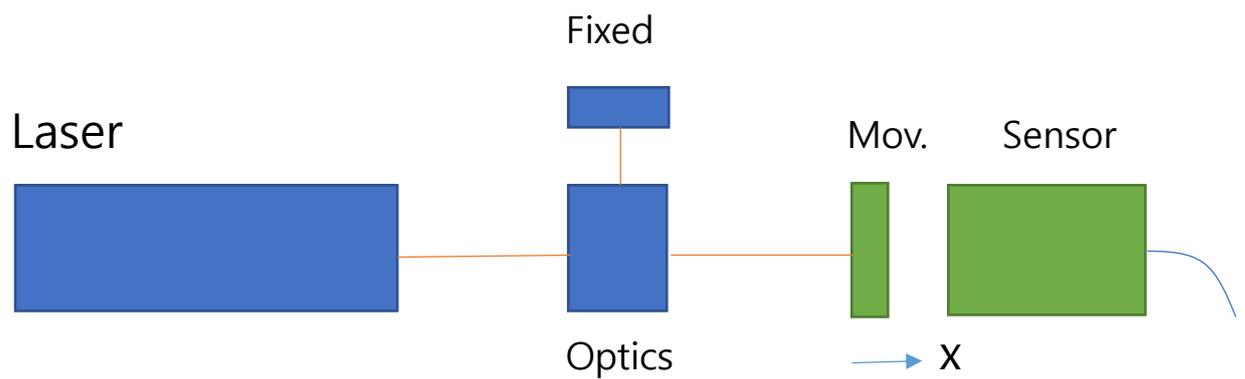


Optical Triangulation Sensor (Source:Keyence.com)

Triangulation gives linear relationship. $\therefore D \propto X$

Sensor Calibration for Small Range

: To calibrate the linearity, deviation, and repeatability of the sensor with respect to the reference length measurement



Procedures for sensor calibration

- 1) Divide the range into N-1 sections, typically, N=5 or more such as -E,M,O,M,E
- 2) Perform the measurement at least 3-5 times repeatedly for every points
- 3) Evaluate the linearity error, maximum linearity error, repeatability, and maximum repeatability

Least Squares Fit, $Y=aX+b$

where a=slope, b=offset

Based on (X_i, Y_i) measurement data, for $i=1,2..N$;

And N is the number of locations dividing the range.

The sum of squares of deviations, I, is

$$I = \sum (Y_i - aX_i - b)^2 \text{ be minimum}$$

$$\frac{\partial I}{\partial a} = 2 \sum (Y_i - aX_i - b)(-X_i) = 0$$

$$\therefore a \sum X_i^2 + b \sum X_i = \sum Y_i X_i$$

$$\frac{\partial I}{\partial b} = 2 \sum (Y_i - aX_i - b)(-1) = 0$$

$$\therefore a\sum X_i + bN = \sum Y_i$$

$$\text{Thus, } a = (N\sum X_i Y_i - (\sum X_i)^2) / (N\sum X_i^2 - (\sum X_i)^2)$$

$$b = (\sum X_i^2 \sum Y_i - \sum X_i Y_i \sum X_i) / (N\sum X_i^2 - (\sum X_i)^2)$$

Linearity error at ith position,

$$\delta_i = (Y_i - (aX_i + b)) / a, \quad i=1,2..$$

$$\text{Maximum linearity error} = \text{Max } \delta_i, \quad i=1,2..$$

The repeatability can be assessed with at least 3-5 times repeat measurement;

$$\text{Repeatability at ith position} = 3\sigma_i, \quad i=1,2..$$

$$\text{Maximum repeatability} = \text{Max } 3\sigma_i, \quad i=1,2..$$

HW7: For the given the measurement data from the small sensor measurement, calibrate the sensor, and evaluate the parameters.