

## Precision Metrology 9

Positional Error Calibration and Practical length measurement:

3-5 Cycles of bi-directional measurement from 0mm to 1000mm at 100mm step, allowing over-travel at the edge of machine travel

<u>Target</u>	<u>Nom</u>	<u>Laser</u>	<u>Error[mm]</u>	
0	0	0 (reset)	0	forward
1	100	100.001	0.001	
2	200	200.003	0.003	
3	300	300.004	0.004	
4	400	400.005	0.005	
5	500	500.007	0.007	
6	600	600.008	0.008	
7	700	700.009	0.009	
8	800	800.010	0.010	
9	900	900.012	0.012	

<u>Target</u>	<u>Nom</u>	<u>Laser</u>	<u>Error[mm]</u>	
10	1000	1000.014	0.014	overtravel
10	1000	1000.016	0.016	backward
9	900	900.014	0.014	
8	800	800.012	0.012	
7	700	700.010	0.010	
6	600	600.009	0.009	
5	500	500.008	0.008	
4	400	400.007	0.007	
3	300	300.005	0.005	
2	200	200.004	0.004	
1	100	100.003	0.003	
0	0	0.002	0.002	overtravel

1<sup>st</sup> cycle-----

0	0	0.001	0.001	forward
1	100	100.002	0.002	

<u>Target</u>	<u>Nom</u>	<u>Laser</u>	<u>Error[mm]</u>	
2	200	200.004	0.004	
3	300	300.005	0.005	
4	400	400.007	0.007	
5	500	500.009	0.009	
6	600	600.010	0.010	
7	700	700.011	0.011	
8	800	800.012	0.012	
9	900	900.014	0.014	
10	1000	1000.016	0.016	overtravel
10	1000	1000.018	0.018	backward
9	900	900.017	0.017	
8	800	800.015	0.015	
7	700	700.013	0.013	
6	600	600.011	0.011	
5	500	500.009	0.009	

<u>Target</u>	<u>Nom</u>	<u>Laser</u>	<u>Error[mm]</u>	
4	400	400.008	0.008	
3	300	300.007	0.007	
2	200	200.005	0.005	
1	100	100.004	0.004	
0	0	0.003	0.003	overtravel
2 <sup>nd</sup> cycle-----				
0	0	0.002	0.002	forward
1	100	100.002	0.002	
2	200	200.005	0.005	
3	300	300.006	0.006	
4	400	400.008	0.008	
5	500	500.010	0.010	
6	600	600.011	0.011	
7	700	700.013	0.013	
8	800	800.014	0.014	

Target	Nom	Laser	Error[mm]	
9	900	900.016	0.016	
10	1000	1000.018	0.018	overtravel
10	1000	1000.020	0.020	backward
9	900	900.019	0.019	
8	800	800.017	0.017	
7	700	700.015	0.015	
6	600	600.014	0.013	
5	500	500.012	0.011	
4	400	400.010	0.009	
3	300	300.009	0.007	
1	100	100.007	0.005	
0	0	0.005	0.004	

3<sup>rd</sup> cycle-----

...

In this way the number of cycle can be increased, and the positional error calibration is performed at every target of machine travel for the bidirectional movement.

### Positional Error Calibration

#### Forward measurement

The  $i$ th repeat forward positional error at the  $j$ th target,

$$X_{ji} \uparrow$$

$X_{ji} \uparrow$  = the  $i$ th measurement error at the  $j$ th target

$$= \text{Laser} - \text{Nominal} = L_{ji} \uparrow - N_j$$

where  $L_{ji}$  is the  $i$ th repeat laser measurement at the  $j$ th target, and  $N_j$  is the nominal data.

The forward mean positional error at the  $j$ th target is

$$\underline{X}_j \uparrow = \sum X_{ji} \uparrow / N$$

where  $N$  = number of cycle = 1, 2, ..

The forward standard deviation at the  $j$ th target is

$$\sigma_j \uparrow = \sqrt{\sum (X_{ji} \uparrow - \underline{X}_j \uparrow)^2 / N - 1}$$

The uni-directional repeatability for forward direction at the jth target,

$$UR_j \uparrow = 6 \sigma_j \uparrow$$

And the maximum uni-directional repeatability for the forward direction is,

$$UR_{\max} \uparrow = \text{Max } UR_j \uparrow \text{ (for } j=1,2,\dots)$$

### Backward measurement

The ith repeat backward positional error at the jth target,  $X_{ji} \downarrow$

$X_{ji} \downarrow$  = the ith measurement error at the jth target

$$= \text{Laser} - \text{Nominal} = L_{ji} \downarrow - N_j$$

where  $L_{ji}$  is the ith repeat laser measurement at the jth target, and  $N_j$  is the nominal data.

The backward mean positional error at the jth target is

$$\underline{X}_j \downarrow = \Sigma X_{ji} \downarrow / N$$

where  $N$  = number of cycle = 1, 2, ..

The backward standard deviation at the jth target is

$$\sigma_j \downarrow = \sqrt{\sum (X_{ji} \downarrow - \underline{X}_j \downarrow)^2 / N - 1}$$

The uni-directional repeatability for backward direction at the jth target,

$$UR_j \downarrow = 6 \sigma_j \downarrow$$

And the maximum uni-directional repeatability for the forward direction is,

$$UR_{\max} \downarrow = \text{Max } UR_j \downarrow \text{ (for } j=1,2,\dots)$$

### Mean positional error

The mean positional error or system positional error at the jth target

$$\underline{X}_j = (\text{Forward mean positional error} + \text{Backward mean positional error}) / 2$$

$$= (\underline{X}_j \uparrow + \underline{X}_j \downarrow) / 2$$



The reversal error (hysteresis error, or backlash error) at the  $j$ th target is,

$R_j = \text{Backward mean positional error} - \text{Forward mean positional error}$

$$= \underline{X}_j \downarrow - \underline{X}_j \uparrow$$

The maximum reversal error,  $R_{\max}$ , is

$$R_{\max} = \text{Max } R_j \text{ (} j=1,2,\dots \text{)}$$

The maximum uni-directional repeatability is,

$$UR_{\max} = \text{Max } (UR_j \uparrow, UR_j \downarrow) = \text{Max}(6\sigma_j \uparrow, 6\sigma_j \downarrow), \text{ for } j=1,2,\dots$$

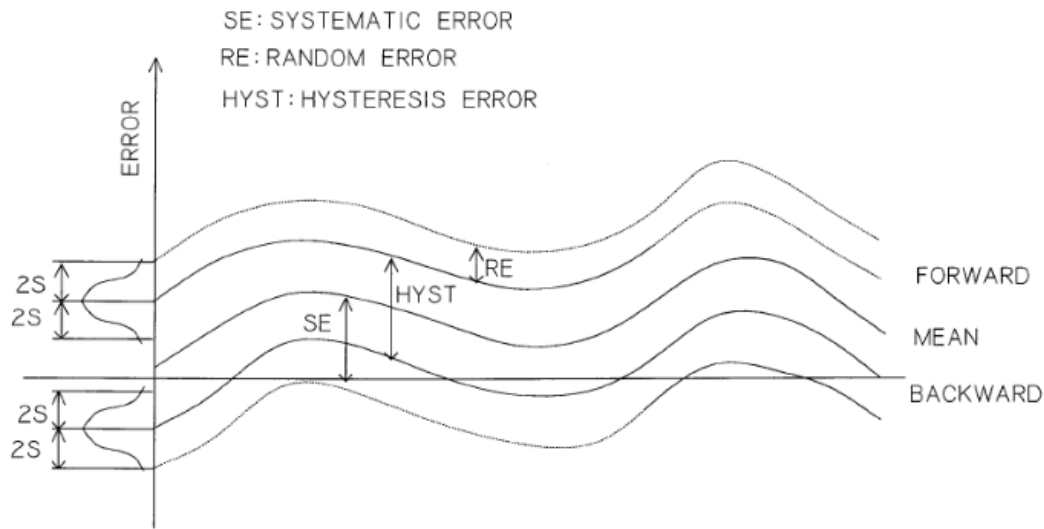
The Bi-directional repeatability at the  $j$ th target is

$$BR_j = |R_j| + 3(\sigma_j \uparrow + \sigma_j \downarrow)$$

The maximum bi-directional repeatability is

$$BR_{\max} = \text{Max } BR_j = \text{Max } (|R_j| + 3\sigma_j \uparrow + 3\sigma_j \downarrow), \text{ for } j=1,2,\dots$$

## Positional error calibration



### Error Calibration along a machine axis

(Note:  $s = \sigma$ , sample standard deviation, and  $2\sigma$  for 95% probability,  $3\sigma$  for 99% probability)

### Evaluation of Accuracy

#### 1. Bandwidth Method(ANSI)

:The greatest difference in the positional error representation,

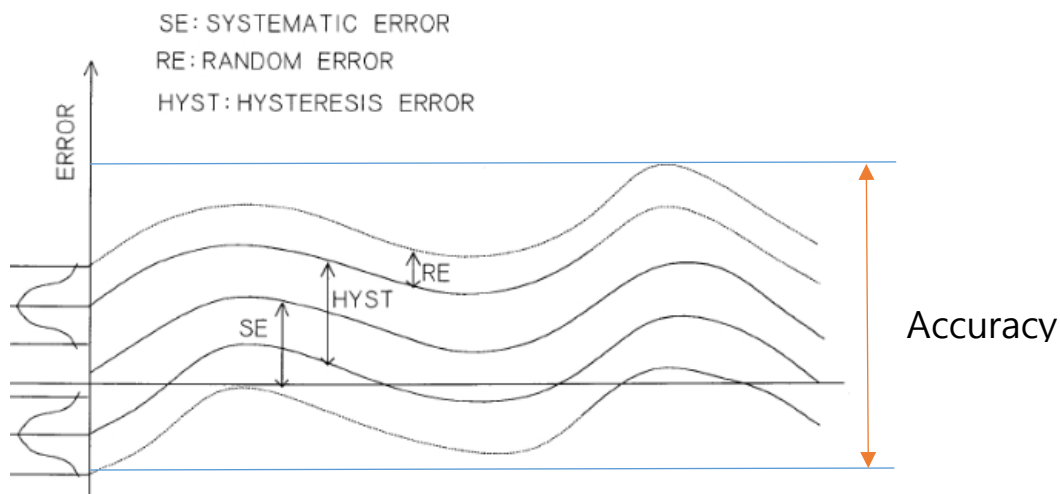
Linear Displacement Accuracy, or Accuracy

=Maximum error – Minimum error

=Max ( $X_j \uparrow \pm 3\sigma_j \uparrow$ ,  $X_j \downarrow \pm 3\sigma_j \downarrow$ ) ( $j=1,2\dots$ )

- Min ( $X_j \uparrow \pm 3\sigma_j \uparrow$ ,  $X_j \downarrow \pm 3\sigma_j \downarrow$ ) ( $j=1,2\dots$ )

(\*Discuss pros. and cons.)



The arrow indicates the Linear Displacement Accuracy, or Accuracy.

## 2. Tolerance Template Method (DIN, Europe)

:To describe the machine accuracy in relation to the length of travel.

The accuracy or uncertainty can be expressed as the function of the distance travelled, as indicated as red line.

$$U = A + KL \leq B$$

where A= offset, [um]

= maximum uni/bi-directional repeatability

=Max ( $6\sigma_j \uparrow$  ,  $6\sigma_j \downarrow$  ) for uni-directional

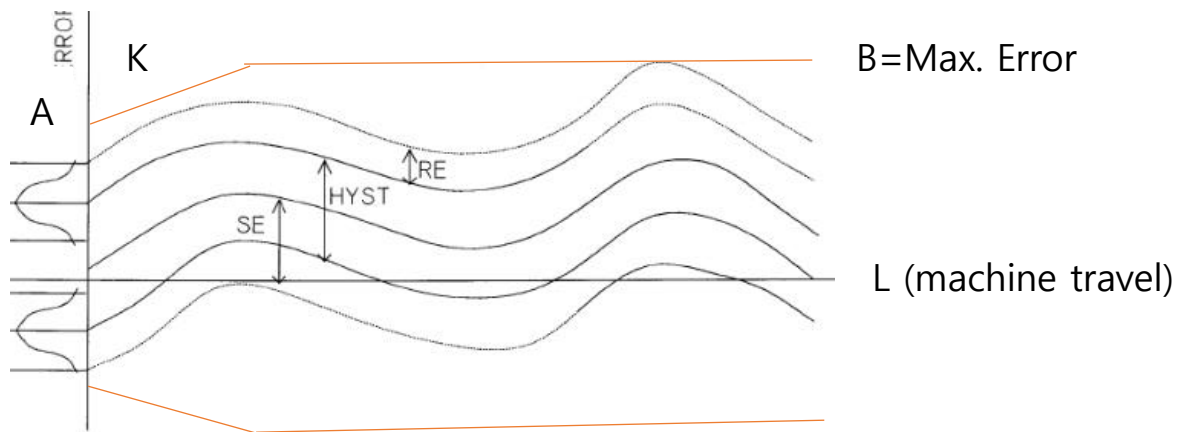
=Max (  $|R_j| + 3\sigma_j \uparrow + 3\sigma_j \downarrow$  ) for bi-directional

B=Maximum error, [um]

K=slope to just enclose the error data over the machine travel, [um/m]

L=length of machine travel, [m]

Offset=A, Slope=K, Maximum Error=B

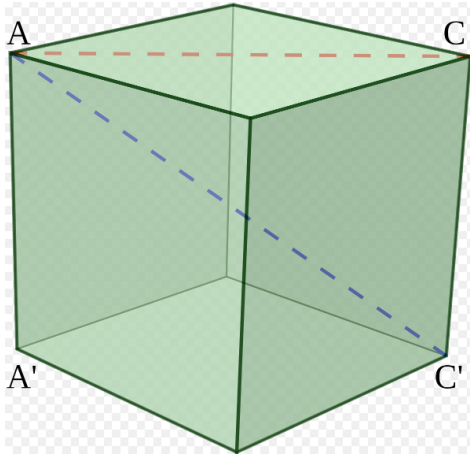


This is a most widely accepted method, and many industry currently use it as the accuracy or uncertainty evaluation.

Ex)  $U=2.0+L/100 \leq 10$  [um]

HW6) For the given positional error data,

- (1) Calculate the positional errors, and
- (2) Evaluate the positional accuracy



Source:wikipedia

Face diagonals and space diagonals in the 2D/3D working space

The tolerance template can be extended to the 2D;

$$U_2 = A_2 + K_2 L \leq B_2$$

Where  $U_2$  = accuracy (or uncertainty) in 2D work space

$A_2$  = maximum repeatability in the length measurement error in 2D work space, [ $\mu\text{m}$ ]

$K_2$  = slope to just enclose the length measurement error data over the 2D work space, [ $\mu\text{m}/\text{m}$ ]

$B_2$  = maximum length measurement error in the 2D work space, [ $\mu\text{m}$ ]

L=length of machine travel along the 2D work space,  
such as face diagonals, [m]

Also in the 3D working space;

$$U_3 = A_3 + K_3 L \leq B_3$$

where  $U_3$ =accuracy (or uncertainty) in 3D space, [um]

$A_3$ =maximum repeatability in the length measurement  
error in 3D work space, [um]

$K_3$ =slope to just enclose the length measurement error  
data over the 3D work space, [um/m]

$B_3$ =maximum length measurement error in the 3D work  
space, [um]

L=length of machine travel along the 3D work space,  
such as space diagonals, [m]