Precision Metrology 12-Straightness Measurement

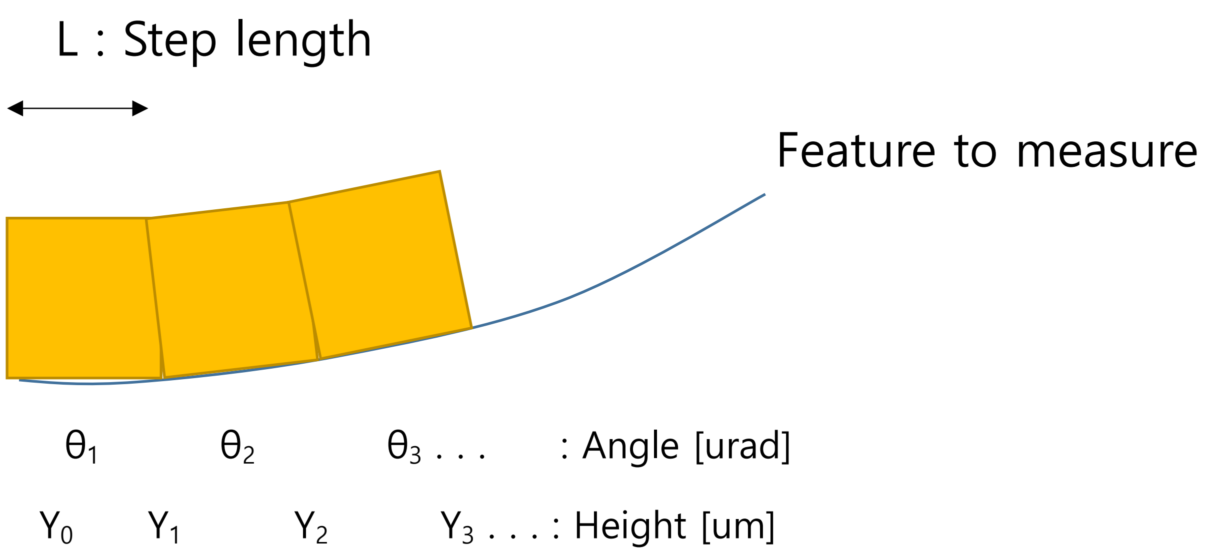
Straightness Error Measurement

Straightness error:

:Deviation from the ideal reference straight line

: ⏤ 0.02 Straightness Tolerance (ISO)

Incremental angular measuring devices are very useful such as Precision Level, Autocollimator, Laser Interferometer with angular optics



$Y_i = Y_{i-1} + L \cdot \sin\theta_i \quad i=1,2,3\ldots$

$\fallingdotseq Y_{i-1}+L\cdot\theta_i$ for θ in [urad]

$= Y_{i-1}+4.8\cdot L\cdot\theta_i$ for θ in [sec]

Thus, with $Y_0=0$

$Y_i = \Sigma L \cdot \theta_i$ for θ [urad], $Y_i = 4.8 \cdot \Sigma L \cdot \theta_i$ for θ [sec]

where i=1,2,3…N, N=number of steps; L=0.1m

Straightness Error based on the End Points Fit

i	Xi	θ_i	$4.8L\theta_i$	Y_i	δ_i
0	0	--	0	0	0
1	0.1	3	1.44	1.44	-1.2
2	0.2	5	2.4	3.84	-1.44
3	0.3	7	3.36	7.2	-0.72
4	0.4	8	3.84	11.04	0.48
5	0.5	10	4.8	15.84	2.64
6	0.6	9	4.32	20.16	4.32
7	0.7	6	2.88	23.04	4.56
8	0.8	5	2.4	25.44	4.32
9	0.9	3	1.44	26.88	3.12
10	1.0	-1	-0.48	26.4	0

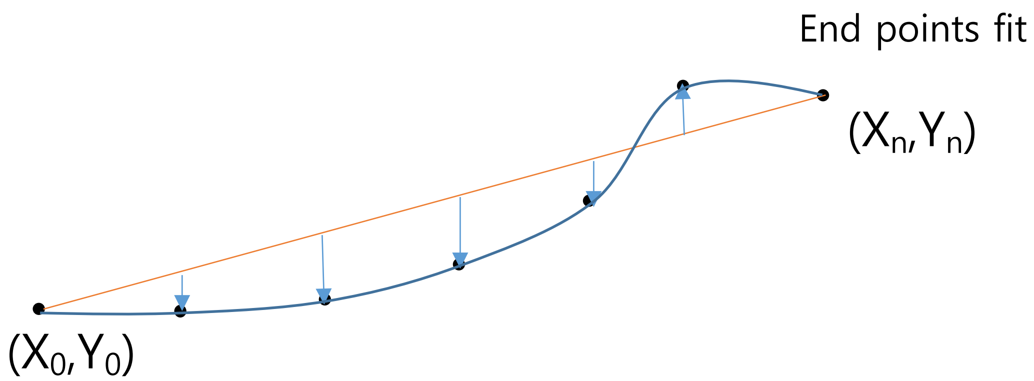
For straightness error calculation, the ideal reference straight line is to be evaluated.

3 criteria for reference straight line

1. End points fit

:The reference line is determined by the two end points

Given (X_i, Y_i)



For measured (X_i, Y_i) data;

Slope, $a = (Y_n - Y_0)/nL = (26.4-0)/10(0.1) = 26.4$um/m

Offset, $b = Y_0 = 0$

Thus the reference line, $Y = aX + b = 26.4X$

Straightness deviation, δ_i, for i=0,1,2..n

$\delta_i = Y_i - (aX_i + b)/\sqrt{1+a^2} \fallingdotseq Y_i - (aX_i + b)$ $(\because a \ll 1)$

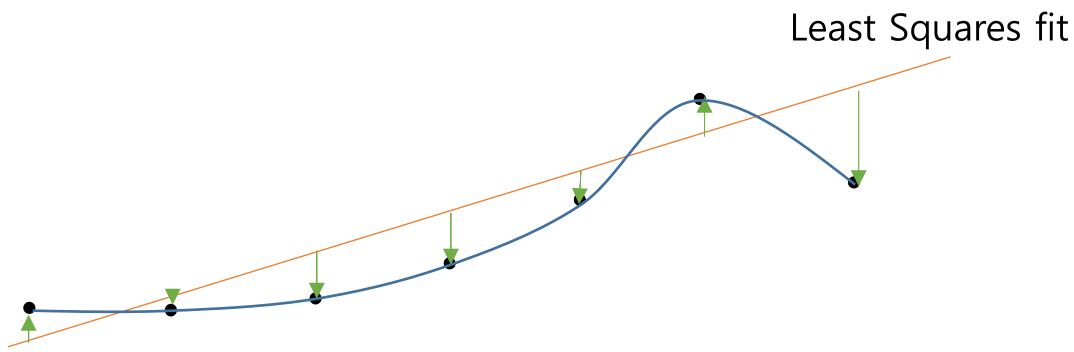
The straightness error is the difference between the maximum and minimum deviation.

Straightness error=max δ_i – min δ_i

=4.56-(-1.44)=6.0 [um]

2. Least squares fit

:The reference line is determined such that the sum of squares of deviations from the line is minimum.



For measured (X_i,Y_i) data,

a, b : unknowns to be determined

Straightness deviation $\delta_i=Y_i-aX_i-b$

$E=\Sigma\delta_i^2=\Sigma(Y_i-aX_i-b)^2$ be minimum

$\partial E/\partial a=2\Sigma(Y_i-aX_i-b)(-X_i)=0$

$\therefore a\Sigma X_i^2+b\Sigma X_i=\Sigma Y_iX_i$

$\partial E/\partial b=2\Sigma(Y_i-aX_i-b)(-1)=0$

$\therefore a\Sigma X_i+bN=\Sigma Y_i$

Thus,

Slope$=a=(N\Sigma X_iY_i-\Sigma X_i\Sigma Y_i)/(N\Sigma X_i^2-(\Sigma X_i)^2)$

$=[11(114.576)-(5.5)(161.28)]/[11(3.85)-5.5^2]$

$=373.296/12.1=30.8509$

Offset$=b=(\Sigma X_i^2\Sigma Y_i-\Sigma X_iY_i\Sigma X_i)/(N\Sigma X_i^2-(\Sigma X_i)^2)$

$=[(3.85)(161.28)-(114.576)(5.5)]/12.1$

$=-9.24/12.1=-0.7636$

$\delta_i=Y_i-aX_i-b=Y_i - 30.8509X_i + 0.7636$

Straightness error$=$ max δ_i-min δ_i

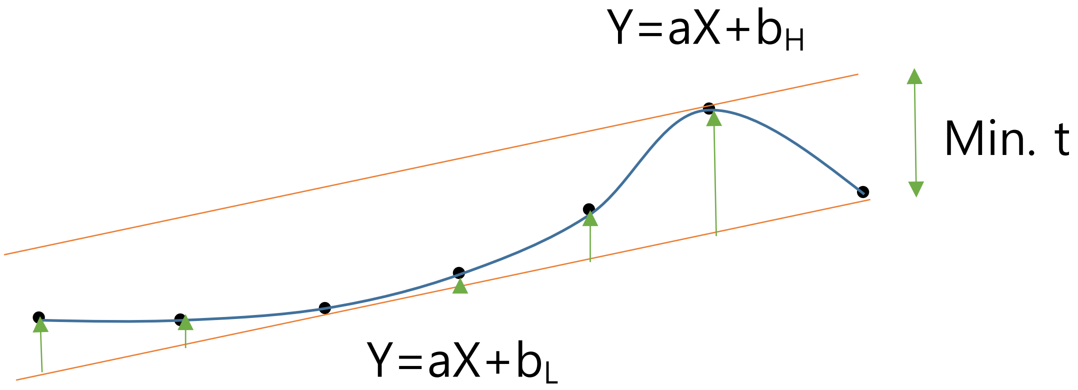
$=2.4131-(-3.6873)=6.100$ (um)

Straightness Error based on the Least Squares Fit

i	X_i	Y_i	X_i^2	Y_i^2	X_iY_i	δ_i
0	0	0	0	0	0	0.7636
1	0.1	1.44	0.01	2.0736	0.144	-0.8815
2	0.2	3.84	0.04	14.7456	0.768	-1.5666
3	0.3	7.2	0.09	51.84	2.16	-1.2917
4	0.4	11.04	0.16	121.8816	4.416	-0.5368
5	0.5	15.84	0.25	250.9056	7.92	1.1782
6	0.6	20.16	0.36	406.4256	12.096	2.4131
7	0.7	23.04	0.49	530.8416	16.128	2.2080
8	0.8	25.44	0.64	647.1936	20.352	1.5229
9	0.9	26.88	0.81	722.5344	24.192	-0.1222
10	1.0	26.4	1.0	696.96	26.4	-3.6873
∑	5.5	161.28	3.85	3445.4016	114.576	

3. <u>Minimum Zone Fit or Minimum Separation Fit</u>

:The parallel straight lines containing the measured data to give minimum distance, t



Two parallel lines containing the measured data, and the minimum distance $t=b_H-b_L$ (that is, minimum separation, or minimum zone).

By the 1-2 criterion of ETT, or intuition,

a= slope joining the (X_2,Y_2) to the end point(X_{10},Y_{10})

=(26.4-3.84)/8(0.1)=28.2 [um/m]

Low offset occurs when passing through (X_2,Y_2)

Low offset, $b_L=Y_2 - aX_2$=3.84-28.2(0.2)=-1.8 [um]

High offset occurs when passing through (X_7,Y_7)

High offset, $b_H=Y_7 - aX_7$=23.04-28.2(0.7)=3.3[um]

Straightness deviation, $\delta_i = Y_i - (aX_i + b_L)$

<u>Straightness Error based on the Minimum Zone Fit</u>

i	X_i	Y_i	aX_i+b_L	δ_i
0	0	<u>0</u>	0	1.8
1	0.1	1.44	1.02	0.42
2	0.2	3.84	3.84	<u>0</u>
3	0.3	7.2	0.09	0.54
4	0.4	11.04	9.48	1.56
5	0.5	15.84	12.3	3.54
6	0.6	20.16	15.12	5.04
7	0.7	23.04	17.94	<u>5.1</u>
8	0.8	25.44	20.76	4.68
9	0.9	<u>26.88</u>	23.58	3.3
10	1.0	26.4	26.4	<u>0</u>

The minimum zone straightness error=5.1[um]

(Discuss 6.0um vs 6.1um vs 5.1um)

Minimum Zone Evaluation

(1)'Enclose and Tilt' technique: Graphical and geometrical solution for the minimum zone, using the intuitive '2-1' or '1-2' criterion for the minimum separation, developed by Pahk,H.

The application of a micro computer to the on-line calibration of the flatness of engineering surfaces, BURDEKIN,M.& PAHK,H., Proceedings of Institution of Mechanical Engineers,1989, Vol. 203 B,127-137

(2)Linear Programming method: Simplex search based optimization algorithm

General LP formulation by simplex search

MIN (or MAX) **CX**

Subject to **AX** ≤ (≥) **B**

Where **C** is constant vector, **X** is unknown vector

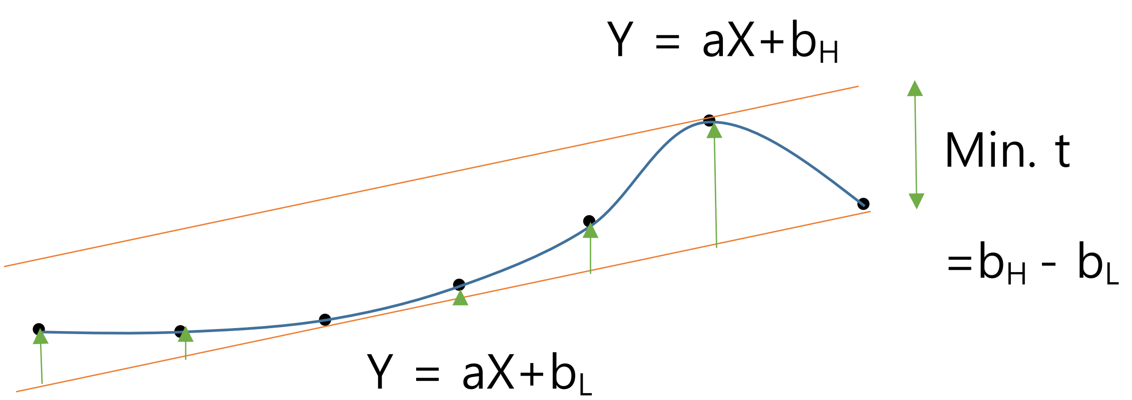
To find the two parallel straight lines that give the minimum distance ->

Minimize $b_H - b_L$; Object Function

Subject to

$aX_i + b_H \leq Y_i$ (i=0,1,2…N) ; Constraints

$aX_i + b_L \geq Y_i$ (i=0,1,2,…N) ; Constraints



Let **C**=[0, 1, -1], **X**=[a, b_H, b_L]^T

Minimize **CX=** [0,1,-1][a, b_H, b_L]^T=b_H-b_L

such that **AX** ≥(≤) **B**

where

$$\mathbf{A}=\begin{bmatrix} X_0 & 1 & 0 \\ X_1 & 1 & 0 \\ \cdots & \cdots & \cdots \\ X_N & 1 & 0 \\ \hline X_0 & 0 & 1 \\ X_1 & 0 & 1 \\ \cdots & \cdots & \cdots \\ X_N & 0 & 1 \end{bmatrix} \begin{matrix} \\ \\ \geq \\ \\ \\ \\ \leq \\ \\ \end{matrix} \mathbf{B}=\begin{bmatrix} Y_0 \\ Y_1 \\ \cdots \\ Y_N \\ \hline Y_0 \\ Y_1 \\ \cdots \\ Y_N \end{bmatrix}$$

No. of Variables = 3

No. of Constraints = 2(N+1)

No. of '≤' Constraints = N+1

No. of '≥' Constraints = N+1

No. of '=' Constraints = 0

Solution is ;

$a=28.2$, $b_H=3.3$, $b_L=-1.8$

and the minimum distance, $t=b_H-b_L=5.1$

HW8) Evaluate the straightness error for the data given in the lecture in terms of the end points fit, the least squares fit, and the minimum zone fit.