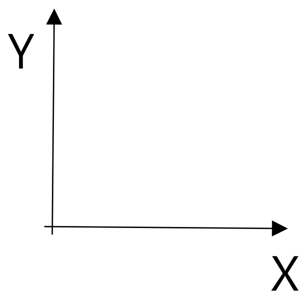
Precision Metrolgy 15: Flatness Calculation

Thus all the height data are determined, and the height measurement procedures are completed.

The measured height data are;

B	3.833	5.791	7.25	*A*
2.875				
1.416	1.846	2.777	5.208	
0.958	1.361	2.264	4.167	
O	0.542	2.083	2.625	*C*



The flatness is the deviation from the ideal reference plane, and there are 3 reference surfaces; 3 points surface, least squares surface, and the minimum zone surface.

①3 points surface

Based on the 3 points, the reference plane can be calculated. The 3 points are preferred as the points on the edge to cover the whole measurement datum.

The reference plane, $Z=aX+bY+C$

The 3 points are chosen as O, A, C points;

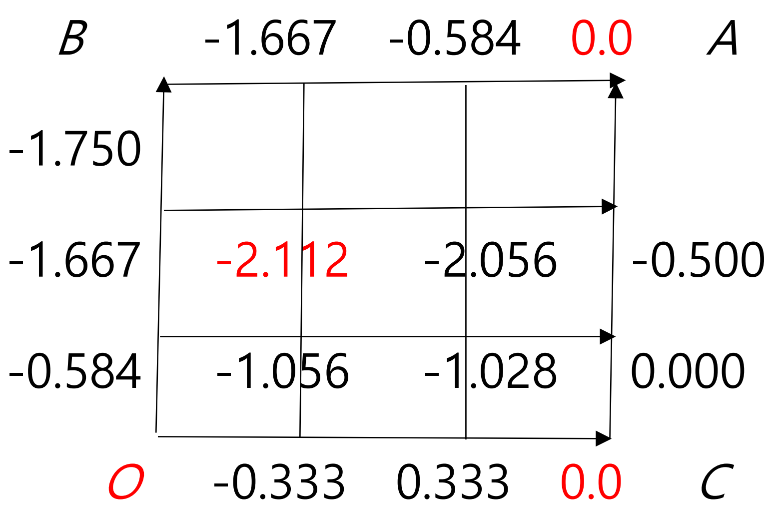
Slope a= (2.625-0)/0.3=8.75 [urad]

Slope b= (7.25-2.625)/0.3=15.417 [urad]

Offset C=0 [um]

Flatness deviation, h*=h - (aX+bY+C)

And Flatness error =max h*- min h*



∴Flatness error = 0-(-2.112)=2.112 um

in terms of 3 points surface

②Least Squares surface

The perpendicular distance, d, from the surface $Z=aX+bY+c$ to the point $P_i(X_i,Y_i,Z_i)$ is;

$Di= | aX_i+bY_i+C-Z_i) | /\sqrt{(1+a^2+b^2)}$

$\fallingdotseq | aX_i+bY_i+C-Z_i) | (\because a, b \ll 1)$

The sum of squares of distance, J, is

$J=\Sigma(aX_i+bY_i+C-Z_i)^2$ be minimum

$\partial J/\partial a=2\Sigma(aX_i+bY_i+C-Z_i)(X_i)=0$

$\therefore a\Sigma X_i^2+b\Sigma X_iY_i+C\Sigma X_i=\Sigma Z_iX_i$

$\partial J/\partial b=2\Sigma(aX_i+bY_i+C-Z_i)(Y_i)=0$

$\therefore a\Sigma X_iY_i+b\Sigma Y_i^2+C\Sigma Y_i=\Sigma Z_iY_i$

$\partial J/\partial C=2\Sigma(aXi+bYi+C-Zi)(1)=0$

$\therefore a\Sigma X_i+b\Sigma Y_i+C\Sigma=\Sigma Z_i$

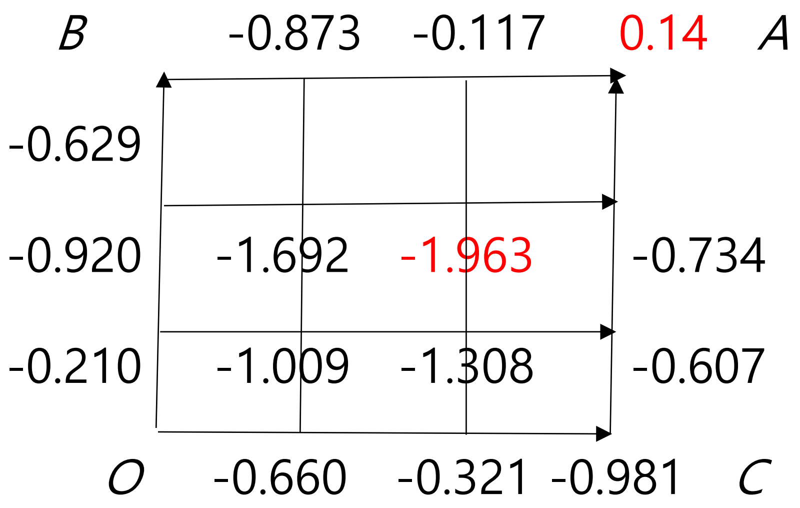
Three unknows, a,b,c can be calculated by the Gauss Elimination method or other numerical equation solver.

Slope a=12.02 urad, Slope b=11.68 urad

Offset C=0 (assigned)

Flatness deviation, h*=h - (aX+bY+C)

Flatness error = max h*- min h*

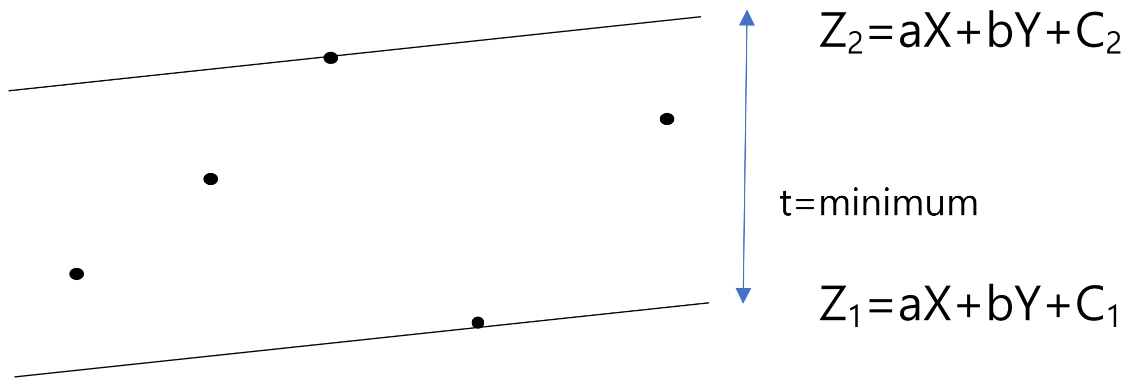


Flatness error = 0.14-(-1.963)=2.103

in terms of the least squares plane

②Minimum Zone Surface

:To find the two parallel planes that gives the minimum distance between the two



Linear Programming such as Simplex Search

Min C_2-C_1

such that $C_1 \le Z_i - aX_i - bY_i \le C_2$, that is,

$aX_i+bY_i+C_1 \le Z_i$

$aX_i+bY_i+C_2 \ge Z_i$

Let $\mathbf{C}=[a,b,C_2,C_1]$, $\mathbf{X}=[0,0,1,-1]^T$

Min $\mathbf{CX}$

s.t. $\mathbf{A}_1\mathbf{X}\leq\mathbf{B}$, $\mathbf{A}_2\mathbf{X}\geq\mathbf{B}$

$$\mathbf{A}_1=\begin{bmatrix} X_1 & Y_1 & 0 & 1 \\ X_2 & Y_2 & 0 & 1 \\ \multicolumn{4}{c}{\cdots\cdots\cdots\cdots} \\ X_N & Y_N & 0 & 1 \end{bmatrix} \quad \mathbf{B}=\begin{bmatrix} Z_1 \\ Z_2 \\ \cdots \\ Z_N \end{bmatrix}$$

$$\mathbf{A}_2=\begin{bmatrix} X_1 & Y_1 & 1 & 0 \\ X_2 & Y_2 & 1 & 0 \\ \multicolumn{4}{c}{\cdots\cdots\cdots\cdots\cdots} \\ X_N & Y_N & 1 & 0 \end{bmatrix}$$

Alternative geometric solution Enclose Tilt Technique* gives, the surface passing OAB gives the minimum zone surface.

For flatness; 3-1 or 2-2 criterion

For straightness; 2-1 criterion

Thus a=(7.25-3.833)/0.3=11.39 [urad]

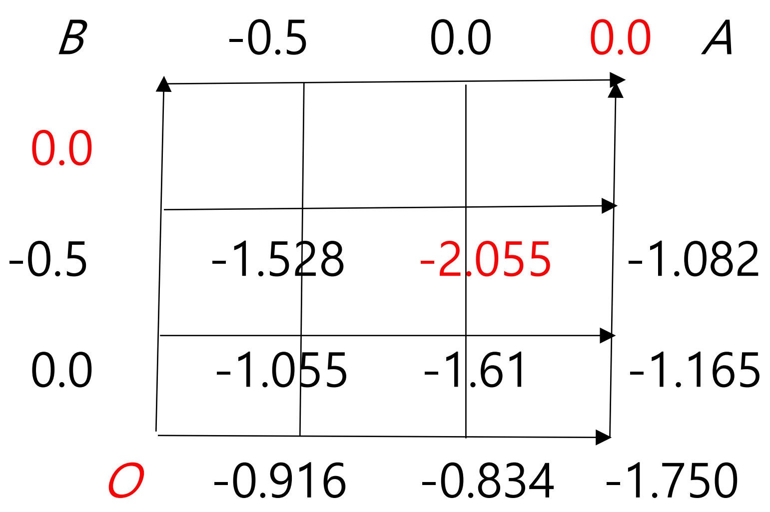
B=(2.875-0)/0.3=9.583 [urad]

C=0

Thus the flatness deviation is

h*=h-(aX+bY+C)

Flatness error= max h*- min h*

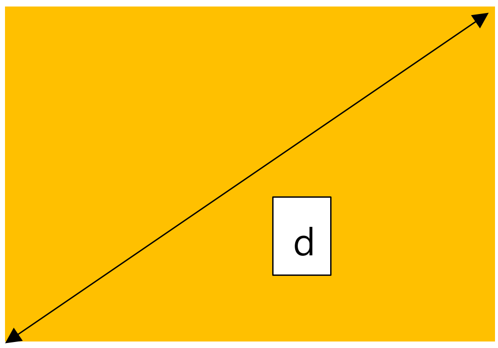


Thus Flatness error= 0.0-(-2.055)=2.055 in terms of the minimum zone surface

*The application of a micro-computer to the on-line calibration of the flatness of engineering surfaces, BURDEKIN,M.& PAHK,H., Proceedings of Institution of Mechanical Engineers,1989, Vol. 203 B,127-137

Table Grade

:To grade the surface table according to the Flatness error and the Size



Permitted tolerance, t_0, for Grade 0 table

$t_0=2.5(1+d/1000)$

where d=nominal length of the diagonal in mm rounded up-to the nearest 100mm t is rounded up-to the nearest 0.5um

Each succeeding grade has the double t of preceding grade, i.e. $t_1=2t_0$, $t_2=2t_1$, $t_3=2t_2$

Ex) 1000mm by 1000mm granite surface plate

$d=1000\sqrt{2} \fallingdotseq 1400$

$t_0=2.5(1+1400/1000)=6.0$ [um] for Grade0,

$t_1=12$[um] for Grade 1, $t_2=24$[um] for Grade 2, etc.

HW)Given flatness measurement data, write a computer code for the 3 points surface, least squares surface, and the minimum zone surface.