Convection-Diffusion Equation

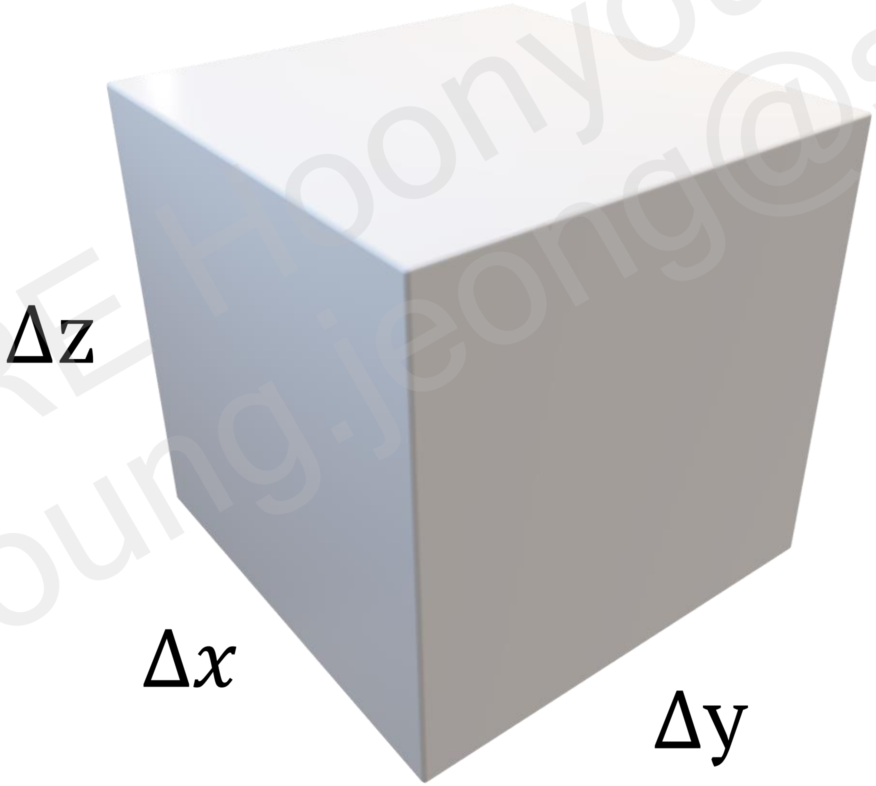
Hoonyoung Jeong
Department of Energy Resources Engineering
Seoul National University

Transport Equation

- Generic Scalar Transport Equation
- Convection-Diffusion
- Advection-Diffusion Equation
- This describes the transport phenomena of particles, energy, or other physical quantities by convection and diffusion
- In a control volume, the accumulation rate of a physical quantity equals the external source rate of the physical quantity minus the transport (outflow by convection and diffusion) rate of the physical quantity through the neighboring control surfaces

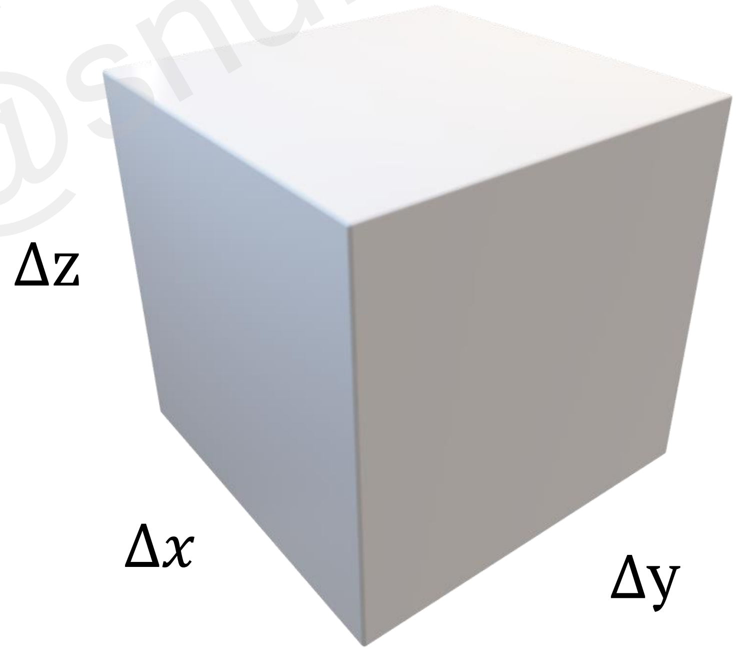
System

Control volume during a very short time, Δt



System

- Physical quantity ψ
  - ✓Concentration C_ψ
  - ✓Amount per unit mass of a fluid
- Control volume
  - ✓Control volume (ΔxΔyΔz) during a very short time, Δt
  - ✓Control surfaces
    - 2ΔxΔy, 2ΔyΔz, 2ΔzΔx
- Assume the fluid flows along only the x direction (1-dimension)



Source

- S_ψ
  - ✓Supply of ψ per unit fluid volume per unit time
- Supply rate to the control volume
  - ✓$S_\psi \Delta x \Delta y \Delta z$

Outflow by Convection and Diffusion

- Transport by convection and diffusion
- Through spatially neighboring control surfaces
- Convection
  - ✓ Volumetric rate of the fluid = $u_x \Delta y \Delta z$
  - ✓ Mass rate of the fluid = $\rho u_x \Delta y \Delta z$
  - ✓ Mass rate of ψ = $C_\psi \rho u_x \Delta y \Delta z$
- Diffusion
  - ✓ Diffusivity, D
  - ✓ Mass rate of $\psi = -\Delta y \Delta z D \partial C_\psi / \partial x$
  - ✓ C_ψ can be different for ψ
    - Ex) ψ is thermal energy, C_ψ should be temperature

Accumulation Term

- Accumulation of ψ per unit time in a control volume
  - ✓Mass of the fluid = $\rho\Delta x\Delta \mathrm{y}\Delta z$
  - ✓Mass of $\psi = C_\psi \rho\Delta x\Delta \mathrm{y}\Delta z$
  - ✓Accumulation rate = $\frac{\partial(\rho C_\psi)}{\partial t}\Delta x\Delta \mathrm{y}\Delta z$

Accumulation = Source - Outflow

- $\frac{\partial(\rho C_\psi)}{\partial t}\Delta x \Delta \mathrm{y} \Delta z = S_\psi \Delta x \Delta \mathrm{y} \Delta z - (C_\psi \rho u_x - D\frac{\partial C_\psi}{\partial x})\Delta \mathrm{y} \Delta z$
- Divide by $\Delta x \Delta \mathrm{y} \Delta z$ and $\Delta x \rightarrow 0$
- $\frac{\partial(\rho C_\psi)}{\partial t} + \frac{\partial\left(C_\psi \rho u_x - D\frac{\partial C_\psi}{\partial x}\right)}{\partial x} = S_\psi$
- Expand to 3D
- $\frac{\partial(\rho C_\psi)}{\partial t} + \nabla\cdot\left(C_\psi \rho u - D\nabla C_\psi\right) = S_\psi$
- $\frac{\partial(\rho C_\psi)}{\partial t} + \nabla\cdot\left(C_\psi \rho u\right) = \nabla\cdot\left(D\nabla C_\psi\right) + S_\psi$
- Transient term + Convection term = Diffusion term + Source term