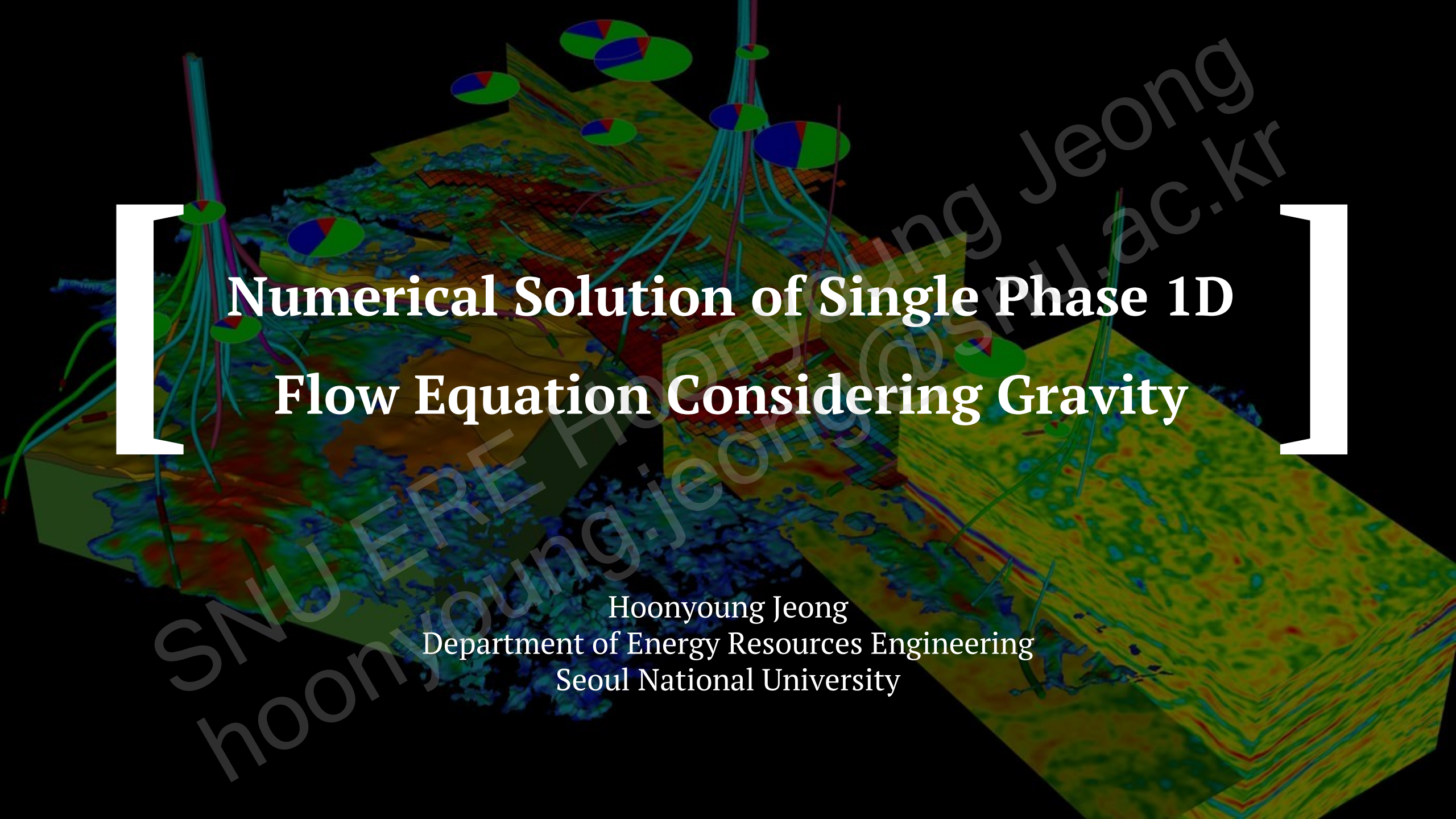

Numerical Solution of Single Phase 1D Flow Equation Considering Gravity

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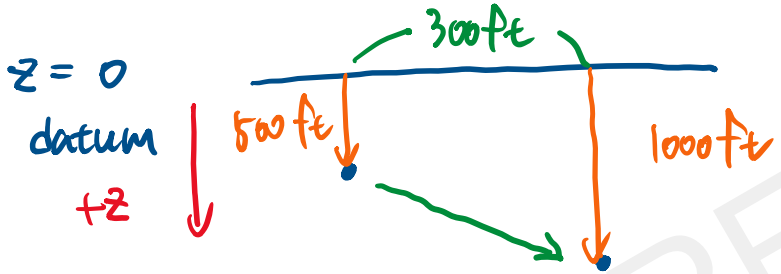
How to Consider Gravity

$$v = -\frac{k}{\mu}\nabla P \xrightarrow{\text{Gravity}} v = -\frac{k}{\mu}\nabla\Phi \; where \; \Phi = \mathrm{P} - \rho g z$$



z : height from a datum
$\mu = 1cp$
$\rho = 8.33ppg$
$k = 100md$

$$v = -\frac{100md}{1cp}\frac{(1000 - 0.052 \cdot 8.33 \cdot 1000) - (900 - 0.052 \cdot 8.33 \cdot 500)}{300ft}$$

$$\nabla \cdot (\rho\vec{v}) + \frac{\partial(\rho\phi)}{\partial t} = 0$$

$$\vec{v} = -\frac{k}{\mu}\nabla\Phi$$

$$\rightarrow \frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial\Phi}{\partial x}\right) + \frac{Q}{\partial x \partial y \partial z} = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)$$

Formulation

$$T_{i+1/2}\left(P_{i+1}^{t+\Delta t}-P_i^{t+\Delta t}\right)+T_{i-1/2}\left(P_{i-1}^{t+\Delta t}-P_i^{t+\Delta t}\right)+\frac{Q_i}{A\Delta x_i}$$
$$=C_{P,i}\left(P_i^{t+\Delta t}-P_i^{t}\right)$$

Considering gravity,

$$T_{i+1/2}\left(\Phi_{i+1}^{t+\Delta t}-\Phi_i^{t+\Delta t}\right)+T_{i-1/2}\left(\Phi_{i-1}^{t+\Delta t}-\Phi_i^{t+\Delta t}\right)+\frac{Q_i}{A\Delta x_i}$$
$$=C_{P,i}\left(P_i^{t+\Delta t}-P_i^{t}\right)$$

$$\Phi_i^{t+\Delta t}=P_i^{t+\Delta t}-\rho_i^{t+\Delta t}gz_i\approx P_i^{t+\Delta t}-\rho_i^{t}gz_i$$
$$\Phi_{i+1}^{t+\Delta t}=P_{i+1}^{t+\Delta t}-\rho_{i+1}^{t}gz_{i+1}$$
$$\Phi_{i-1}^{t+\Delta t}=P_{i-1}^{t+\Delta t}-\rho_{i-1}^{t+\Delta t}gz_{i-1}$$

$\rho_{i+1/2}^{t}$

$$\approx P_{i+1}^{t+\Delta t}-P_i^{t+\Delta t}-\frac{\rho_{i+1}^{t}+\rho_i^{t}}{2}g(z_{i+1}-z_i)$$

$$\approx P_{i-1}^{t+\Delta t}-P_i^{t+\Delta t}-\frac{\rho_{i-1}^{t}+\rho_i^{t}}{2}g(z_{i-1}-z_i)$$

$\rho_{i-1/2}^{t}$

a_i b_i c_i

$$T_{i-1/2}P_{i-1}^{t+\Delta t}+\left(-T_{i+1/2}-T_{i-1/2}-C_{P,i}\right)P_i^{t+\Delta t}+T_{i+1/2}P_{i+1}^{t+\Delta t}$$
$$=-C_{P,i}P_i^{t}-\frac{Q_i}{A\Delta x}+T_{i+1/2}\rho_{i+1/2}g(z_{i+1}-z_i)+T_{i-1/2}\rho_{i-1/2}g(z_{i-1}-z_i)$$

d_i