

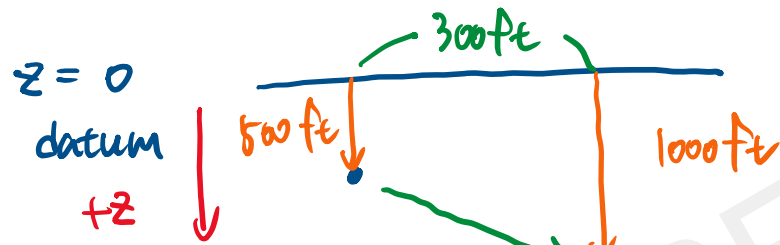
A 3D visualization of a reservoir with several wells. The reservoir is shown in a cross-section with different layers. The wells are represented by vertical lines with pie charts at the top, indicating the production or injection rates of different fluids. The background is dark, and the reservoir is illuminated with a color gradient from blue to green to yellow.

Numerical Solution of Single Phase 1D Flow Equation Considering Gravity

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How to Consider Gravity

$$v = -\frac{k}{\mu} \nabla P \quad \xrightarrow{\text{Gravity}} \quad v = -\frac{k}{\mu} \nabla \Phi \text{ where } \Phi = P - \rho g z$$



z : height from a datum

$$\mu = 1cp$$

$$\rho = 8.33ppg$$

$$k = 100md$$

$$v = -\frac{100md (1000 - 0.052 \cdot 8.33 \cdot 1000) - (900 - 0.052 \cdot 8.33 \cdot 500)}{1cp \cdot 300ft}$$

$$\nabla \cdot (\rho \vec{v}) + \frac{\partial(\rho\phi)}{\partial t} = 0$$

$$\vec{v} = -\frac{k}{\mu} \nabla \Phi$$

$$\rightarrow \frac{\partial}{\partial x} \left(\frac{k}{\mu B} \frac{\partial \Phi}{\partial x} \right) + \frac{Q}{\partial x \partial y \partial z} = \frac{\partial}{\partial t} \left(\frac{\phi}{B} \right)$$

Formulation

$$T_{i+1/2}(P_{i+1}^{t+\Delta t} - P_i^{t+\Delta t}) + T_{i-1/2}(P_{i-1}^{t+\Delta t} - P_i^{t+\Delta t}) + \frac{Q_i}{A\Delta x_i}$$

$$= C_{P,i}(P_i^{t+\Delta t} - P_i^t)$$

Considering gravity,

$$T_{i+1/2}(\Phi_{i+1}^{t+\Delta t} - \Phi_i^{t+\Delta t}) + T_{i-1/2}(\Phi_{i-1}^{t+\Delta t} - \Phi_i^{t+\Delta t}) + \frac{Q_i}{A\Delta x_i}$$

$$= C_{P,i}(P_i^{t+\Delta t} - P_i^t)$$

$$\Phi_i^{t+\Delta t} = P_i^{t+\Delta t} - \rho_i^{t+\Delta t} g z_i \approx P_i^{t+\Delta t} - \rho_i^t g z_i$$

$$\Phi_{i+1}^{t+\Delta t} = P_{i+1}^{t+\Delta t} - \rho_{i+1}^t g z_{i+1}$$

$$\Phi_{i-1}^{t+\Delta t} = P_{i-1}^{t+\Delta t} - \rho_{i-1}^{t+\Delta t} g z_{i-1}$$

$$\approx P_{i+1}^{t+\Delta t} - P_i^{t+\Delta t} - \frac{\rho_{i+1}^t + \rho_i^t}{2} g (z_{i+1} - z_i)$$

$$\approx P_{i-1}^{t+\Delta t} - P_i^{t+\Delta t} - \frac{\rho_{i-1}^t + \rho_i^t}{2} g (z_{i-1} - z_i)$$

$$a_i P_{i-1}^{t+\Delta t} + (b_i - C_{P,i}) P_i^{t+\Delta t} + c_i P_{i+1}^{t+\Delta t}$$

$$= -C_{P,i} P_i^t - \frac{Q_i}{A\Delta x} + T_{i+1/2} \rho_{i+1/2} g (z_{i+1} - z_i) + T_{i-1/2} \rho_{i-1/2} g (z_{i-1} - z_i)$$

d_i