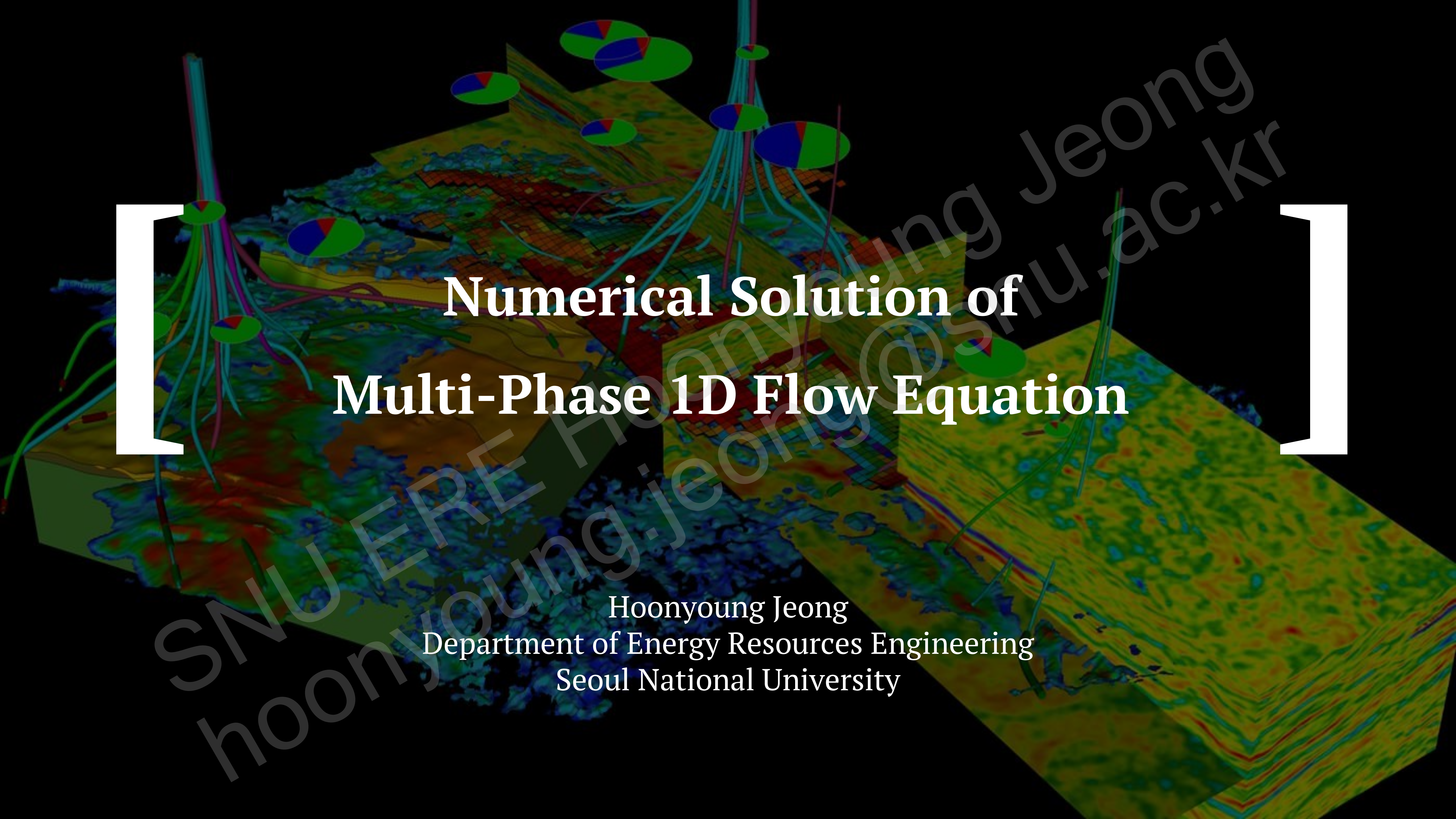

Numerical Solution of Multi-Phase 1D Flow Equation

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Multiphase

- More than two phases that are immiscible
  - ✓ What does immiscible flow mean?
    - They flow separately
    - But it doesn't mean they have independent flow
      - ❖ Competition
      - ❖ Relative permeability
        - ➢ k_r is a function of saturation
        - ➢ More oil ➔ Higher So ➔ Oil flows better
        - ➢ More water ➔ Higher Sw ➔ Water flows better

- Wettability
  - ✓ Wetting / Non-wetting phases
    - In water-wet rock, wettability btn water and rock > wettability btn oil and rock
  - ✓ Capillary pressure

Oil and Water

- Immiscible oil and water
  - ✓Relative permeability
  - ✓Caprillary pressure

Continuity Equations

- $\nabla \cdot (\rho \vec{V}) + \frac{\partial}{\partial t}(\rho \phi) = 0$
- Both oil and water mass balances are satisfied separately.
- $\nabla \cdot (\rho_o \vec{V}_o) + \frac{\partial}{\partial t}(\rho_o \phi S_o) = 0$
- $\nabla \cdot (\rho_w \vec{V}_w) + \frac{\partial}{\partial t}(\rho_w \phi S_w) = 0$

Darcy's Law

- $\vec{V} = -\frac{k}{\mu}\nabla P$
- Oil and water move separately, but their k_r depends on S_o and S_w.
- $\overrightarrow{V_o} = -\frac{k k_{ro}}{\mu_o}\nabla P_o$
- $\overrightarrow{V_w} = -\frac{k k_{rw}}{\mu_w}\nabla P_w$
- $P_c = P_o - P_w$ for water-wet rock
- $P_c = P_w - P_o$ for oil-wet rock

] $P_c = f(S_w \; or \; S_o)$

Constitutive Equations

- $C_f = \frac{1}{\rho_f}\frac{\partial \rho_f}{\partial P}$, $C_r = \frac{1}{\phi}\frac{\partial \phi}{\partial P}$
- $C_o = \frac{1}{\rho_o}\frac{\partial \rho_o}{\partial P_o} = B_o \frac{\partial}{\partial P_o}\left(\frac{1}{B_o}\right)$
- $C_w = \frac{1}{\rho_w}\frac{\partial \rho_w}{\partial P_w} = B_w \frac{\partial}{\partial P_w}\left(\frac{1}{B_w}\right)$
- $C_r = \frac{1}{\phi}\frac{\partial \phi}{\partial P_o}$
- $C_t = C_r + S_o C_o + S_w C_w$

Initial & Boundary Conditions

- Initial pressure : $\rho_o gh, \rho_w gh$
- Initial saturation : calculate S_w using P_c

$$S_o = 1 - S_w$$

→ If heterogeneous rock, this calculation might be inaccurate.
estimate from core samples, well log data, using geostatistics.

Boundary conditions
constant BHP, constant Q
→ BHP, $Q_O, Q_W, Q_O + Q_W$

Multiphase Equations

$S = 1$

$\rightarrow S_o + S_w = 1,$ Two assumptions

1) undersaturated reservoir

$P > P_b \rightarrow$ no gas

2) gas is liberated in surface

$k_{ro}, k_{rw} = f(S_o \ or\ S_w)$

$P_w = P_o - P_c, \quad P_c = f(S_w)$

1) Approximation of Spatial Term

Approximate using $\begin{cases} \left.\dfrac{kk_{ro}}{\mu_o B_o}\dfrac{\partial P_o}{\partial x}\right|_{i-1/2} \\ \left.\dfrac{kk_{ro}}{\mu_o B_o}\dfrac{\partial P_o}{\partial x}\right|_{i+1/2} \end{cases}$

Calculated using upstream

based on the current pressure

$$\lambda_{o,i}^t = \frac{k_{ro,i}^t}{\mu_{o,i}^t B_{o,i}^t}$$

$$\lambda_{o,i+1/2}^t = \begin{cases} \lambda_{o,i}^t, & if\ P_{o,i}^t > P_{o,i+1}^t \\ \lambda_{o,i+1}^t, & if\ P_{o,i}^t < P_{o,i+1}^t \end{cases}$$

Approximate using $P_{o,i-1}^{t+\Delta t}, P_{o,i}^{t+\Delta t}, P_{o,i+1}^{t+\Delta t}$

same for the water flow equation.

2) Approximation of Temporal Term

In a single phase,

$$\frac{\phi_i^t C_{t,i}}{B_i^t \Delta t}\left(P_i^{t+\Delta t} - P_i^t\right)$$

Using analogy,

$$\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)$$

$\phi_i^t, C_{r,i}, C_{o,i}, B_{o,i}^t, \Delta t$

$P_{o,i}^{t+\Delta t}, P_{o,i}^t$

$S_{o,i}^{t+\Delta t}, S_{o,i}^t$ are needed.

3) Build Oil and Water Equations

Obtain the approximate oil flow eq.

Obtain the approximate water flow eq.

2n unknown : $P_{o,i}^{t+\Delta t}, S_{o,i}^{t+\Delta t} \rightarrow n\ eqs.$

2n unknown : $P_{w,i}^{t+\Delta t}, S_{w,i}^{t+\Delta t} \rightarrow n\ eqs.$

4) Combine Oil and Water Equations

Combine the two approximate oil and water flow eqs.

i) $P_{w,i}^{t+\Delta t} = P_{o,i}^{t+\Delta t} - \underline{P_{c,i}^{t+\Delta t}}$

$= P_c\left(\underline{S_{w,i}^{t+\Delta t}}\right)$ unknown, cannot solve

$\approx P_c\left(S_{w,i}^{t}\right)$

ii) $S_{w,i}^{t+\Delta t} = 1 - S_{o,i}^{t+\Delta t}$

4n unknown and 2n eqs.

→ Water flow eq : expressed in terms of $P_{o,i}^{t+\Delta t}$ and $S_{o,i}^{t+\Delta t}$

2n unknown and 2n eqs.

→ n unknown and n eqs. by cancelling $S_{o,i}^{t+\Delta t}$

Others

5) Build a matrix and calculate $P_{o,i}^{t+\Delta t}$

6) Calculate $S_{o,i}^{t+\Delta t}$ by substituting $P_{o,i}^{t+\Delta t}$ into the oil flow eq.

7) Calculate $S_{w,i}^{t+\Delta t}$, $P_{w,i}^{t+\Delta t} = P_{o,i}^{t+\Delta t} - P_c\left(S_{w,i}^{t+\Delta t}\right)$

8) Update $B_o, B_w, \phi, \mu_o, \mu_w$

→ IMPES (Implicit Pressure Explicit Saturation)