[Geomechanical Simulation]

Hoonyoung Jeong
Department of Energy Resources Engineering
Seoul National University

Thermo-Poro-Elastic System

- The rock is
  - ✓Heated
  - ✓Porous
  - ✓Elastic
  - ✓What’s gonna happen?

Rock is Heated

- The rock
  - ✓Wants to expand
  - ✓But it is confined
  - ✓The rock gets a thermal stress
- How much is the rock thermally stressed?
  - ✓β is the linear thermal expansion coefficient
  - ✓The volumetric expansion coefficient is 3β
  - ✓Volumetric strain $= 3\beta(T - T_{ref})$
  - ✓Thermal stress $= 3\beta K(T - T_{ref})$

Rock is Porous

- The rock
  - ✓Has fluid in the porous space
  - ✓And the fluid defies external stresses
- How much is the rock effectively stressed?
  - ✓Total (external) stress – Biot's coefficient * Pore pressure
  - ✓$\sigma' = \sigma - \alpha P$

Rock is Elastic

- The rock deforms elastically
- What is the relation between stresses and elastic strains?
  - ✓ $\sigma_{xx} = 2G\varepsilon_{xx} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
  - ✓ $\sigma_{yy} = 2G\varepsilon_{yy} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
  - ✓ $\sigma_{zz} = 2G\varepsilon_{zz} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
  - ✓ $\sigma_{xy} = \sigma_{yx} = 2G\varepsilon_{xy} = 2G\varepsilon_{yx}$
  - ✓ $\sigma_{yz} = \sigma_{zy} = 2G\varepsilon_{yz} = 2G\varepsilon_{zy}$
  - ✓ $\sigma_{zx} = \sigma_{xz} = 2G\varepsilon_{zx} = 2G\varepsilon_{xz}$

Thermo-Poro-Elastic System

- How much stressed is the rock effectively when the rock is heated, porous, and elastic?
  - ✓ $\sigma' = \sigma - \alpha P + 3\beta K(T - T_{ref})$
  - ✓ Effective stress = Total stress – Pore pressure + Thermal stress
- What is the relation between effective stress and elastic strain when the rock is heated, porous, and elastic?
  - ✓ $\sigma_{xx} - \alpha P + 3\beta K(T - T_{ref}) = 2G\varepsilon_{xx} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
  - ✓ $\sigma_{yy} - \alpha P + 3\beta K(T - T_{ref}) = 2G\varepsilon_{yy} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
  - ✓ $\sigma_{zz} - \alpha P + 3\beta K(T - T_{ref}) = 2G\varepsilon_{zz} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
  - ✓ $\sigma_{xy} = \sigma_{yx} = 2G\varepsilon_{xy} = 2G\varepsilon_{yx}$
  - ✓ $\sigma_{yz} = \sigma_{zy} = 2G\varepsilon_{yz} = 2G\varepsilon_{zy}$
  - ✓ $\sigma_{zx} = \sigma_{xz} = 2G\varepsilon_{zx} = 2G\varepsilon_{xz}$

Mean Normal Stress and Volumetric Strain

1) $\sigma_{xx} - \alpha P + 3\beta K(T - T_{ref}) = 2G\varepsilon_{xx} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
2) $\sigma_{yy} - \alpha P + 3\beta K(T - T_{ref}) = 2G\varepsilon_{yy} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
3) $\sigma_{zz} - \alpha P + 3\beta K(T - T_{ref}) = 2G\varepsilon_{zz} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
4) $\sigma_m = \frac{\sigma_{xx}+\sigma_{yy}+\sigma_{zz}}{3}$
5) Substitute 4) into 1) + 2) + 3)
6) $\sigma_m - \alpha P + 3\beta K(T - T_{ref}) = \left(\lambda + \frac{2}{3}G\right)(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
7) $\sigma_m = \alpha P - 3\beta K(T - T_{ref}) + \left(\lambda + \frac{2}{3}G\right)(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$
8) $\varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\sigma_m - \alpha P + 3\beta K(T - T_{ref})}{\lambda + \frac{2}{3}G}$
9) $\sigma_m = \varepsilon_v(\lambda + \frac{2}{3}G) + \alpha P - 3\beta K(T - T_{ref})$
10) $\Delta\sigma_m = \Delta\varepsilon_v(\lambda + \frac{2}{3}G) + \alpha \Delta P - 3\beta K \Delta T$

Force Equilibrium Equation

- $\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + \rho_b B_x = 0$
- $\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + \rho_b B_y = 0$
- $\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho_b B_z = 0$

Substitution the Constitutive Relation into the Force Equilibrium Equation

- $\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + \rho_b B_x = 0$
- $\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + \rho_b B_y = 0$
- $\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho_b B_z = 0$
- $\sigma_{xx} = \alpha P - 3\beta K\left(T - T_{ref}\right) + 2G\varepsilon_{xx} + \lambda\left(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\right)$
- $\sigma_{yx} = 2G\varepsilon_{yx}$
- $\sigma_{zx} = 2G\varepsilon_{zx}$
- $\alpha \frac{\partial P}{\partial x} - 3\beta K \frac{\partial T}{\partial x} + 2G \frac{\partial \varepsilon_{xx}}{\partial x} + \lambda \frac{\partial\left(\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}\right)}{\partial x} + 2G \frac{\partial \varepsilon_{yx}}{\partial y} + 2G \frac{\partial \varepsilon_{zx}}{\partial z} + \rho_b B_x = 0$
- $\alpha \frac{\partial P}{\partial y} - 3\beta K \frac{\partial T}{\partial y} + 2G \frac{\partial \varepsilon_{yy}}{\partial y} + \lambda \frac{\partial\left(\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}\right)}{\partial y} + 2G \frac{\partial \varepsilon_{xy}}{\partial x} + 2G \frac{\partial \varepsilon_{zy}}{\partial z} + \rho_b B_y = 0$
- $\alpha \frac{\partial P}{\partial z} - 3\beta K \frac{\partial T}{\partial z} + 2G \frac{\partial \varepsilon_{zz}}{\partial z} + \lambda \frac{\partial\left(\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}\right)}{\partial z} + 2G \frac{\partial \varepsilon_{yz}}{\partial y} + 2G \frac{\partial \varepsilon_{xz}}{\partial x} + \rho_b B_z = 0$

Relation between Strain and Displacement

$$\epsilon = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

$\epsilon_{xy} = \epsilon_{yx}$

$$= \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) & \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) \\ \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) & \frac{\partial u_y}{\partial y} & \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) \\ \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

$$\nabla u = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} u_x & u_y & u_z \end{bmatrix} = \begin{bmatrix} \partial u_x/\partial x & \partial u_y/\partial x & \partial u_z/\partial x \\ \partial u_x/\partial y & \partial u_y/\partial y & \partial u_z/\partial y \\ \partial u_x/\partial z & \partial u_y/\partial z & \partial u_z/\partial z \end{bmatrix}$$

$$\epsilon = \frac{1}{2}[\nabla u + \nabla u^T]$$

Strains can be expressed using displacements

Substitution Relation between Strain and Displacement (1)

$$\alpha\frac{\partial P}{\partial x} - 3\beta K\frac{\partial T}{\partial x} + (2G+\lambda)\frac{\partial^2 u_x}{\partial x^2} + \lambda\frac{\partial^2 u_y}{\partial x\partial y} + \lambda\frac{\partial^2 u_z}{\partial x\partial z} + G\left(\frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_y}{\partial x\partial y}\right) + G\left(\frac{\partial^2 u_x}{\partial z^2} + \frac{\partial^2 u_z}{\partial x\partial z}\right) + \rho_b B_x = 0$$

$$\alpha\frac{\partial P}{\partial y} - 3\beta K\frac{\partial T}{\partial y} + (2G+\lambda)\frac{\partial^2 u_y}{\partial y^2} + \lambda\frac{\partial^2 u_x}{\partial x\partial y} + \lambda\frac{\partial^2 u_z}{\partial y\partial z} + G\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_x}{\partial x\partial y}\right) + G\left(\frac{\partial^2 u_y}{\partial z^2} + \frac{\partial^2 u_z}{\partial y\partial z}\right) + \rho_b B_y = 0$$

$$\alpha\frac{\partial P}{\partial z} - 3\beta K\frac{\partial T}{\partial z} + (2G+\lambda)\frac{\partial^2 u_z}{\partial z^2} + \lambda\frac{\partial^2 u_y}{\partial z\partial y} + \lambda\frac{\partial^2 u_x}{\partial x\partial z} + G\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_x}{\partial x\partial z}\right) + G\left(\frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_y}{\partial y\partial z}\right) + \rho_b B_z = 0$$

Substitution Relation between Strain and Displacement (2)

$$\alpha\frac{\partial P}{\partial x} - 3\beta K\frac{\partial T}{\partial x} + (G+\lambda)\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial^2 u_z}{\partial x \partial z}\right) + G\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right) + \rho_b B_x = 0$$

$$\alpha\frac{\partial P}{\partial y} - 3\beta K\frac{\partial T}{\partial y} + (G+\lambda)\left(\frac{\partial^2 u_x}{\partial x \partial y} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_z}{\partial y \partial z}\right) + G\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right) + \rho_b B_y = 0$$

$$\alpha\frac{\partial P}{\partial z} - 3\beta K\frac{\partial T}{\partial z} + (G+\lambda)\left(\frac{\partial^2 u_x}{\partial x \partial z} + \frac{\partial^2 u_y}{\partial y \partial z} + \frac{\partial^2 u_z}{\partial z^2}\right) + G\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right) + \rho_b B_z = 0$$

$$\alpha\nabla P - 3\beta K\nabla T + (\lambda + G)\nabla(\nabla\cdot\bar{u}) + G\nabla^2\bar{u} + \rho_b\bar{B} = 0$$

Rock Deformation and Porosity

Mass Balance : During a very short time,
inlet fluid mass = outlet fluid mass
in a very small rock volume

mathematically →

$$\nabla \cdot (\rho V) + \frac{\partial}{\partial t}(\phi\rho) = \frac{1}{\partial x \partial y \partial z}\frac{\partial(\rho V_{ext})}{\partial t}$$

However, if we consider rock deformation caused by effective stress changes in the rock, then the bulk volume changes.

In the mass balance equation, we assume that the bulk volume(=very small rock volume) is constant, but this is not valid any longer if the rock is deformed.

References

- Pruess, K., Oldenburg, C., Moridis, G., 2012, TOUGH2 USER’S GUIDE, VERSION 2