Constitutive Equations

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Constitutive Equation

- Definition
  - ✓Relation that describes the response of a material that is exposed to external stimuli
- Normally, it is referred to the relation between the deformation of a material and external forces acting on the material
- What are there in rock formations?
  - ✓Rock
  - ✓Fluid
  - ✓Deformation of a rock and a fluid for external forces

External and Internal Forces for Subsurface Rock

- What is the external force for a rock?
  - ✓ Overburden stress
  - ✓ Pore pressure
- What is the internal force for a rock?
  - ✓ Effective stress
  - ✓ The rock is deformed by the internal force
  - ✓ $\sigma_{eff} = \mathrm{E}\varepsilon$
- What is the relation between the internal and external forces?
  - ✓ Overburden stress = Pore pressure + Effective stress

출처: CMG education material

응력이란?

- 강체는 변형되지 않으므로 강체 내부 힘을 고려하지 않고 단순히 무게 중심에 미치는 힘으로 운동을 모사할 수 있음
- Deformable bodies는 외부 힘에 의해 내부 힘이 발생하고 내부 힘으로 인해 변형이 발생하므로 내부 힘을 고려해야 함
- 응력
  - ✓외부 힘에 의해 물질 내부에 발생하는 단위 면적당 내력

$$\text{stress} = \lim_{\Delta A \to 0} \frac{\Delta \mathbf{F}}{\Delta A}$$

출처: 고체역학, 포항공과대학교, 박성진 교수

Rock Deformation By Change of Pore Pressure

- Ex)
  - ✓CO_2 injection ➔ Pore pressure increases ➔ Effective stresses decrease (total stresses are constant) ➔ Rock deformation
  - ✓Rock deformation
    - Deformation of rock grains
    - Deformation of pores ➔ Porosity changes
    - Which one is larger?
      - ❖ Pore volume change is significantly larger
  - ✓Which one of pore pressure and effective stress is easier to measure?
    - Pore pressure
    - Effective stress = Total stress (constant) – Pore pressure
    - Easier to express the change of porosity as a function of pore pressure