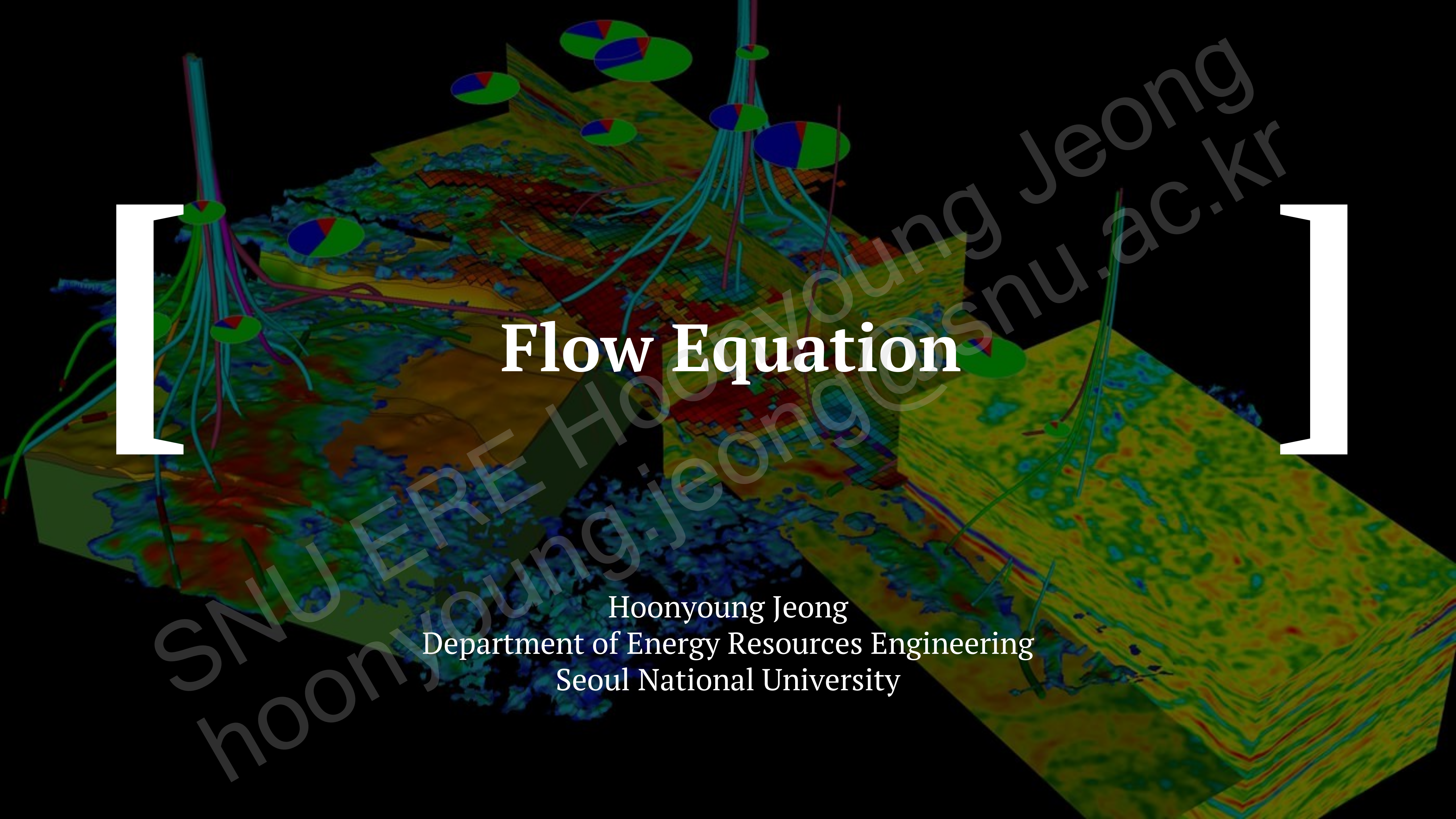

Flow Equation

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Things that are needed to Derive a Flow Equation (for Reservoir Simulation)

- Reservoir structure
  - ✓1-dimension
- Fluid properties
  - ✓Single phase
  - ✓Fluid compressibility (slightly compressible fluid)
- Rock properties
  - ✓Porosity
  - ✓Permeability
  - ✓Rock compressibility (slightly compressible rock)
- Properties of rock and fluid
  - ✓Capillary pressure
  - ✓Relative permeability
- Basic equations
  - ✓Continuity equation
  - ✓Constitutive equation
  - ✓Darcy's law
- Boundary and initial conditions

1-dimension

- $\nabla \cdot (\rho u) + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x \partial y \partial z} \frac{\partial(\rho V_{ext})}{\partial t}$
- $u = -\frac{k}{\mu} \nabla P$
- $\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x \partial y \partial z} \frac{\partial(\rho V_{ext})}{\partial t}$
- $u_x = -\frac{k}{\mu} \frac{\partial P}{\partial x}$

Constitutive Equations

- $\mathrm{C_r} = \frac{1}{\phi}\frac{\partial \phi}{\partial P}$
- $\mathrm{C_f} = \frac{1}{\rho}\frac{\partial \rho}{\partial P}$

Substitution of Darcy's Law

- $\frac{\partial}{\partial x}\left(-\frac{k\rho}{\mu}\frac{\partial P}{\partial x}\right)+\frac{\partial(\phi\rho)}{\partial t}=\frac{1}{\partial x\partial y\partial z}\frac{\partial(\rho V_{ext})}{\partial t}$
- $\frac{\partial}{\partial x}\left(\frac{k\rho}{\mu}\frac{\partial P}{\partial x}\right)+\frac{1}{\partial x\partial y\partial z}\frac{\partial(\rho V_{ext})}{\partial t}=\frac{\partial(\phi\rho)}{\partial t}$

Flow Rate

- $\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right) + \frac{1}{\partial x \partial y \partial z}\frac{\partial\left(\frac{V_{ext}}{B}\right)}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)$
- $\partial y \partial z = A$
- $\frac{V_{ext}}{B} = V_{ext}\ at\ surface$
- $\frac{\partial V_{ext}\ at\ surface}{\partial t} = Q$
  - ✓Surface flow rate: inj +, prod -
- $\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right) + \frac{Q}{A \partial x} = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)$