

A 3D visualization of a reservoir with multiple wells and flow paths. The reservoir is shown as a layered structure with different colors representing different rock types or fluid saturations. Wells are represented by vertical lines with colored tops. Flow paths are shown as colored lines connecting the wells. The visualization is framed by large white brackets on the left and right sides.

Flow Equation

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Things that are needed to Derive a Flow Equation (for Reservoir Simulation)

- Reservoir structure
 - ✓ 1-dimension
- Fluid properties
 - ✓ Single phase
 - ✓ Fluid compressibility (slightly compressible fluid)
- Rock properties
 - ✓ Porosity
 - ✓ Permeability
 - ✓ Rock compressibility (slightly compressible rock)
- Properties of rock and fluid
 - ✓ Capillary pressure
 - ✓ Relative permeability
- Basic equations
 - ✓ Continuity equation
 - ✓ Constitutive equation
 - ✓ Darcy's law
- Boundary and initial conditions

1-dimension

$$\bullet \nabla \cdot (\rho u) + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x \partial y \partial z} \frac{\partial(\rho V_{ext})}{\partial t}$$

$$\bullet u = -\frac{k}{\mu} \nabla P$$

$$\bullet \frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x \partial y \partial z} \frac{\partial(\rho V_{ext})}{\partial t}$$

$$\bullet u_x = -\frac{k}{\mu} \frac{\partial P}{\partial x}$$

Constitutive Equations

- $C_r = \frac{1}{\phi} \frac{\partial \phi}{\partial P}$

- $C_f = \frac{1}{\rho} \frac{\partial \rho}{\partial P}$

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Substitution of Darcy's Law

$$\bullet \frac{\partial}{\partial x} \left(-\frac{k\rho}{\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial(\phi\rho)}{\partial t} = \frac{1}{\partial x \partial y \partial z} \frac{\partial(\rho V_{ext})}{\partial t}$$

$$\bullet \frac{\partial}{\partial x} \left(\frac{k\rho}{\mu} \frac{\partial P}{\partial x} \right) + \frac{1}{\partial x \partial y \partial z} \frac{\partial(\rho V_{ext})}{\partial t} = \frac{\partial(\phi\rho)}{\partial t}$$

Flow Rate

- $$\frac{\partial}{\partial x} \left(\frac{k}{\mu B} \frac{\partial P}{\partial x} \right) + \frac{1}{\partial x \partial y \partial z} \frac{\partial \left(\frac{V_{ext}}{B} \right)}{\partial t} = \frac{\partial}{\partial t} \left(\frac{\phi}{B} \right)$$

- $\partial y \partial z = A$

- $\frac{V_{ext}}{B} = V_{ext}$ at surface

- $$\frac{\partial V_{ext} \text{ at surface}}{\partial t} = Q$$

✓ Surface flow rate: inj +, prod -

- $$\frac{\partial}{\partial x} \left(\frac{k}{\mu B} \frac{\partial P}{\partial x} \right) + \frac{Q}{A \partial x} = \frac{\partial}{\partial t} \left(\frac{\phi}{B} \right)$$