A 3D visualization of a reservoir with multiple wells. The reservoir is shown as a layered block with a color gradient from blue to green. Several wells are depicted as vertical lines with pie charts at their tops, representing different flow components. The pie charts are divided into green, blue, and red segments. The entire scene is set against a black background.

Analytical Solution of 1D Single Phase Flow Equation

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Assumptions in Analytical Solution of 1D Flow Equation

- Homogeneous rock
 - ✓ Constant porosity and permeability
 - ✓ How different from isotropy?
- Constant fluid viscosity

Things that are needed to Derive a Flow Equation (for Reservoir Simulation)

- Reservoir structure
 - ✓ 1-dimension
- Fluid properties
 - ✓ Single phase
 - ✓ Fluid compressibility (slightly compressible fluid)
- Rock properties
 - ✓ Porosity
 - ✓ Permeability
 - ✓ Rock compressibility (slightly compressible rock)
- Properties of rock and fluid
 - ✓ Capillary pressure
 - ✓ Relative permeability
- Governing equation
 - ✓ Continuity equation
 - ✓ Constitutive equation
 - ✓ Darcy's law
- Boundary and initial conditions

1D Flow Equation

- $$\frac{\partial}{\partial x} \left(\frac{k\rho}{\mu} \frac{\partial P}{\partial x} \right) + \frac{1}{\partial x \partial y \partial z} \frac{\partial(\rho V_{ext})}{\partial t} = \frac{\partial(\phi\rho)}{\partial t}$$
- Use the Dirichlet condition for P
 - ✓ A location with V_{ext} is assumed to have infinite volume
 - ✓ V_{ext} does not affect P, so it can be ignored
- $$\frac{\partial}{\partial x} \left(\frac{k\rho}{\mu} \frac{\partial P}{\partial x} \right) = \frac{\partial(\phi\rho)}{\partial t}$$
- $$\frac{k}{\mu} \left(\frac{\partial\rho}{\partial x} \frac{\partial P}{\partial x} + \rho \frac{\partial^2 P}{\partial x^2} \right) = \rho \frac{\partial\phi}{\partial t} + \phi \frac{\partial\rho}{\partial t}$$

Analytical Solution

If $P(x = 0, t) = P_0$

$$P(x = L, t) = P_L$$

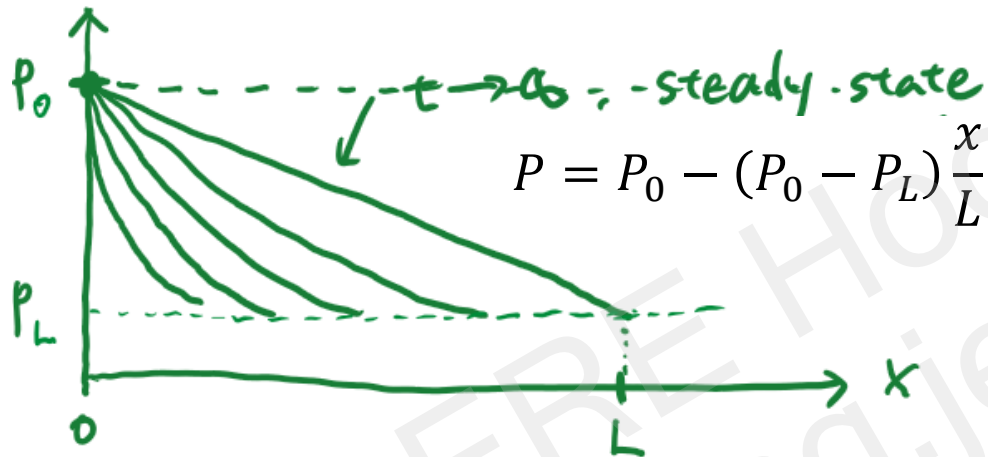
$$P(x, t = 0) = P_L$$



$$P(x, t) = P_0 + (P_L - P_0) \left[\frac{x}{L} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \exp\left(-\frac{n^2 \pi^2 k}{L^2 \phi \mu C_t} t\right) \sin\left(\frac{n\pi x}{L}\right) \right]$$

Sensitivity Analysis

- If $P_0 > P_L$



- Guess physically and mathematically
 - ✓ What if k is larger?
 - ✓ What if porosity is larger?
 - ✓ What if C_t is larger?
 - ✓ What if μ is larger?

$$\frac{\partial^2 P}{\partial x^2} = \frac{\phi \mu C_t}{k} \frac{\partial P}{\partial t}$$