[Analytical Solution of 1D Single Phase Flow Equation]

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Assumptions in Analytical Solution of 1D Flow Equation

- Homogeneous rock
  - ✓Constant porosity and permeability
  - ✓How different from isotropy?
- Constant fluid viscosity

Things that are needed to Derive a Flow Equation (for Reservoir Simulation)

- Reservoir structure
  - ✓1-dimension
- Fluid properties
  - ✓Single phase
  - ✓Fluid compressibility (slightly compressible fluid)
- Rock properties
  - ✓Porosity
  - ✓Permeability
  - ✓Rock compressibility (slightly compressible rock)
- Properties of rock and fluid
  - ✓Capillary pressure
  - ✓Relative permeability
- Governing equation
  - ✓Continuity equation
  - ✓Constitutive equation
  - ✓Darcy's law
- Boundary and initial conditions

1D Flow Equation

- $\frac{\partial}{\partial x}\left(\frac{k\rho}{\mu}\frac{\partial P}{\partial x}\right) + \frac{1}{\partial x \partial y \partial z}\frac{\partial(\rho V_{ext})}{\partial t} = \frac{\partial(\phi\rho)}{\partial t}$
- Use the Dirichlet condition for P
  - ✓A location with V_{ext} is assumed to have infinite volume
  - ✓V_{ext} does not affect P, so it can be ignored

- $\frac{\partial}{\partial x}\left(\frac{k\rho}{\mu}\frac{\partial P}{\partial x}\right) = \frac{\partial(\phi\rho)}{\partial t}$

- $\frac{k}{\mu}\left(\frac{\partial\rho}{\partial x}\frac{\partial P}{\partial x} + \rho\frac{\partial^2 P}{\partial x^2}\right) = \rho\frac{\partial\phi}{\partial t} + \phi\frac{\partial\rho}{\partial t}$

Analytical Solution

If $P(x = 0, t) = P_0$

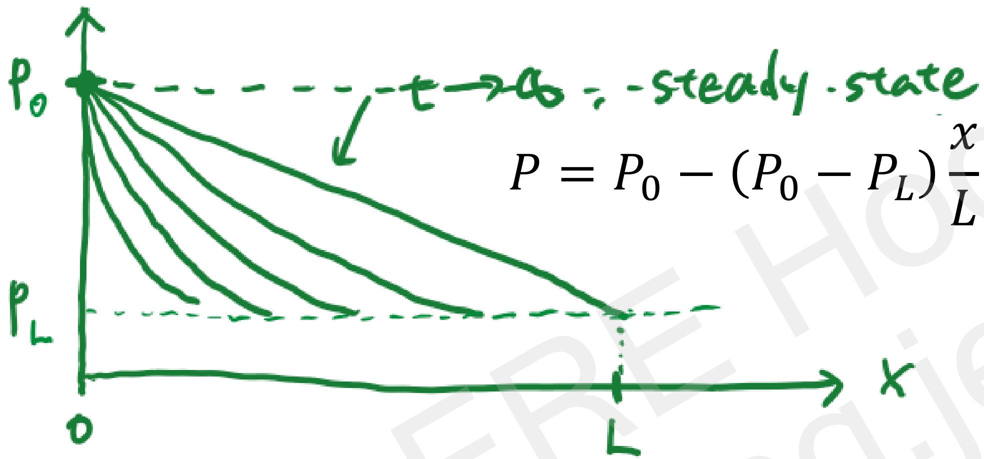
$P(x = L, t) = P_L$

$P(x, t = 0) = P_L$

$$P(x,t) = P_0 + (P_L - P_0)\left[\frac{x}{L} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2}{L^2}\frac{k}{\phi\mu C_t}t\right)\sin\left(\frac{n\pi x}{L}\right)\right]$$

Sensitivity Analysis

- If $P_0 > P_L$



$$P = P_0 - (P_0 - P_L)\frac{x}{L}$$

- Guess physically and mathematically
  - ✓ What if k is larger?
  - ✓ What if porosity is larger?
  - ✓ What if Ct is larger?
  - ✓ What if μ is larger?

$$\frac{\partial^2 P}{\partial x^2} = \frac{\phi \mu C_t}{k}\frac{\partial P}{\partial t}$$