Containers, Relations, and ADTs

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Outline (Book chapter 3.1)

- This topic will discuss
  - The storage of objects in *containers*
  - We will focus on linear orderings:
    - Implicitly defined linear orderings (sorted lists)
    - Explicitly defined linear orderings
  - We will summarize this information
  - We will also look briefly at:
    - Hierarchical orderings
    - Partial orderings
    - Equivalence relations
    - Adjacency relations

Outline

- Any form of information processing or communication requires that **data** must be **stored** in and **accessed** from either main or secondary **memory**

- This topic will cover Abstract Data Types:
  - Models of the storage and access of information
- The next topic will cover data structures and algorithms:
  - The concrete methods for organizing and accessing data in the computer

Containers

- The most general Abstract Data Type (ADT) is that of a *container*
  - The Container ADT

- A container describes structures that store and give access to objects

- The queries and operations of interest may be defined on:
  - The container as an entity, or
  - The objects stored within a container

Operations on a Container

- The operations we may wish to perform on a container are:
  - Create a new container
  - Copy or destroy an existing container
  - Empty a container
  - Query how many objects are in a container
  - Query what is the maximum number of objects a container can hold
  - Given two containers:
    - Find the union (merge), or
    - Find the intersection

Operations on a Container

- Many of these operations on containers are in the Standard Template Library

Constructor	`Container()`
Copy Constructor	`Container( Container const & )`
Destructor	`~Container()`
Empty it	`void clear()`
How many objects are in it?	`int size() const`
Is it empty?	`bool empty() const`
How many objects can it hold?	`int max_size() const`
Merge with another container	`void insert( Container const & )`

Check more: https://en.cppreference.com/w/cpp/container

Operations on Objects Stored in a Container

- Given a container, we may wish to:
  - Insert an object into a container
  - Access or modify an object in a container
  - Remove an object from the container
  - Query if an object is in the container
    - If applicable, count how many copies of an object are in a container
  - Iterate (step through) the objects in a container

Operations on Objects Stored in a Container

- Many of these operations are also common to the Standard Template Library

Insert an object	`void insert( Type const & )`
Erase an object	`void erase( Type const & )`
Find or access an object	`iterator find( Type const & )`
Count the number of copies	`int count( Type const & )`
Iterate through the objects in a container	`iterator begin() const`

Simple and Associative Containers

- We may split containers into two general classifications:

Simple Containers

Containers that store individual objects

Associative Containers

Containers that store keys but also store records associated with those keys

Temperature values

Circular Array

SNU ID Number

mysnu Server

Student Academic Record

Unique or Duplicate Objects

- Another design requirement may be to either:
  - Require that all objects in the container are unique, or
  - Allow duplicate objects
- Many of the containers we will look at will assume uniqueness unless otherwise stated
  - Dealing with duplicate objects is often just additional, and sometimes subtle, code

Standard Template Library

- We will begin by introducing four containers from the C++ Standard Template Library (STL)

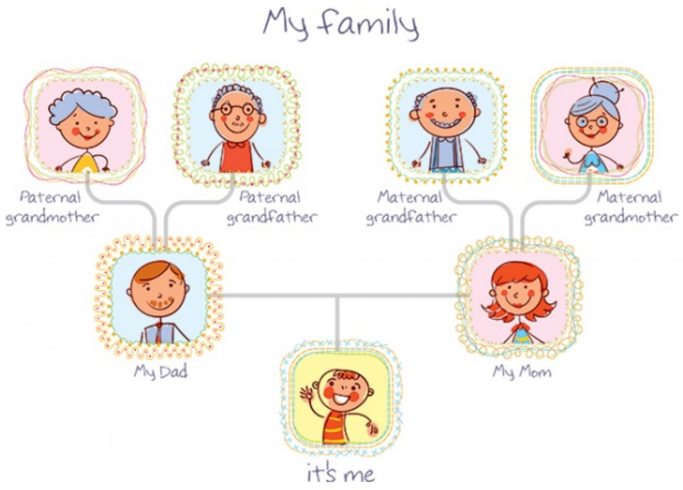
Unique Objects/Keys	Duplicate Objects/Keys
`set<Type>`	`multiset<Type>`
`map<Key_type, Type>`	`multimap<Key_type, Type>`

The STL set Container

```
#include <iostream>
#include <set>

int main() {
    std::set<int> ints;

    for ( int i = -100; i <= 100; ++i ) {
        ints.insert( i*i );         // Ignores duplicates:  (-3)*(-3) == 3*3
    }                               // Inserts 101 values: 0, 1, 4, 9, ..., 10000

    std::cout << "Size of 'is': " << ints.size() << std::endl;  // Prints 101

    ints.erase( 50 );                    // Does nothing
    ints.erase( 9 );                     // Removes 9
    std::cout << "Size of 'is': " << ints.size() << std::endl;  // Prints 100

    return 0;
}
```

Operations with Relationships

- We may want to store not only objects, but relationships between the objects
  - Consequently, we may have additional operations based on the relationships
  - Consider a genealogical database
    - We don't only want to store the people, but we want to also make queries about the relationships between the people



https://www.sciencenewsforstudents.org/

Operations w/o Relationships

- If we are not storing relationships, there is a data structure that is always the same speed no matter how many objects are stored
  - A hash table roughly takes the same time to find an object whether there are 10 or one billion objects in the container
  - Example:
    - Assume a department has 12 staffs that are frequently changing
    - Rather than having a mailbox for each person, have 24 mailboxes and place mail into the *bin* corresponding to the person's last name

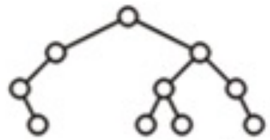
A	E	I	M	R	V
B	F	J	M	S	W
C	G	K	N	T	XY
D	H	L	PQ	U	Z

- This works fine as long as there is not too much collisions

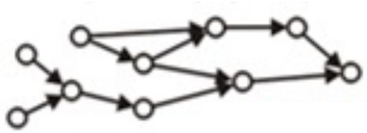
Relationships

- We will look at four relationships:
  - Linear orderings

  - Hierarchical orderings



  - Partial orderings



  - Adjacency relations

Relationships

- Relationships are often Boolean-valued binary operations

- Example: given two integers:
  - Are they equal? $x = y$
  - Is one less than the other? $x < y$
  - Does the first divide the second? $x \mid y$
  - Do they have the same remainder modulo 16? $x \underset{16}{\equiv} y$

Linear Orderings

- A linear ordering is any relationship where any two objects x and y that can be compared, exactly one of:

  $$x < y \text{ , } x = y\text{, or } y < x$$

  is true and where the relation is transitive
  - Such a relation is therefore anti-symmetric
  - Any collection can therefore be **sorted** according to this relation

- Examples of sets which are linearly ordered include:
  - Integers 1, 2, 3, 4, 5, 6, ...
  - Real numbers 1.2, 1.2001, 1.24, 1.35, 2.78, ...
  - The alphabet a, b, c, d, e, f, ..., x, y, z
  - Memory address `0x00000000, 0x00000001, ..., 0xFFFFFFFF`

- We could store linearly ordered sets using arrays or linked lists

Operations on Linear Orderings

- Queries that may be asked about linear orderings:
  - What are the first and last objects (the *front* and the *back*)?
  - What is the k^{th} object?
  - What are all objects in a given interval $[a, b]$
  - Given a reference to one object in the container:
    - What are the previous and next objects?

- Operations that may be performed as a result:
  - Sort a collection of objects
  - Insert an object into a sorted list
  - Insert an object at either the front, the back, or into the k^{th} position

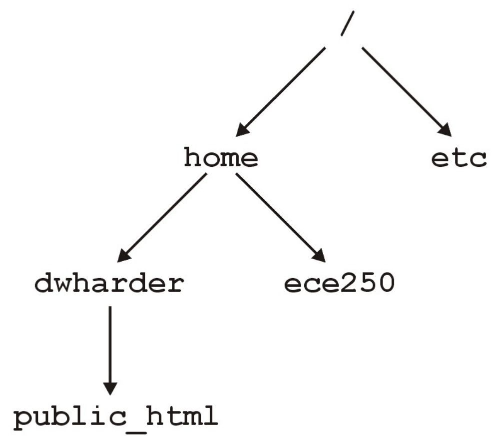
- You will learn how to do this throughout the semester!

Hierarchical Orderings

- Consider directories in a file system:

  $x \prec y$ if x contains y within one of its subdirectories

  - In Unix, there is a single root directory /

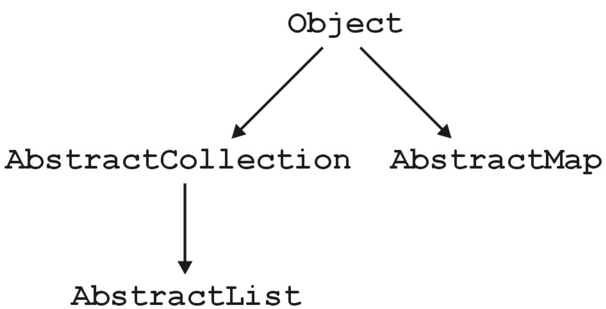


- Such structures allow us to organize information

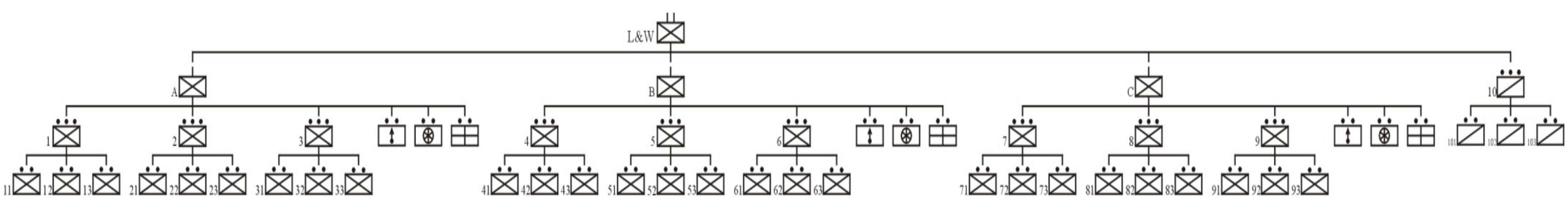
Hierarchical Orderings

- Other examples:

Classes in Java and C#




```
int f() {
    int a;
    {
        int b;
        {
            int c;
        }
    }

    {
        int d;
    }
    return a;
}
```



Operations on Hierarchical Orders

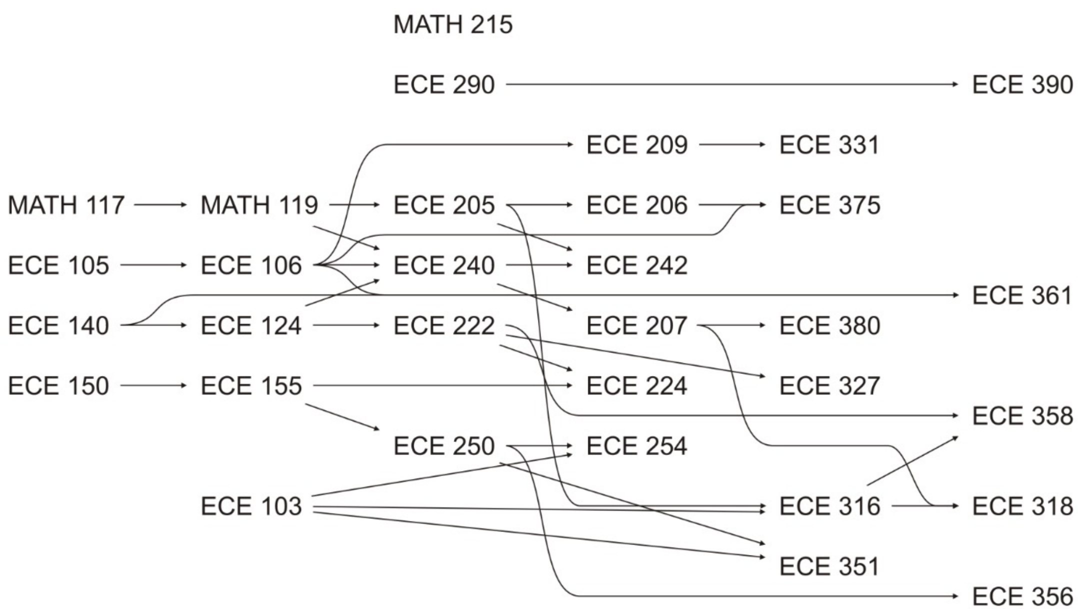
- If the hierarchical order is explicitly defined (the usual case), given two objects in the container, we may ask:
  - Does one object contains the other?
  - Are both objects at the same depth?
  - What is the nearest common predecessor?

Partial Orderings

- Partial orderings are denoted as

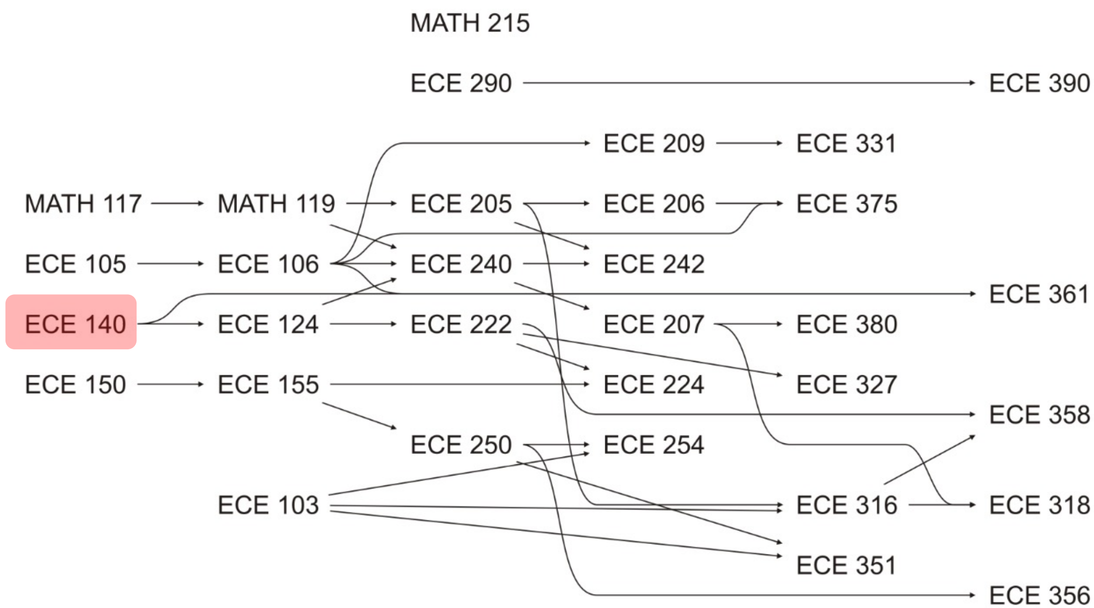
$$x \prec y \text{ if } x \text{ is a prerequisite of } y$$

This is NOT a hierarchy, as there are multiple starting points and one class may have multiple prerequisites



Partial Orderings

- Arrows are necessary to indicate the direction:
  - Having completed ECE 140, you can now take ECE 124 and ECE 361



Operations on Partial Orderings

- Partial orders are similar to hierarchical orders; consequently, some operations are similar:
  - Given two objects, does one precede the other?
  - Which objects have no predecessors?
  - Which objects are immediately preceded by an object?
    - A hierarchical order has only one immediate predecessor
  - Which objects immediately succeed an object?

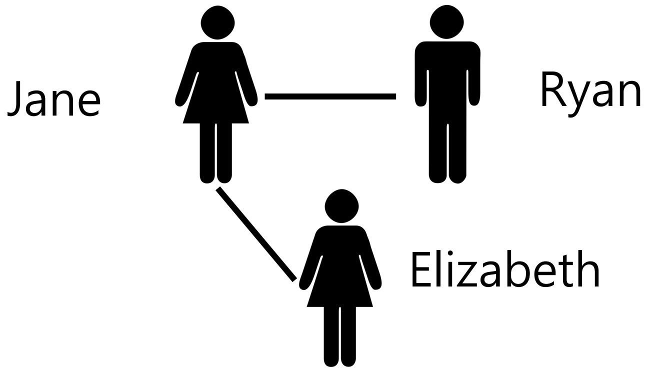
Adjacency Relations

- Adjacency relations can be represented as:

$$x \leftrightarrow y \text{ if } x \text{ and } y \text{ are friends}$$

- Like a tree, we will display such a relationship by displaying a line connecting two individuals if they are friends (a *graph*)

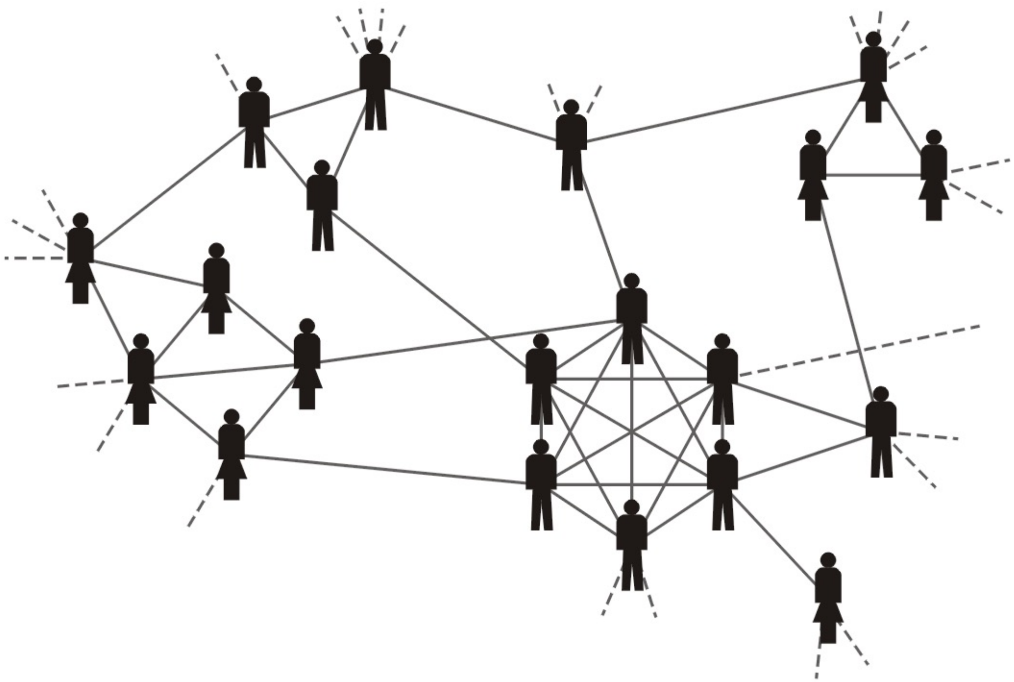
  e.g., Jane and Ryan are friends, Elizabeth and Jane are friends, but Elizabeth thinks Ryan is a little odd...



Adjacency Relations

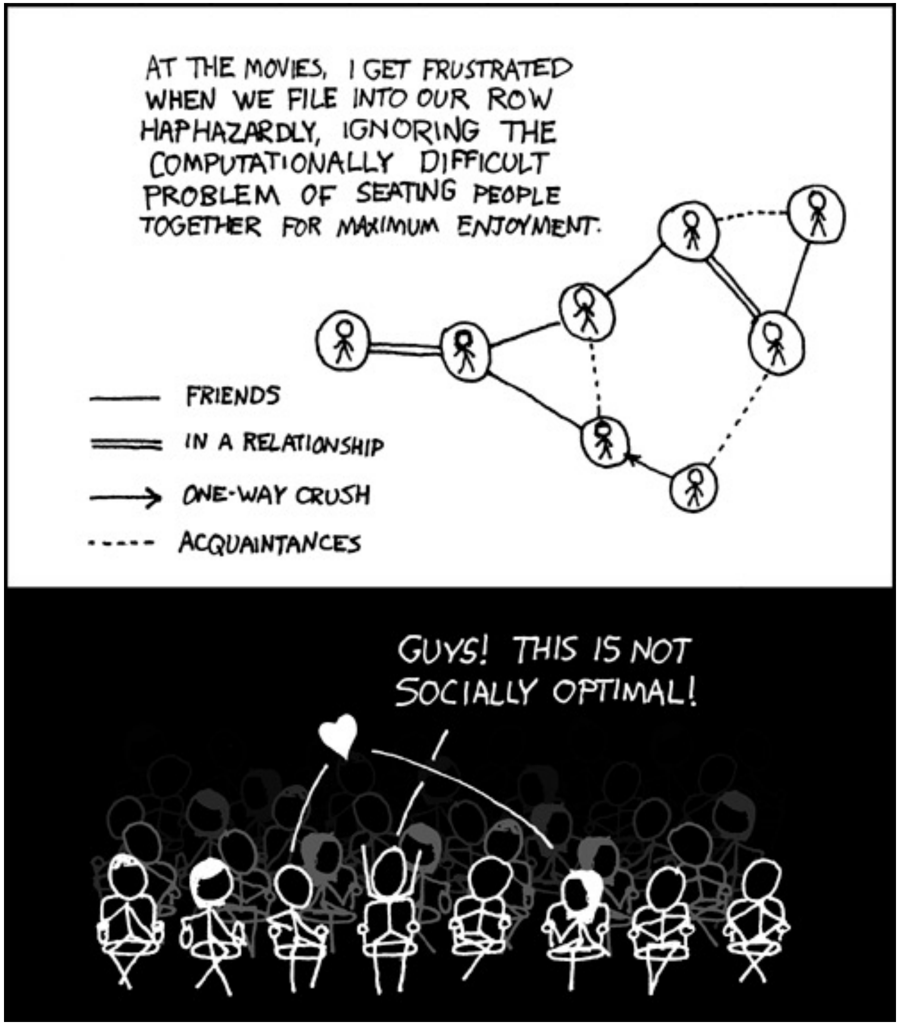
- Such a relationship is termed an *adjacency relationship*
  - Two individuals who are related are also said to be *adjacent* to each other

- Here we see a hockey team and some of their friends



Adjacency Relations

- Alternatively, the graph maybe more complex



http://xkcd.com/173/

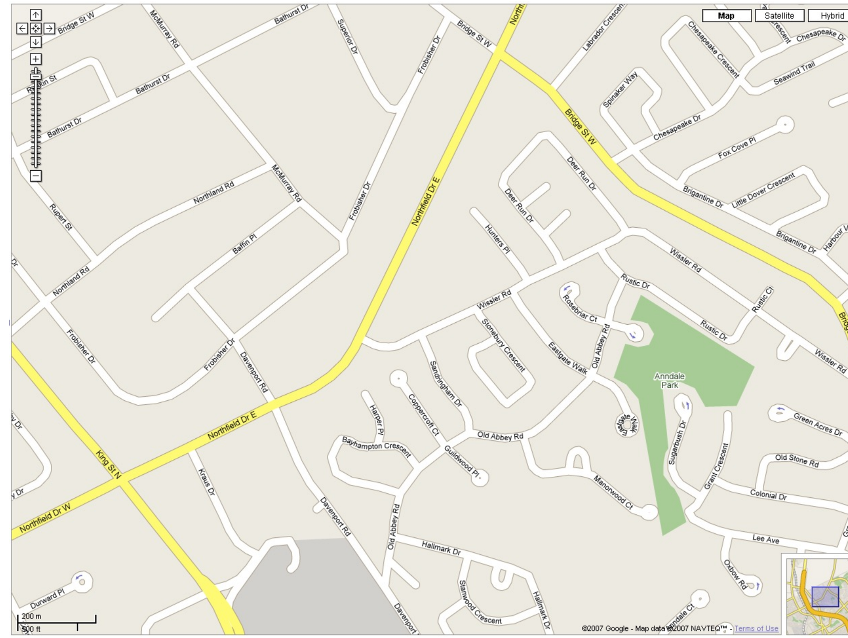
Adjacency Relations

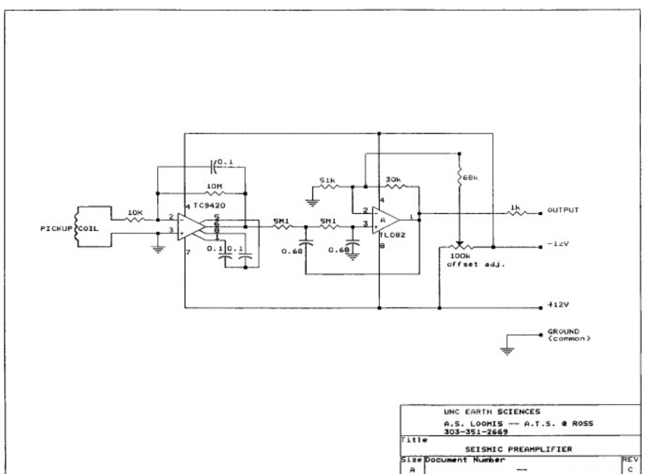
- In some cases, you do not have global relationships, but rather, you are simply aware of neighboring, or adjacent, nodes

- Such a relationship defines a graph where:
  - Nodes are termed vertices
  - Edges denote adjacencies

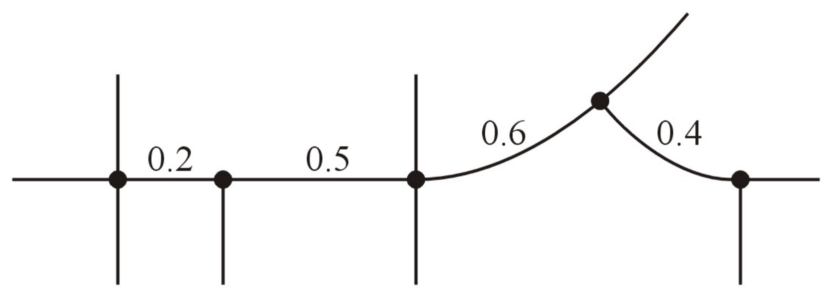
Adjacency Relations

- Two examples:
  - City streets
    - intersections are vertices
    - streets are edges
  - Circuits
    - circuit elements are vertices
    - connections are edges





http://esci.unco.edu/resource/circuit.htm



Operations on Adjacency Relations

- Given an adjacency relation:
  - Are two objects adjacent?
  - Iterate through all objects adjacent to one object

Summary of Relations

- We have now seen four relationships:
  - Linear orderings
  - Hierarchical orderings
  - Partial orderings
  - Adjacency relations

- All of these are relationships that exist on the objects we may wish to store, access, and query

Abstract Data Types

- In engineering, we tend to see certain patterns that occur over and over in applications

- In these circumstances, we first name these patterns and then proceed to define certain standard solutions or implementations

- In software in storing objects and relationships in containers, there are reoccurring containers of objects and associated relationships where the actual queries and operations are restricted
  - We model such containers by *Abstract Data Types* or ADTs

What's next?

- We have discussed containers, relationships, and ADTs
  - What is the thing that we want to store and access
  - What queries and operations are we interested in

- The next question is, how do we implement these efficiently on a computer?

- The next step is to look at *data structures*

Summary

- In this topic, we have covered:
  - The Container ADT as a basic model of organizing data
    - Queries and operations on containers
    - Simple and associative containers
    - Unique or duplicate objects
  - Relationships between data
    - Linear ordering
    - Hierarchical ordering
    - Partial ordering
    - Adjacency relation
  - In each case, we considered relationship-specific queries and operations
  - Abstract Data Types as a model for organizing information

http://en.wikipedia.org/wiki/Container_(abstract_data_type)
http://en.wikipedia.org/wiki/Binary_relation