210909

2021년 9월 7일 화요일 오후 12:07

Last lecture

Total, substantial, material derivative

$$\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + \vec{V}\cdot\nabla Q \qquad Q \to \text{scalar, vector}$$

$\frac{D\vec{V}}{Dt} \to$ local vs. convective acceleration

Properties — p, ρ, T, μ

Reynolds Number $= \rho V L/\mu = VL/\nu \qquad \nu \equiv \mu/\rho$

Newtonian fluid $\tau = \mu\, du/dy$

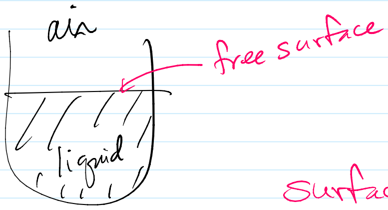
$\boxed{\text{HW \# 1} \quad 1-3, 12, 49, 63, 72, 73 \quad \text{Due } 9/14}$ 8th edition

$\hookrightarrow$ these problems will be posted on ETL

any edition ok.

Surface tension γ (N/m)

attractive force among liquid molecules near the free surface to maintain volume.



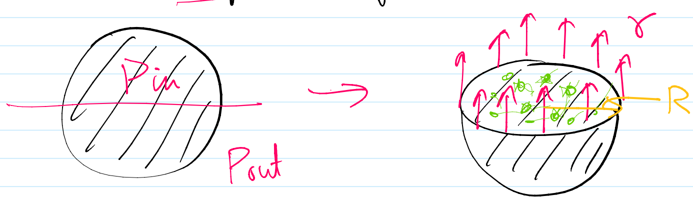
curved free surface can support pressure difference.

Surface tension effect between liquid & gas.

e.g. water droplet (sphere of water)

water $\quad$ air

P_{in} — P_{out}



$\Delta p = P_{in} - P_{out}$

$\Delta p\, \pi R^2 = \gamma\, 2\pi R$

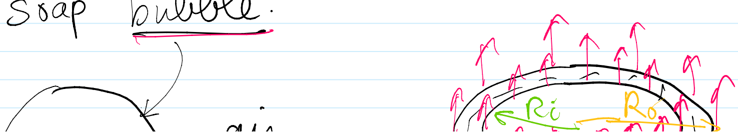
$\Delta p = \frac{2\gamma}{R}.$

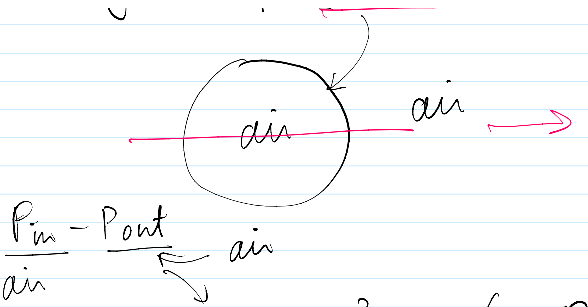
$\Delta p \propto \gamma$

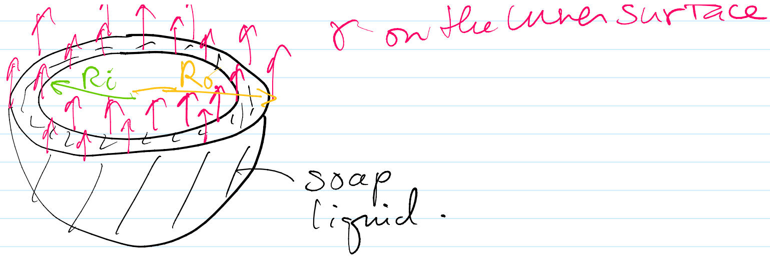
$\propto \frac{1}{R} \to$ smaller the drop (R), higher Δp.

e.g. soap bubble.

γ on the outer surface & γ on the inner surface





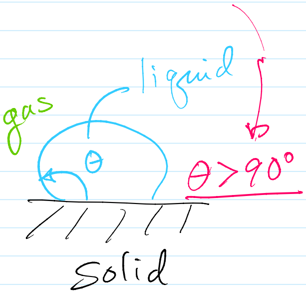


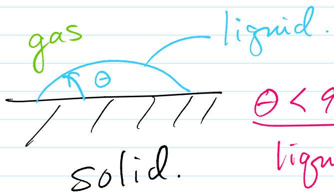
$$\Delta p\, \pi R^2 = \gamma (2\pi R \times 2)$$ → Assume $R_i \approx R_o$ (thin bubble)

$$\Delta p = \frac{4\gamma}{R}$$

"non-wetting"

Surface tension effect at boundary of [liquid, solid, gas





"wetting"

$\theta < 90°$ liquid "wets" the surface.

θ = contact angle between liquid & solid at the liquid-solid boundary
measured from the solid surface inside the liquid.

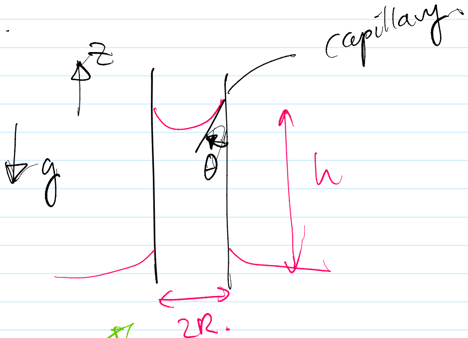
$\theta \rightarrow$ s on the liquid-solid combination.

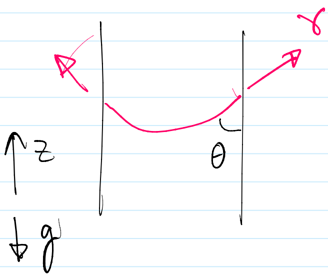
$\theta < 90°$ wetting
$\theta > 90°$ non-wetting.

} depends on balance between intermolecular force (among liquid molecules) and adhesion force (between liquid & solid)

google images

Capillary effect.





upward force due to γ — weight of the column of fluid.

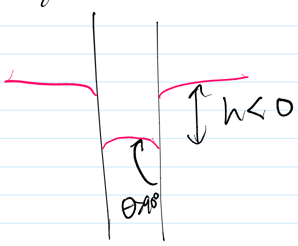
$$2\pi R \gamma \cos\theta = \rho g \pi R^2 h$$

$$\boxed{h = \frac{2\gamma\cos\theta}{\rho g R}}$$

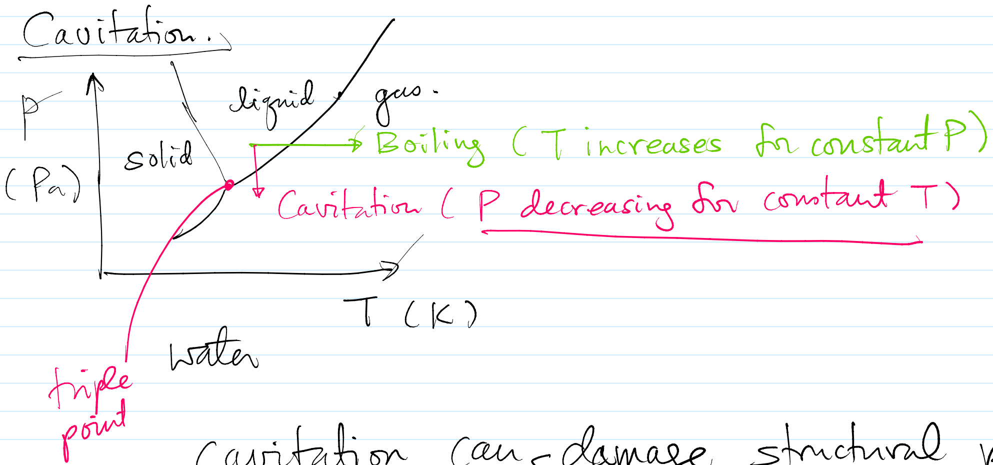
$h \propto \frac{1}{R}$

$h > 0$ if $\theta < 90°$ (wetting) → capillary rise

$h < 0$ if $\theta > 90°$ (non-wetting) → capillary depression.



Cavitation.



} vaporization bubbles form

Cavitation can [damage structural parts / cause instability

Cavitation number

$$Ca \equiv \frac{P_a - P_v}{\frac{1}{2}\rho V^2}$$

P_a = ambient pressure of liquid

P_v = vapor pressure of liquid at the given temperature

ρ = liquid density

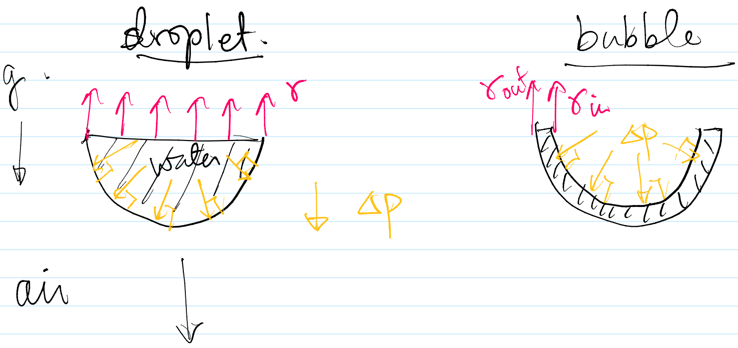
V = reference fluid velocity.

if $Ca < Ca_{critical}$,

then cavitation occurs.

↳ 2-phase (gas & liquid) flow.

back to γ. surface tension.



force balance

$2\pi R\gamma$ upward force due surface tension.

$\Delta p \; \pi R^2$.

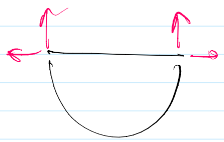
Categories of flows.

compressible vs. incompressible $\rightarrow \rho \rightarrow \nabla \cdot \vec{V}$

viscous vs. inviscid $\rightarrow \mu, \tau$

steady vs. unsteady $\rightarrow \frac{\partial}{\partial t}$

rotational vs. irrotational $\nabla \times \vec{V}$



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2021년 9월 14일 화요일 오전 9:16

Last lecture

surface tension γ

contact angle θ

capillary rise / depression

cavitation vs. boiling

$Ca \# = (P_a - P_v) / (\frac{1}{2}\rho V^2)$

Types of flow
- unsteady vs. steady ($\frac{\partial}{\partial t}$)
- invisid vs. viscous (τ)
- incompressible vs. compressible (ρ, $\nabla \cdot \vec{V}$)
- irrotational vs. rotational ($\nabla \times \vec{V}$)

Flow lines.

1. streamline → a line tangent to the velocity vector everywhere at a given time.

$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$; $\left.\frac{dy}{dx}\right|_{\text{streamline}} = \frac{v}{u}$; . . .

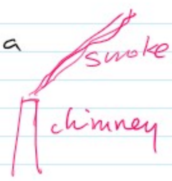
$\frac{u}{dx} = \frac{v}{dy} = \frac{w}{dz}$

ψ = constant

2. pathline → actual trajectory (path travelled) of a fluid particle

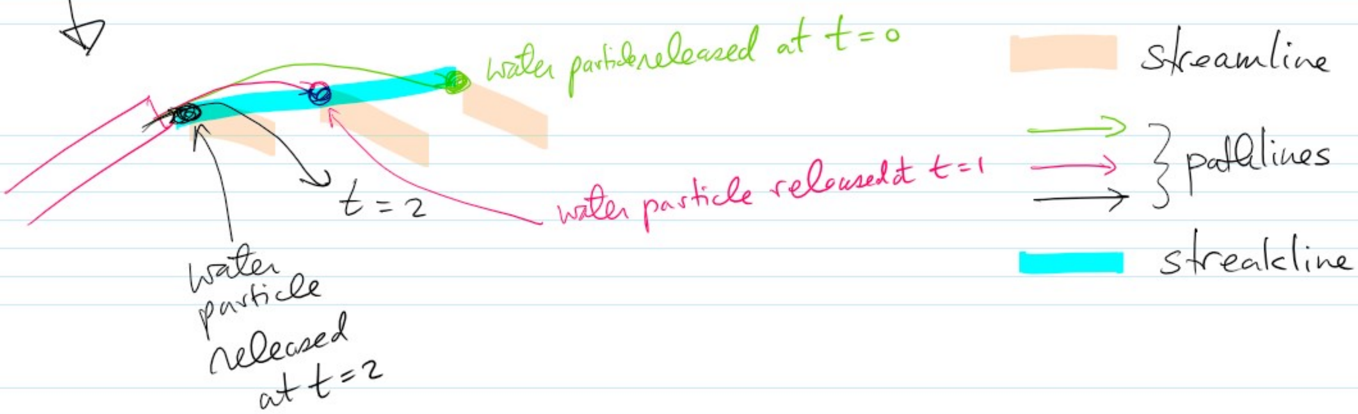
→ $x_k(t)$ → trajectory x of kth particle over time

3. streakline → locus of particles which have earlier passed through a common location.



in steady flow, all 3 lines are the same.

unsteady flow, 3 lines are different.



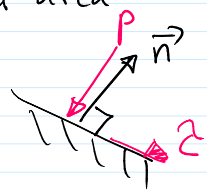
Fluid statics. ($\vec{V} = 0$)

Fluid statics ($\vec{V}=0$)

Forces on Fluid

Surface force
(requires contact)
$\propto$ contact area

surface defined by normal vector



body force
(can act remotely)
$\propto$ mass
e.g. gravity
electromagnetic
etc.

pressure → normal compressive stress on any plane through a fluid at rest.
p acts perpendicular to surface

viscous shear stress.
τ acts parallel surface

Pascal's Principle.

At a point, pressure is the same in all directions.

$\vec{F} = m\vec{a} = \frac{d(m\vec{v})}{dt} =$

$\vec{f}$: force per unit volume.

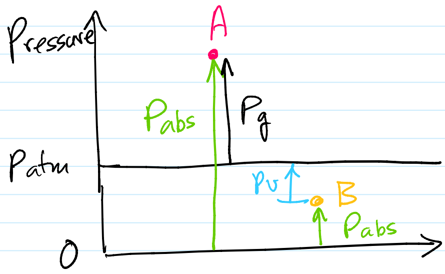
$\vec{f} = \rho\vec{a} = \vec{f}_p + \vec{f}_\tau + \vec{f}_g$

$\rho\vec{a} = -\nabla p + \mu\nabla^2\vec{V} + \rho\vec{g}$

$\boxed{\nabla p = \rho(\vec{g}-\vec{a}) + \mu\nabla^2\vec{V}}$

1. $\vec{V}=0$ → $\vec{a}=0$ & $\nabla^2\vec{V}=0$ → $\boxed{\nabla p = \rho\vec{g}}$
   ↑ Fluid statics
   ↳ stationary fluid.

2. Rigid body rotation $\vec{V}=r\omega$ → $\mu\nabla^2\vec{V}=0$ → $\boxed{\nabla p = \rho(\vec{g}-\vec{a})}$



$P_{atm} \simeq 10^5\ N/m^2$
$\simeq 10^5\ Pa$
$\simeq 1\ bar$

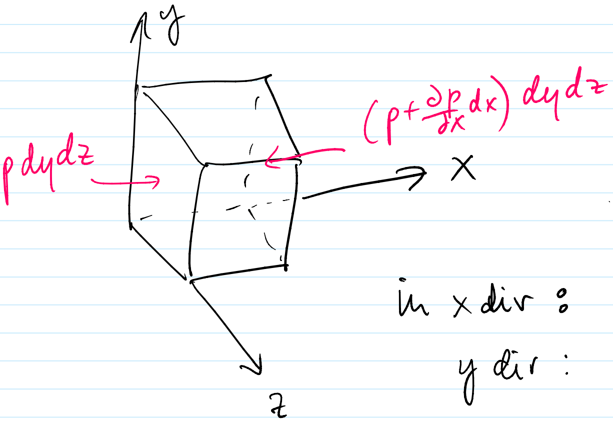
↑ Atmospheric pressure

Pressure A can be indicated in 2 ways.

1. Absolute pressure P_{abs}
2. Gage pressure $P_g = P_{abs} - P_{atm}$

For pressure B (which is below P_{atm}), 2 ways to indicate.

1. P_{abs}
2. Vacuum $P_{v_B} = P_{atm} - P_{abs}$ of B



Net force due to pressure

in x dir : $p\,dy\,dz - \left(p + \frac{\partial p}{\partial x} dx\right) dy\,dz = -\frac{\partial p}{\partial x} dx\,dy\,dz$

y dir : $-\frac{\partial p}{\partial y} dy\,dz\,dx$

z dir : $-\frac{\partial p}{\partial z} dz\,dy\,dx$

$\therefore \vec{f}$ = force per unit volume $\rightarrow \vec{f}_p = -\nabla p$.

to have net pressure force, pressure needs to be non-uniform ($\nabla p \neq 0$)

if uniform pressure ($\nabla p = 0$), then $\vec{f}_p = 0$

$\vec{f}_v = \mu\left(\frac{\partial^2 \vec{V}}{\partial x^2} + \frac{\partial^2 \vec{V}}{\partial y^2} + \frac{\partial^2 \vec{V}}{\partial z^2}\right)$ → cover later in Ch. 4.

$\vec{f}_g \rightarrow \vec{F}_g = m\vec{g}$ due to gravity

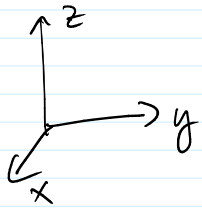
$f_g \rightarrow \quad F_g = m\vec{g}$ due to gravity

$$\vec{f}_g = \rho \vec{g}$$

Pressure distribution in static fluid
- incompressible (ρ = constant) → water → "hydro"
- compressible ($\rho \neq$ constant) → air → "aero"

Hydrostatic pressure distribution

in stationary water (or liquid) ($\vec{V}=0$)

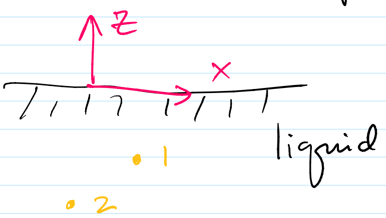


$\downarrow \vec{g}$

$$\nabla p = \rho \vec{g}$$

$$\vec{g} = -g\vec{k}$$

$$\underbrace{\frac{\partial p}{\partial x} = 0 \; ; \; \frac{\partial p}{\partial y} = 0}_{\text{no pressure gradient in horizontal plane.}} \; ; \qquad \frac{\partial p}{\partial z} = -\rho g \rightarrow \frac{dp}{dz} = -\rho g$$



$$\boxed{p_2 - p_1 = -\int_1^2 \rho g \, dz = \rho g (z_1 - z_2)}$$

pressure only changes with z

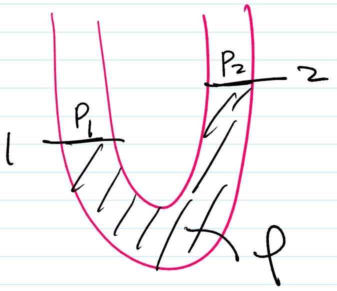
$$\boxed{z_1 - z_2 = \frac{P_2 - P_1}{\rho g}.}$$

ρ = constant.

$\rho g \rightarrow$ specific weight.

Manometer:

device to measure $(P_1 - P_2)$ from z_1 & z_2 (read) for a given liquid with ρ



What if $\rho \neq$ constant

$$\frac{dp}{dz} = -\rho g$$

$\rho = \frac{p}{RT}$ ideal gas

$$\int_1^2 \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R}\int_1^2 \frac{dz}{T(z)}$$

$T(z)$ for atmosphere varies with z

between $z = 10$ km and $z = 20$ km, $T \approx$ constant at T_0

in that case, $p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$

or if $T(z) = T_0 - Bz$ (0 – 10 km)

$p(z)$