210916

2021년 9월 16일 목요일 오전 10:39

Last lecture

Flow lines → streamlines, pathlines, streaklines

Forces on fluids $\begin{cases} \text{surface} \to p, \tau \\ \text{body} \to \text{gravity} \end{cases}$

pressure → absolute, gage, vacuum

Pasal's Principle

Hydrostatic formula → incompressible → $p(z)$ $\uparrow z$ $\downarrow \vec{g}$

water $\vec{V}=0$

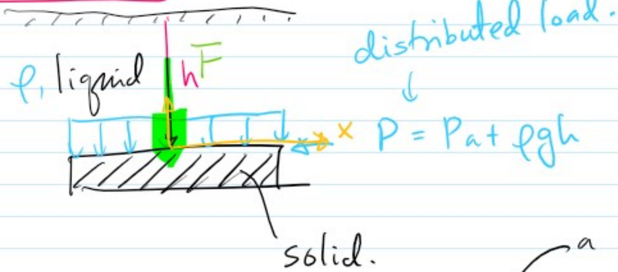
$p(z)$ for compressible fluid (atmosphere)

$\boxed{\text{HW\#2} \quad 2\text{-}5,\ 35,\ 68,\ 85 \qquad \text{Due } 9/27}$

No class on 9/21

## FORCES ON PLANE SURFACES.

HORIZONTAL $P_a = P_{atm}$





Vertical force due to pressure acting on a horizontal solid surface $= pA = (p_a + \rho g h)A = F$

uniform pressure acting

$x_{cp}$ center of pressure = center of gravity.

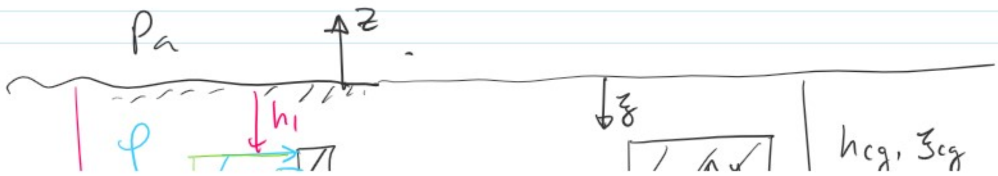
location where F acts.

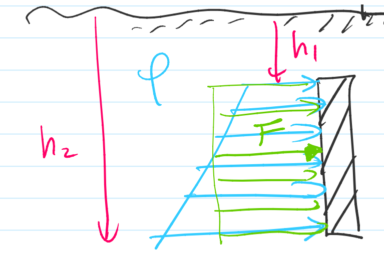
x, y → distance from center of gravity in x, y dir

$$F x_{cp} = \int x\, p\, dA = 0 \rightarrow x_{cp} = 0$$

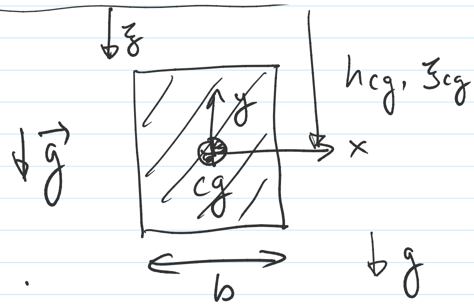
$$F y_{cp} = \int y\, p\, dA = 0 \rightarrow y_{cp} = 0$$

## VERTICAL SURFACE.






$\vec{P}$ at $h_{cg}$ $\quad p(z) = P_a + \rho g h$.




$cg \to$ center of gravity of plate.

Assume $\underline{P_a = 0}$

$$F = \left(\frac{\rho g h_1 + \rho g h_2}{2}\right) b(h_2 - h_1) = \rho g h_{cg} b (h_2 - h_1)$$

$$h_{cg} = \frac{(h_2 + h_1)}{2}$$

$F x_{cp} = 0 \quad p$ uniform along $x. \to \underline{x_{cp} = 0.}$

$$F y_{cp} = \int y p \, dA = \int y \rho g (\xi_{cg} - y) \, dA.$$

$$\xi_{cg} = \frac{1}{A}\int \xi \, dA \quad \& \quad \xi = \xi_{cg} - y.$$

$$F y_{cp} = -\rho g \int y^2 dA = -\rho g I_{xx}$$

$P_{cg} A$

$I_{xx} = \dfrac{b(h_2 - h_1)^3}{12}$ for a rectangular plate.

$$y_{cp} = -\frac{\rho g I_{xx}}{P_{cg} A}$$

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Force on an inclined surface.

$$\xi = \frac{h}{\sin\theta} \qquad \xi_{cg} = \frac{1}{A}\int \xi \, dA \to \xi_{cg} A = \int \xi \, dA$$

$$F = P_a A + \rho g \xi_{cg} \sin\theta \, A \to \int h \, dA = \int \xi \sin\theta \, dA = \xi_{cg} \sin\theta \, A.$$

$F = P_a A + \rho g h_{cg} A \to$ independent of shape of $A$.

$$F y_{cp} = \int y p \, dA \qquad\qquad F x_{cp} = \int x p \, dA$$

$$y_{cp} = \frac{-\rho g \sin\theta \, I_{xx}}{P_{cg} A}.$$

$$x_{cp} = -\frac{\rho g \sin\theta \, I_{xy}}{P_{cg} A}.$$

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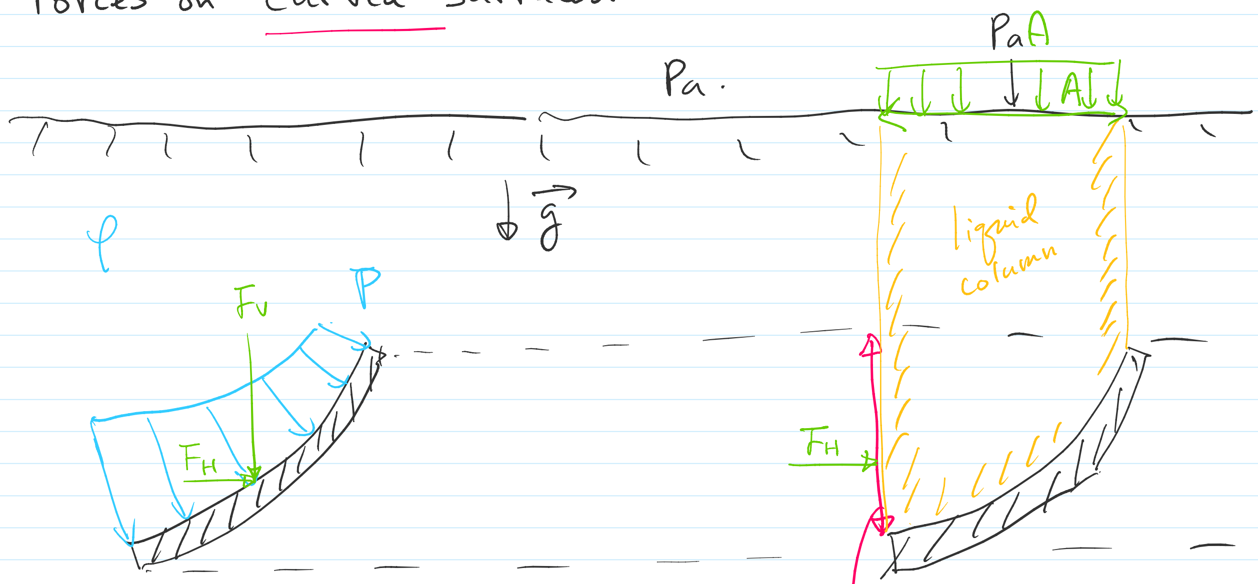
## Forces on curved surfaces.




vertical surface of projection of original surface onto the vertical axis

$F_H$ is equivalent to the force on the vertical plane area formed by the projection of the curved surface onto the vertical plane.

//// → liquid column above the plate.

$F_V$ equals the weight of the entire column of fluid above the curved plate.

( in case of air-water combination, $\rho_{water} \sim 1000 \times \rho_{air}$)

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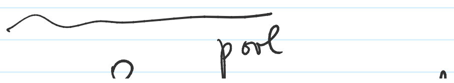
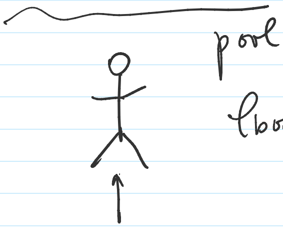
## Buoyancy → Archimedes.

1. A submerged body (e.g. person submerged in a pool) is subject to a buoyant force equivalent to the weight of the fluid displaced by the body

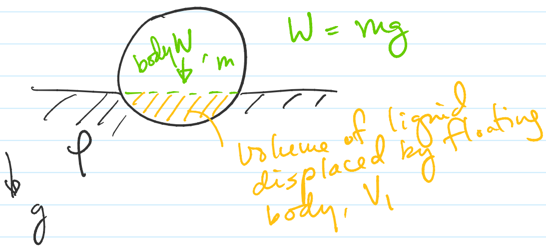





$\rho_{body} > \rho_{liquid}$

$F_B$ = buoyant force

2. A floating body displaces exactly the volume of fluid whose weight is equivalent to the body's weight.




$\rho_{body} < \rho_{liquid}$

$$F_B = \rho g V_1 = mg$$

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$$F_B = \int (p_2 - p_1)\, dA$$

$$= -\int \rho g (z_2 - z_1)\, dA_H$$ horizontal projected area.

$$= \rho g \underbrace{\int (z_1 - z_2)\, dA_H}_{\text{volume of body.}}$$

$$\underline{F_B = \rho g \, \text{Volume}_{body}.}$$

↑ for a submerged body.

On 9/23
↓
$p$ in <u>moving</u> fluids

210923

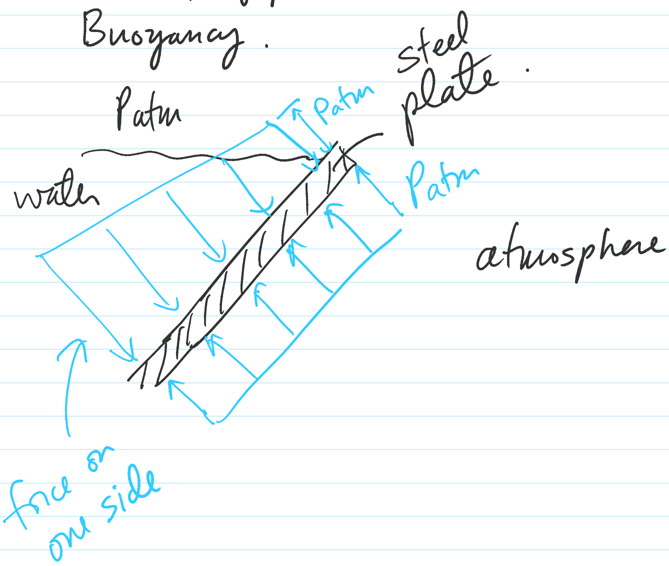
2021년 9월 23일 목요일 오전 10:29

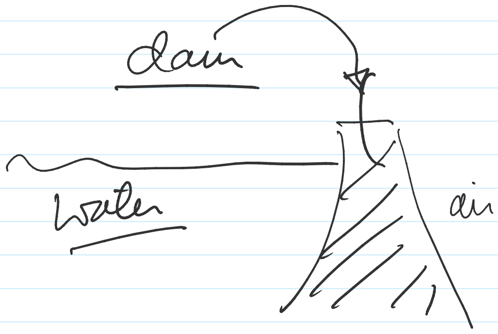
Last lecture

forces on due to p
- plane surfaces – vertical / horizontal / inclined
- curved surfaces

F, $x_{cp}$, $y_{cp}$, $I_{xx}$, etc.

Buoyancy.




Pressure distribution in Rigid Body Motion

$\tau = 0$

$\mu \nabla^2 \vec{V} = 0$ → no shear force.

no deformation

$\vec{V} \neq 0$

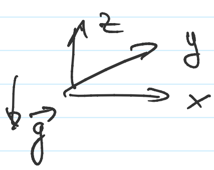
1. translation (linear)
2. rotation

Translation

Uniform linear $\vec{a}$

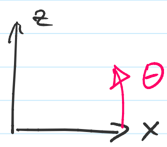
Rigid body motion

$\tau = 0$




$$\rho \vec{a} = -\nabla p + \rho \vec{g} + \cancel{\mu \nabla^2 \vec{V}}$$

$$\nabla p = \rho(\vec{g} - \vec{a})$$




$\rho\, dx\, dy\, dz$

x: $p\, dy\, dz - (p + \frac{\partial p}{\partial x} dx)\, dy\, dz = dm \cdot a_x \rightarrow -\frac{\partial p}{\partial x} = \rho a_x$

y: $-\frac{\partial p}{\partial y} = \rho a_y$

z: $p\, dx\, dy - (p + \frac{\partial p}{\partial z} dz)\, dx\, dy - g\, dm = dm \cdot a_z \rightarrow -\frac{\partial p}{\partial z} = \rho(g + a_z)$

$$dp = \frac{\partial p}{\partial x} dx + \cancel{\frac{\partial p}{\partial y} dy} + \frac{\partial p}{\partial z} dz$$

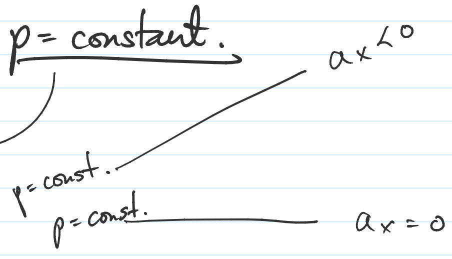
$$dp = \frac{\partial p}{\partial x}dx + \cancel{\frac{\partial p}{\partial y}dy} + \frac{\partial p}{\partial z}dz$$

1. Assume $a_y = 0$

$$dp = -\rho a_x\, dx - \rho(g + a_z)dx.$$

isobar → surface of equivalent pressure → p = constant.
line

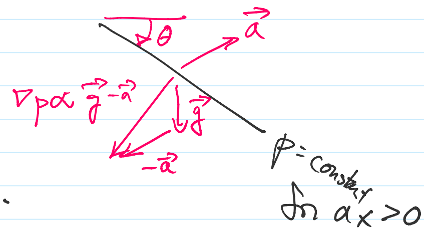
equal pressure




$$dp = 0 = -\rho a_x dx - \rho(g + a_z)\, dz.$$

slope of the isobar

$$\left.\frac{dz}{dx}\right|_{p=\text{constant}} = \tan\theta = \frac{-a_x}{g + a_z}.$$



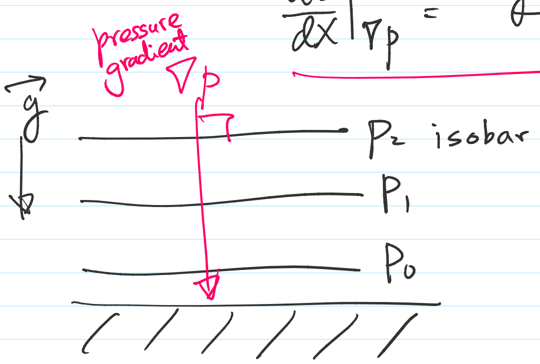

$$p = -\rho a_x x - \rho(g + a_z)z + C_0$$

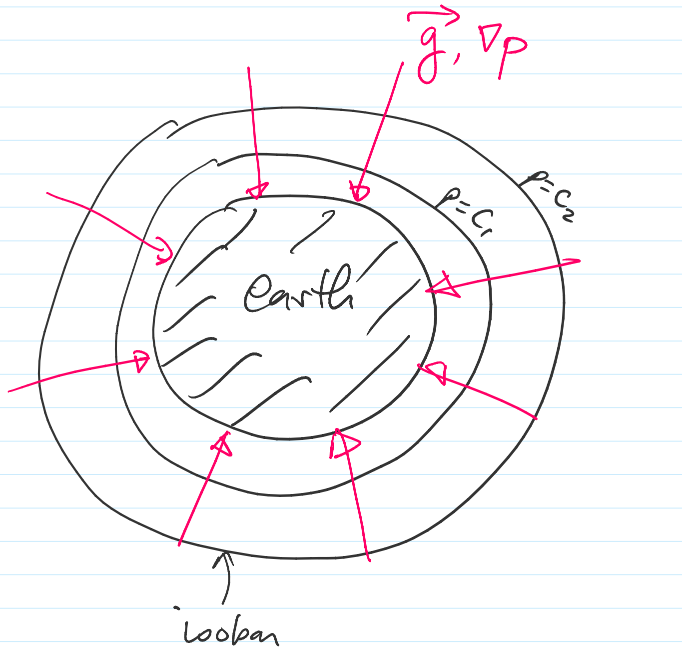
B.C. at $x = 0$, $z = 0$, $p = p_0$

$$p = p_0 - \rho a_x x - \rho(g + a_z)z.$$

$\left.\frac{dz}{dx}\right|_{\nabla p}$ is perpendicular to $\left.\frac{dz}{dx}\right|_{p=\text{constant}}$.

$$\left.\frac{dz}{dx}\right|_{\nabla p} = \frac{g + a_z}{a_x}$$

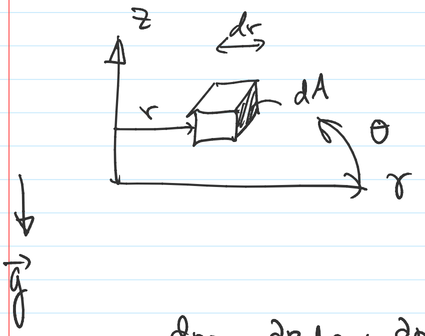



Rigid Body Rotation (at constant $\omega$ = angular velocity).

# Rigid Body Rotation (at constant $\omega$ = angular velocity).

$\tau = 0$




centripetal $\vec{a}$

$$r: \quad p\,dA - \left(p + \frac{\partial p}{\partial r}dr\right)dA = \rho\, dA\, dr\,(-r\omega^2)$$

$$z: \quad \frac{\partial p}{\partial z} = -\rho g$$

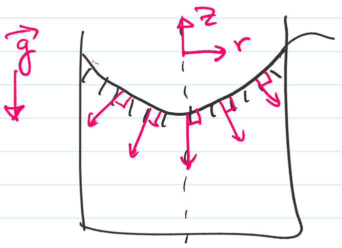
$$dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz \;\longrightarrow\; dp = 0 \rightarrow \left.\frac{dz}{dr}\right|_{isobar} = \frac{\omega^2 r}{g}$$

$$p = \frac{\rho\omega^2 r^2}{2} - \rho g z + C \qquad \left.\frac{dz}{dr}\right|_{\nabla p} = \frac{-g}{\omega^2 r}$$

B.C. @ $r = 0$ & $z = 0$, $p = p_0$

$$p(r,z) = \frac{\rho\omega^2 r^2}{2} - \rho g z + p_0 \qquad \text{isobar}$$

circular beaker rotating about its center

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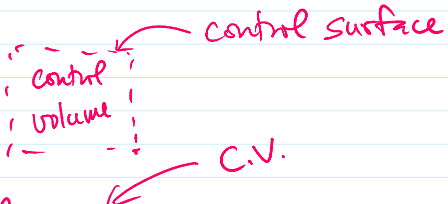
## Ch. 3. Integral Control Volume Analysis. MOST IMPORTANT CHAPTER.

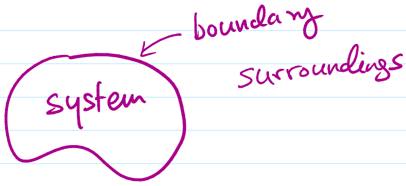
Integral ← large scale (e.g. car compressor etc.)

Control Volume ← Space (not mass) selected for analysis.

"control volume" (Eulerian view)

"system" (Lagrangian view)




control volume = region or space selected for analysis.

C.S.

control surface separates the control volume from the rest.

system ≡ mass [ selected for analysis / of interest

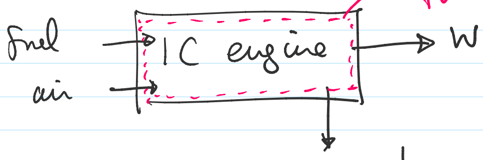
surroundings ≡ mass not included in the system

boundary separates system from surroundings.

Control volume from the rest.

interested in space

interested in the engine and NOT particular fluid particles




apply conservation laws to C.V.

**Reynolds Transport Theorem**

## Conservation Laws. (Lagrangian)

1. Conservation of mass

$m_{sys}$ = constant.

$$\frac{dm_{sys}}{dt} = \frac{Dm}{Dt} = 0$$

2. Momentum conservation.

$$\frac{D(m\vec{V})}{Dt} = m\vec{a} = \Sigma\vec{F}$$

3. Energy conservation (1st Law of Thermo)

$$\frac{DE}{Dt} = \dot{E} = \dot{Q} - \dot{W}$$

$\dot{E}$: rate of change in system energy

$\dot{Q}$: heat transferred from the surroundings to the system

$\dot{W}$: work done by the system

## Reynolds Transport Theorem