210928

2021년 9월 28일 화요일 오전 10:40

Last lecture

Rigid body motion → linear $\vec{a}$
rotational $\vec{a}$

isobar (p = constant) $\perp$ ∇p

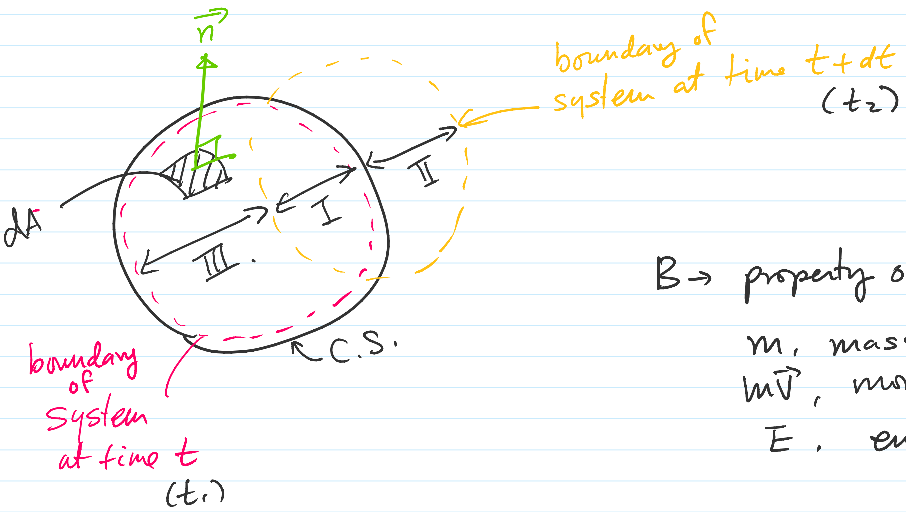
Reynolds Transport Theorem (RTT) → enables application of conservations (for Lagrangian system) to control volume (Eulerian view)

Today: RTT
mass conservation to C.V.
momentum " "

Test #1 Oct 14 Thurs. in class offline } → will cover what we cover upto Oct 7

RTT.

C.V. fixed in space.



B → property of system. $\beta = \frac{dB}{dm}$

m, mass — 1

$m\vec{V}$, momentum — $\vec{V}$

E, energy — e

System

B at t_1 $\quad B_{I\,t_1} + B_{III\,t_1}$

B at t_2 $\quad B_{I\,t_2} + B_{II\,t_2}$

$$\frac{dB_{system}}{dt} = \frac{DB}{Dt} = \lim_{\underbrace{(t_2-t_1)}_{\Delta t}\to 0} \frac{B_{I t_2} + B_{II t_2} - (B_{I t_1} + B_{III t_1})}{\underbrace{(t_2 - t_1)}_{\Delta t}}$$

$$= \lim_{\Delta t \to 0} \frac{B_{I t_2} + B_{III t_2} - (B_{I t_1} + B_{III t_1}) + B_{II t_2} - B_{III t_2}}{\Delta t}$$

amount of B which has left C.V.

amount of B which came into C.V.

$$= \frac{dB_{C.V.}}{dt} + \frac{B_{II t_2} - B_{III t_2}}{\Delta t}$$

Reynolds Transport Theorem

$$\frac{dB_{system}}{dt} = \boxed{\frac{DB}{Dt} = \frac{d}{dt}\int_{C.V.} \beta \rho \, dVol + \int_{C.S.} \beta \rho (\vec{V}\cdot\vec{n})\, dA}$$

$$\boxed{B_{C.V.} = \int_{C.V.} \beta \rho \, dVol}$$

$$\frac{dB_{system}}{dt} = \boxed{\underbrace{\frac{DB}{Dt}}_{\text{Lagrangian}} = \underbrace{\frac{d}{dt}\int_{C.V.} \beta\rho\, dVol + \int_{C.S.} \beta\rho(\vec{V}\cdot\vec{n})dA}_{\text{Eulerian}}}$$

Rate of change in system's B.

Rate of change in B inside C.V.

Flux of B into and out of C.V.

$$B_{C.V.} = \int_{C.V.} \beta\rho\, dVol$$

$$\beta = \frac{dB}{dm}$$

1. $\frac{Dm}{Dt} = 0$
2. $\frac{D(m\vec{V})}{Dt} = \Sigma\vec{F}$
3. $\frac{DE}{Dt} = \dot{Q} - \dot{W}$

Question $\int_{C.S.} \beta\rho(\vec{V}\cdot\vec{n})dA$?

Net Volume flow rate $= \int_{C.S.} (\vec{V}\cdot\vec{n})dA = Q$

Mass flow rate $= \int_{C.S.} \rho(\vec{V}\cdot\vec{n})dA = \dot{m}$

Net flux of B $= \int_{C.S.} \beta\rho(\vec{V}\cdot\vec{n})dA$

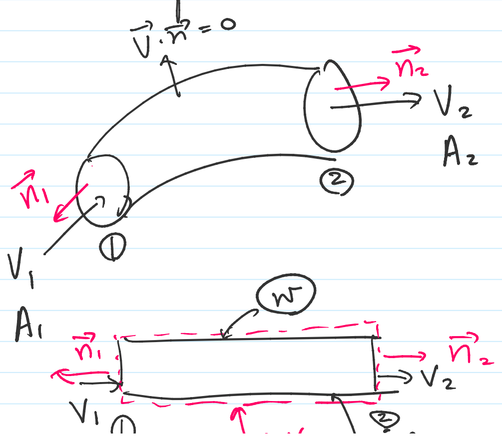
$\vec{n}$ = unit normal vector pointing outward from dA
↳ (perpendicular to dA)

IT IS VERY IMPORTANT TO CLEARLY SHOW THE C.V. THAT YOU CHOOSE

Mass conservation.

$$\frac{Dm}{Dt} = 0 = \frac{d}{dt}\int_{C.V.} \rho\, dVol + \int_{C.S.} \rho(\vec{V}\cdot\vec{n})dA$$

examples:

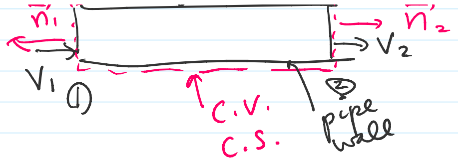


Assumptions.
1. steady
2. 1-D (velocity has one component)
3. no leakage thru walls ($\vec{V}\cdot\vec{n} = 0$)

$$\frac{Dm}{Dt} = 0 = \cancel{\frac{d}{dt}\int_{C.V.} \rho\, dVol} + \int_{C.S.} \rho(\vec{V}\cdot\vec{n})dA$$

$$0 = \int_{A_2} \rho V_2\, dA_2 - \int_{A_1} \rho V_1\, dA_1 + \cancel{\int_W \rho(\vec{V}_w\cdot\vec{n}_w)dA_w}$$

$$\dot{m}_1 = \dot{m}_2 = \rho V A$$



| inlet
| exit

A_2 A_1

$$\therefore \dot{m}_1 = \dot{m}_2 = \rho V A$$

$Q_1 = Q_2$ if $\rho =$ constant.

$V_1 = V_2$ if $A_1 = A_2$

examples with multiple inlets & exits

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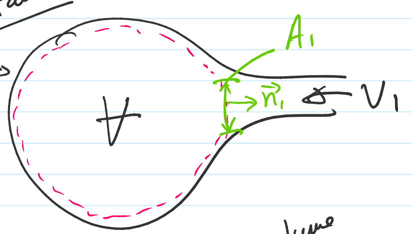
example: filling a tank with gas.

1. uniform velocity at 1

metal tank.



constant volume

$\frac{dVol}{dt} = 0$

$\forall =$ tank volume

$$\frac{Dm}{Dt} = 0 = \frac{d}{dt}\int_{C.V.} \rho\, dVol + \int \rho(\vec{V}\cdot\vec{n})dA$$

$$0 = \frac{d}{dt}\int_{C.V.} \rho\, dVol - \rho_1 V_1 A_1$$

ρ of gas in the tank

$$\frac{d\rho}{dt} = \frac{\rho_1 V_1 A_1}{\forall}$$

Unsteady problem.

Momentum Conservation for a fixed C.V.

$B = m\vec{V}$

$\beta = \vec{V}$

$$\frac{D(m\vec{V})}{Dt} = \Sigma \vec{F} = \frac{d}{dt}\left[\int_{c.v.} \rho \vec{V} dVol\right] + \int_{c.s.} \rho\vec{V}(\vec{V}\cdot\vec{n})dA.$$

$\vec{V} \to$ absolute velocity in fixed frame.

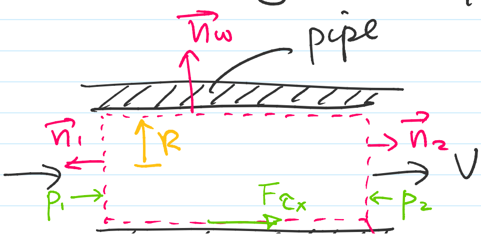
$\vec{F} = \vec{F}_B + \vec{F}_p + \vec{F}_\tau + \vec{F}_{solid}$

$\vec{F}_B$: body; $\vec{F}_p$: pressure; $\vec{F}_\tau$: shear.

$\vec{F}_{solid}$ if C.V. cuts through a solid surface.

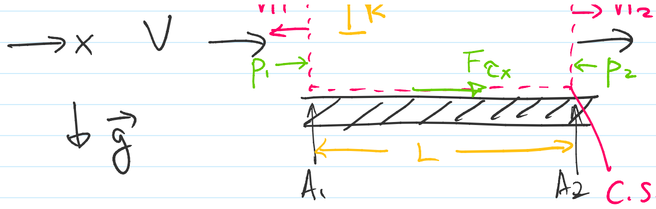
$\to$ External forces acting on the system contained in the C.V.

Example:



Assumptions
1. steady
2. 1-D $\to$ ~~X~~
3. uniform
4. incomp. $\rho =$ const.

$\vec{V} = u\vec{i}$



3. uniform $\vec{V} = u\vec{\imath}$

4. incomp. $\rho = \text{const.}$

5. $(\vec{V}\cdot\vec{n}) = 0$ on wall

C.S. contains only the fluid.

$$\Sigma F_x = F_{px} + F_{\tau x} + \cancel{F_{Bx}}^{\,0} = \cancel{\frac{d}{dt}\int \rho u \, dVol} + \int \rho u (\vec{V}\cdot\vec{n}) dA$$

assume shear force acts in + x dir.

$$P_1A_1 - P_2A_2 + \underline{F_{\tau x}} = \int_{A_2} \rho V_2^2 \, dA - \int_{A_1} \rho V_1^2 \, dA = \dot{m}_2 V_2 - \dot{m}_1 V_1$$

shear force by the wall on fluid

but $\frac{Dm}{Dt} = 0 \Rightarrow \dot{m}_2 = \dot{m}_1 = \dot{m}$
$Q_2 = Q_1$

$$\therefore \quad \underline{P_1A_1 - P_2A_2 + F_{\tau x} = \dot{m}(V_2 - V_1)}$$

Now <u>if</u> assume
5. $A_1 = A_2 \rightarrow V_2 = V_1$
6. $\frac{\partial V}{\partial x} = 0$ (fully developed flow)
7. far away from inlet/exit
8. circular pipe with radius R and C.V. length L

then

$$\underline{(P_1 - P_2)A + \tau \, 2\pi R L = 0}$$

$$\underline{\frac{P_1 - P_2}{L} = -\frac{2\tau}{R} > 0}$$

because flow goes from 1 to 2, $P_1 > P_2$

therefore $\underline{\tau < 0}$. $\rightarrow$ shear stress force acts in direction opposite to the assumed direction.

210930

2021년 9월 30일 목요일 오전 10:18

Last lecture

Reynolds Transport Theorem (RTT)

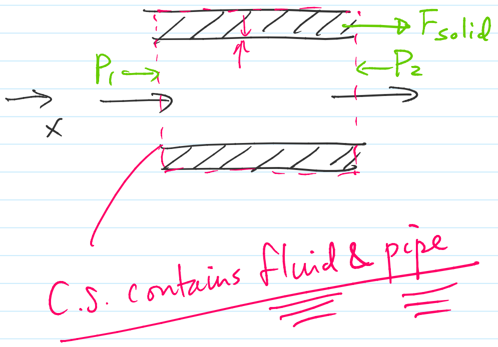
Mass conservation

Momentum conservation

$\boxed{\text{HW \#3} \quad 3-7, 9, 14, 18, 25, 35, 45, 49, 55 \quad \text{Due Oct. 5}}$

Test #1 Oct. 14 in-class OFFLINE!

continuation from previous lecture

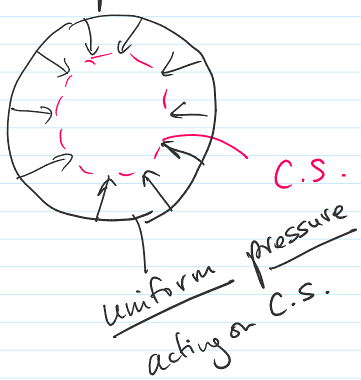


This time, C.V. contains pipe & fluid.

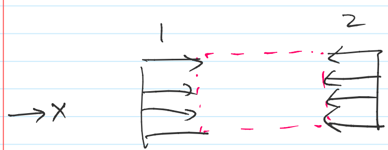
∴ Forces acting on system inside the C.V.

$$P_1A_1 - P_2A_2 + F_{solid} = \dot{m}(V_2 - V_1)$$

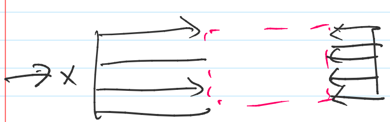
Net pressure force on C.V.



→ no net force due to pressure

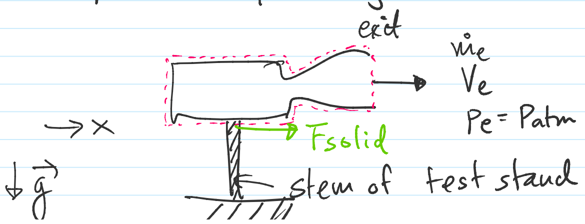


equal uniform pressure $P_1 = P_2$ at 1 & 2 → no net pressure force in x dir.



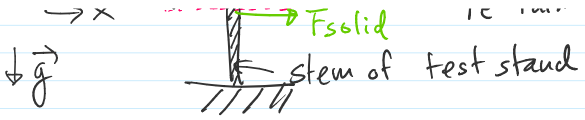
$P_1 > P_2$ → net force due to pressure

Rocket testing.



uniform pressure on C.S. → no net pressure force

no shear force



no shear force

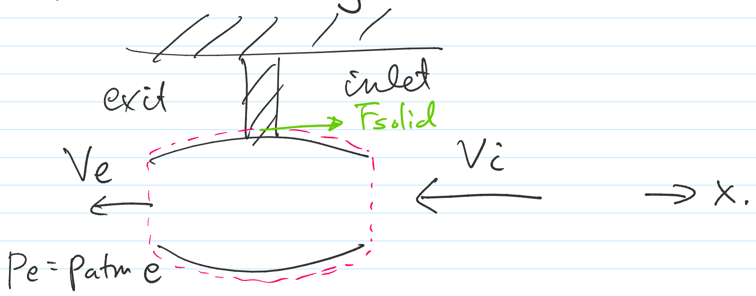
static rocket firing test

Assume
1. 1-D (x dir)
2. steady

$$\Sigma \vec{F} = F_{solid} = \cancel{\frac{d}{dt}\int_{c.v.} \rho \vec{V}\, dVol}^{\,2.} + \int \rho \vec{V}(\vec{V}\cdot\vec{n})\, dA$$

rocket thrust $\quad \underline{F_{solid} = \dot{m}_e V_e}$

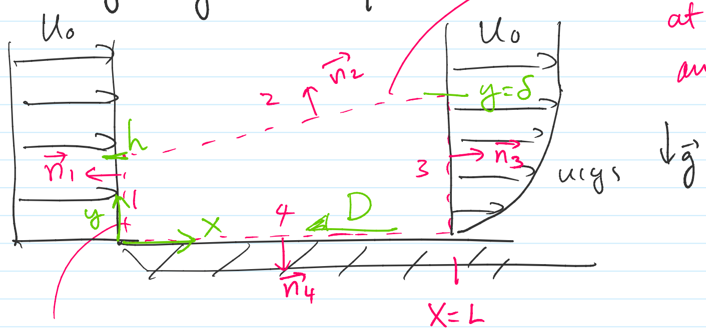
Gas turbine testing



no net pressure force
shear force.

$$\underline{F_{solid} = \underbrace{-\dot{m}_e V_e}_{\text{momentum going out at exit}} + \underbrace{\dot{m}_i V_i}_{\text{momentum coming into inlet}}}$$

Boundary layer analysis.



→ choose the streamline at $y=h$ at $x=0$ and at $y=\delta$ at $x=L$

b = depth into the page.

δ = boundary layer thickness.

C.S. ← contains only the fluid.

Surface 2 → streamline

Assume
1. uniform p. $\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$
2. viscous.
3. steady
4. incomp

drag due to shear stress on fluid by plate

$$x: \quad \Sigma \vec{F} = \cancel{F_p}^{\,1} + \cancel{F_{body}}^{\,0} - D = \cancel{\frac{d}{dt}\int \rho \vec{V}\, dVol}^{\,3.} + \int \rho \vec{V}(\vec{V}\cdot\vec{n})\, dA$$

$$-D = \int_{①} + \cancel{\int_{②}}^{\,0\ \text{streamline}} + \int_{③} + \cancel{\int_{④}}^{\,0\ \text{no leakage}} = -\rho u_o^2 bh + \rho b \int_0^{\delta} u^2 dy. \quad (1)$$

mass conservation

velocity (m/s)

mass conservation

velocity ($\frac{m}{s}$)

$$\boxed{D = \rho b U_0^2 \int_0^\delta \frac{u}{U_0}\left(1-\frac{u}{U_0}\right)dy.}$$

force (N)

h?

von Karman 1921 (2)

θ momentum thickness

Powerful: Can estimate force from velocity.

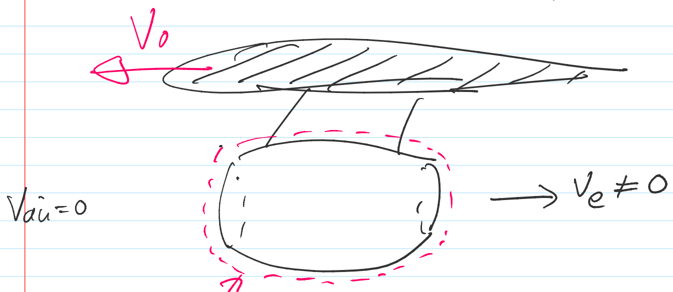
$$\text{mass}: \frac{Dm}{Dt} = 0 = \frac{d}{dt}\cancel{\int_{C.V.}\rho\, dVol}^{\,0} + \int_{C.S.}\rho(\vec{V}\cdot\vec{n})\,dA = \int_{①} + \cancel{\int_{②}} + \int_{③} + \cancel{\int_{④}}$$

$$0 = \int_0^\delta \cancel{\rho}\, u\, b\, dy - \cancel{\rho}\, U_0\, b\, h$$

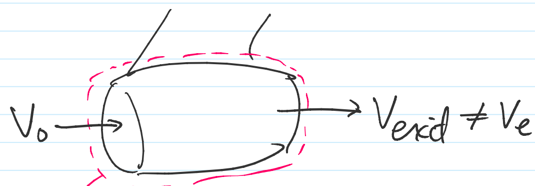
$$\boxed{U_0 h = \int_0^\delta u\, dy.}$$

mass conservation

Momentum theorem referred to coordinate moving at constant velocity.



fixed frame



C.V. moving at V_0

relative frame

(sitting on the wing)

unsteady in fixed frame.

steady in relative frame.

C.V. fixed in space