211005

2021년 10월 5일 화요일 오전 10:17

Last lecture

Momentum conservation for C.V.

Rocket

gas turbine

boundary layer → drag (force) from velocity → von Karman

Test #1 Oct. 14. Thurs 11 am – 12:15 pm

Location: TBA on ETL

cover upto material of Oct. 7.

Momentum Theorem Referred to C.V. moving at constant V_s

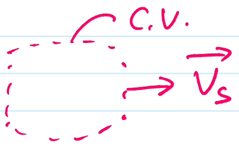
$\vec{V}_s$ = absolute velocity of C.V.

$\vec{V}$ = absolute velocity of fluid

↳ in fixed frame

$\vec{V}_r = \vec{V} - \vec{V}_s$ [velocity of fluid relative to C.V. (which is moving at $\vec{V}_s$) / relative velocity

$\vec{V} = \vec{V}_s + \vec{V}_r$



Then, we replace $\vec{V}$ with $\vec{V}_r$ in mass & momentum theorems.

mass conservation

$$\frac{Dm}{Dt} = 0 = \frac{d}{dt}\int_{C.V.} \rho \, dVol + \int_{C.S.} \rho (\vec{V}_r \cdot \vec{n}) \, dS. \qquad (1)$$

momentum conservation

$$\Sigma \vec{F} = \frac{d}{dt}\int_{C.V.} \rho \vec{V}_r \, dVol + \int_{C.S.} \rho \vec{V}_r (\vec{V}_r \cdot \vec{n}) \, dS \qquad (2)$$

Now,

$\vec{V}_s \times$ Eqn (1) + Eqn (2)

$$0 = \frac{d}{dt} \vec{V}_s \int_{C.V.} \rho \, dVol + \vec{V}_s \int_{C.S.} \rho (\vec{V}_r \cdot \vec{n}) \, dS$$

$$\hookrightarrow \quad 0 = \frac{d}{dt} \vec{V}_s \int_{C.V.} \rho \, dVol + \vec{V}_s \int_{C.S.} \rho (\vec{V}_r \cdot \vec{n}) \, dS$$

$$+ \quad \sum \vec{F} = \frac{d}{dt} \int_{C.V.} \rho \vec{V}_r \, dVol + \int_{C.S.} \rho \vec{V}_r (\vec{V}_r \cdot \vec{n}) \, dS$$

$$\sum \vec{F} = \frac{d}{dt} \int_{C.V.} \rho \vec{V} \, dVol + \int_{C.S.} \rho \vec{V} (\vec{V}_r \cdot \vec{n}) \, dS. \qquad (3)$$

Momentum theorem can be expressed as either Eqn (2) or Eqn. (3)

$\therefore$

$$\boxed{\frac{dB_{system}}{dt} = \frac{DB}{Dt} = \frac{d}{dt} \int_{C.V.} \beta \rho \, dVol + \int_{C.S.} \beta \rho (\vec{V}_r \cdot \vec{n}) \, dS}$$

RTT for C.V. moving at $\vec{V}_s$

So far, we have done conservation of

1. mass
2. momentum (linear) $\quad B = m\vec{V} \quad \beta = \vec{V}$
3. ~~3.~~ momentum (angular) $\quad B = \int (\vec{r} \times \vec{V}) \, dm \; ; \quad \beta = \vec{r} \times \vec{V}$

$$\frac{DH_o}{Dt} = \sum M_o$$

Now, move onto conservation of energy

Aside:

Thermodynamics

mass, energy } conservation.

Fluid Mechanics

mass, momentum, energy } conservation

The Energy Equation

$B = E$ = total energy of system.

$\beta = e = \frac{dE}{dm}$

1st Law of Thermodynamics.

$\dot{E} = \dot{Q} - \dot{W}$

$\cdot \rightarrow$ per unit time or rate of change in

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} \quad \text{(system)}$$

$$\frac{DE}{Dt} = \frac{dQ}{dt} - \frac{dW}{dt} = \frac{d}{dt}\int_{CV} \rho e \, dVol + \int_{CS} e \rho (\vec{V}\cdot\vec{n}) dA$$ Energy Conservation for C.V.

E = total energy of system (in C.V.)

Q = heat transfer to system (in C.V.) from surroundings.

W = work done by the system (in C.V.)

$$e = e_{internal} + e_{kinetic} + e_{potential} + \cancel{e_{other}}$$

$$e = u + \frac{V^2}{2} + gz$$

$$\frac{dQ}{dt} = \dot{Q}$$

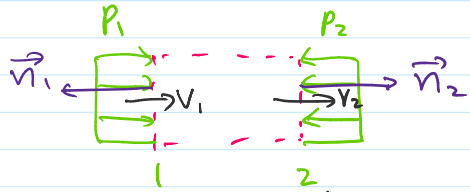
$$\frac{dW}{dt} = \dot{W} = \dot{W}_{shaft} + \underbrace{\dot{W}_{pressure}}_{\dot{W}_p} + \underbrace{\dot{W}_{viscous}}_{\dot{W}_v}$$

$\dot{W}_{shaft}$: work done by a rotating shaft in C.V.

$\dot{W}_{pressure}$: work done by fluid to overcome pressure

$\dot{W}_{viscous}$: work done by fluid to overcome shear stress

$(\frac{J}{s}) \rightarrow \dot{W}_p = \int_{C.S.} p(\vec{V}\cdot\vec{n}) dA.$



fluid gets help from P_1

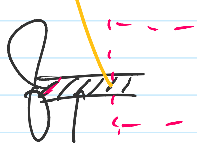
fluid has to work to overcome P_2

$$\dot{W}_v = -\int_{C.S.} \tau \cdot \vec{V} dA$$ τ = stress on fluid element dA.

τ & $\vec{V}$ in the same direction $\rightarrow$ $\tau\cdot\vec{V} > 0$ or $\dot{W}_v < 0$
fluid receives work

τ & $\vec{V}$ " opposite " $\rightarrow$ $\tau\cdot\vec{V} < 0$ or $\dot{W}_v > 0$
fluid has to do work.

$\dot{W}_v$ of shaft included in $\dot{W}_{shaft}$

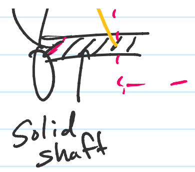


1. $\dot{W}_v = 0$ if $\vec{V} = 0$ (e.g. on wall) stationary

2. $\dot{W}_v = 0$ on solid surface (included in $\dot{W}_{shaft}$)

enthalpy internal energy

$h = u + pv = u + \frac{p}{\rho}$



2. $\dot{W}_\sigma = 0$ on solid surface (included in $\dot{W}_{shaft}$)

3. $\dot{W}_\sigma = 0$ on streamline

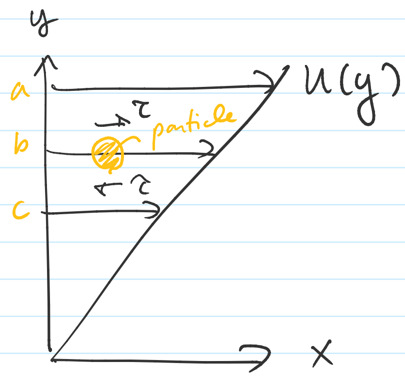
$h = u + pv = u + \frac{P}{\rho}$

Thus, (moving $\dot{W}_p$ to the RHS of Energy Conservation Eqn),

$$\dot{Q} - \dot{W}_s - \dot{W}_\sigma = \frac{d}{dt}\int_{C.V.} \left(u + \frac{V^2}{2} + gz\right)\rho \, dVol + \int_{C.S.} \Big(\underbrace{u + \frac{P}{\rho}}_{h} + \frac{V^2}{2} + gz\Big)\underbrace{\rho(\vec{V}\cdot\vec{n})dA}_{d\dot{m}}$$

$$\dot{Q} - \dot{W}_s - \dot{W}_\sigma = \frac{d}{dt}\int_{C.V.} \left(u + \frac{V^2}{2} + gz\right)\rho \, dVol + \int_{C.S.} \left(h + \frac{V^2}{2} + gz\right)\rho(\vec{V}\cdot\vec{n})dA$$

Energy Equation for Fixed C.V.



particle in layer b is pulled in +x dir by layer a

−x dir " c

If steady flow then,

$$\dot{Q} - \dot{W}_s - \dot{W}_\sigma = \int_{C.S.} \left(h + \frac{V^2}{2} + gz\right)\rho(\vec{V}\cdot\vec{n})dA$$

$q = \frac{\dot{Q}}{\dot{m}}$; $w_s = \frac{\dot{W}_s}{\dot{m}}$; $w_\sigma = \frac{\dot{W}_\sigma}{\dot{m}}$ (from mass conservation $\dot{m} = \sum_{exits} \dot{m} = \sum_{inlets} \dot{m}$)

$$q - w_s - w_\sigma = \sum_{exits}\left(h + \frac{V^2}{2} + gz\right) - \sum_{inlets}\left(h + \frac{V^2}{2} + gz\right)$$

for multiple inlets & exits.

if one exit (2) and one inlet (1), then

$$\boxed{h_1 + \frac{V_1^2}{2} + gz_1 = h_2 + \frac{V_2^2}{2} + gz_2 - q + w_s + w_\sigma}$$

Steady Flow Energy Eqn (SFEE)

$\left(\frac{J}{kg}\right)$ or $\left(\frac{m^2}{s^2}\right)$

Divide SFEE by g, then Head Form of Energy Equation

$$\boxed{z_1 + \frac{P_1}{\rho_1 g} + \frac{u_1}{g} + \frac{V_1^2}{2g} = z_2 + \underbrace{\frac{P_2}{\rho_2 g}}_{\text{pressure head}} + \frac{u_2}{g} + \underbrace{\frac{V_2^2}{2g}}_{\text{velocity head}} - \frac{q}{g} + \frac{W_s}{g} + \frac{W_v}{g}}$$

(m)

In low-speed pipe flows, no shaft work. $W_v = 0$ (on pipe walls)

$$\underbrace{\left(\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1\right)}_{\text{available head in}} = \underbrace{\left(\frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2\right)}_{\text{available head out}} + \underbrace{\left(\frac{u_2 - u_1 - q}{g}\right)}_{\text{frictional head loss}}$$

h_f

211007

2021년 10월 7일 목요일 오전 10:21

Last lecture

Momentum theorem for C.V. moving at constant $\vec{V}_s$

$$\vec{V}_r = \vec{V} - \vec{V}_s$$

Energy equation for C.V. $B = E,\ \beta = e$

$\dot{E} = \dot{Q} - \dot{W}$ $e = u + \frac{V^2}{2} + gz$

$\dot{W} = \dot{W}_{shaft} + \dot{W}_{pressure} + \dot{W}_{viscous}$

Steady Flow Energy Eqn (SFEE)

Head form of Energy Eqn.

HW #4 3-174, 176, 178, 180, 185 Due Oct. 12

Test #1 11:00-12:15 Thurs Oct. 14 in-person (offline)

location posted on ETL.

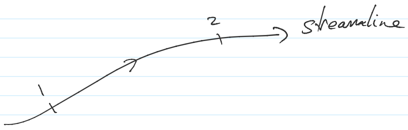
→ cover upto Chapter 3.

Steady Form of Energy Eqn. (SFEE).

$$z_1 + \frac{p_1}{\rho_1 g} + \frac{u_1}{g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho_2 g} + \frac{u_2}{g} + \frac{V_2^2}{2g} - \frac{q}{g} + \frac{w_s}{g} + \frac{w_v}{g}$$

Assumptions

1. steady
2. incompressible ($\rho_1 = \rho_2 = \rho$)
3. inviscid → $W_v = 0$
4. no shaft work → $W_s = 0$
5. adiabatic → $q = 0$
6. follow along a single streamline between 1 and 2



then, SFEE becomes the $\boxed{\text{Bernoulli Equation}}$

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} = \text{Bernoulli constant}$$

$u_1 = u_2$ if inviscid & adiabatic → $u(T)$

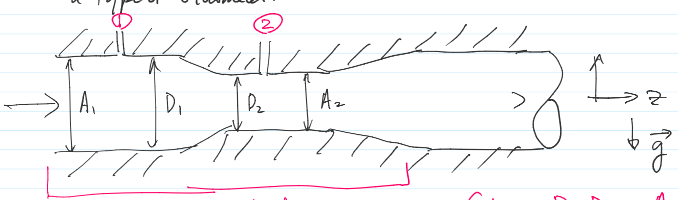
T can change via
1. heat transfer
2. viscous dissipation (e.g. engine oil)

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Applications of Bernoulli Equation.

1. Venturi tube → to measure $\dot{m}$ (low speed → incompressible)

a type of flowmeter.



Given: $D_1, D_2 \to A_1, A_2$

ρ

Bernoulli ↑ inviscid $\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$

$z_1 = z_2$

$$V_2^2 - V_1^2 = \frac{2\Delta p}{\rho}$$

$\Delta p = p_1 - p_2 > 0$

because $V_2 > V_1 \to A_1 > A_2$

mass conservation $\cancel{\rho} A_1 V_1 = \cancel{\rho} A_2 V_2$

$A_1 V_1 = A_2 V_2$

if circular pipe $A = \pi D^2/4$

$D_1^2 V_1 = D_2^2 V_2$

$V_1 = \beta^2 V_2$; $\beta = \frac{D_2}{D_1}$

thus $V_2 = \left[\frac{2\Delta p}{\ldots} \right]^{1/2}$

thus $\quad V_2 = \left[\dfrac{2 \Delta p}{\rho (1-\beta^4)} \right]^{1/2}$

$\dot{m}_{ideal} = \rho A_2 V_2$

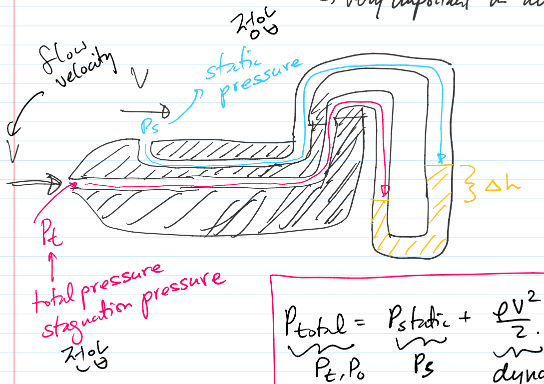
$\dot{m}_{actual} = C_d \, \dot{m}_{ideal}$

$C_d(Re, Ma, \text{geometry})$

discharge coefficient.

2. Pitot tube → measures velocity

↳ very important in airplanes.



$$\boxed{P_{total} = P_{static} + \frac{\rho V^2}{2}}$$

P_{total}: P_t, P_0 $\quad$ P_{static}: P_s $\quad$ $\frac{\rho V^2}{2}$: dynamic pressure (동압.)

if $V=0$ → then $P_t = P_s$

$V \neq 0$ → $P_t > P_s$

Pitot measures $(P_t - P_s)$

Flow Velocity $V = \sqrt{\dfrac{2(P_t - P_s)}{\rho}}$

END of Ch. 3 (Test #1)

Steady flow → $\dfrac{\partial}{\partial t} = 0$ at a given location.

Chapter 4: Differential Relations for a Fluid Particle.

Same as Ch. 3 but Ch. 4 deals with a small C.V. (differential)

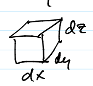
Ch. 3: integral relations, large (integral) C.V.

e.g.: rocket, gas turbine, compressor, turbine

output: $\dot{m}$, power, force — integral quantities.

algebraic eqn

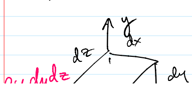
Ch. 4: e.g.: $dx\,dy\,dz$



output: distributions of $\vec{V}(x,y,z)$, $p(x,y,z)$ — more detailed information

differential eqn.

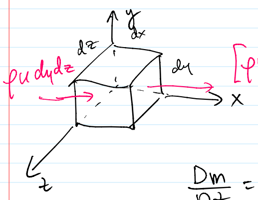
Mass conservation.



$\left[\rho u + \frac{\partial}{\partial x}(\rho u)\,dx\right] dydz$

$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

// Mass Conservation.



$\left[\rho u + \frac{\partial}{\partial x}(\rho u)\, dx\right] dy dz$

$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

on each surface, uniform flow

$$\frac{Dm}{Dt} = 0 = \frac{d}{dt}\int_{C.V.} \rho \, dVol + \int_{C.S.} \rho (\vec{V}\cdot\vec{n})\, dA$$

x: $\left[\rho u + \frac{\partial (\rho u)}{\partial x} dx\right] dy\, dz - \rho u \, dy dz \Rightarrow \frac{\partial}{\partial x}(\rho u)\, dx dy dz$

y: $\Rightarrow \frac{\partial}{\partial y}(\rho v)\, dy dx dz$

z: $\Rightarrow \frac{\partial}{\partial z}(\rho w)\, dz dx dy$

thus $\frac{Dm}{Dt} = 0 = \frac{\partial \rho}{\partial t} dx dy dz + \frac{\partial}{\partial x}(\rho u)\, dx dy dz + \frac{\partial}{\partial y}(\rho v)\, dy dx dz + \frac{\partial}{\partial z}(\rho w)\, dz dx dy$

divide by $dx dy dz$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

Differential form of Mass Conservation
↓
Continuity Eqn.

where $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0}$$

nonlinear differential equation.

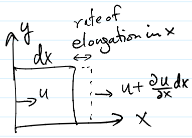
if steady ($\frac{\partial}{\partial t} = 0$)

$\nabla\cdot(\rho\vec{V}) = 0$

if incompressible (ρ = constant)

$\nabla\cdot\vec{V} = 0$ linear eqn.

physical meaning of $\nabla\cdot\vec{V}$



$\nabla\cdot\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

rate of strain in x

rate of change in volume of fluid element.