211012

2021년 10월 12일 화요일 오전 10:14

Last lecture

Bernoulli Equation as a special case of energy equation.

Pitot → Total or stagnation p; static pressure; dynamic pressure

Venturi

Ch. 4: Differential Relations.

Mass conservation.

1. Test #1 Oct 14 11-12:15 301-204 & 301-306
(upto Ch. 3)

2. $\boxed{\text{Final Exam Dec 9 11-13:30 ? Tentative date}}$
Dec 14 11-13:30 ?

Today: Differential form of momentum conservation

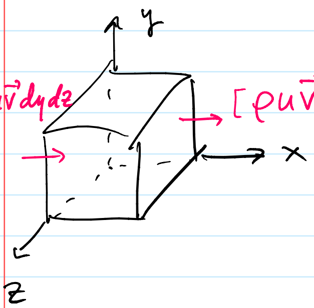
$$\underline{\sum d\vec{F} = \frac{d}{dt}\int_{C.V.}\rho\vec{V}\,dVol + \int_{C.S.}\rho\vec{V}(\vec{V}\cdot\vec{n})\,dA}$$

stationary fixed C.V.

$\frac{\partial}{\partial t}$

$\frac{\partial}{\partial t}(\rho\vec{V})\,dxdydz$

$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$



$\rho u\vec{V}dydz$

$[\rho u\vec{V} + \frac{\partial}{\partial x}(\rho u\vec{V})dx]dydz$

in x: $[\rho u\vec{V} + \frac{\partial}{\partial x}(\rho u\vec{V})dx]dydz - \rho u\vec{V}dydz = \frac{\partial}{\partial x}(\rho u\vec{V})dxdydz$

y: $\frac{\partial}{\partial y}(\rho v\vec{V})dydxdz$

z: $\frac{\partial}{\partial z}(\rho w\vec{V})dzdydx$

$$\sum d\vec{F} = dxdydz\left[\frac{\partial}{\partial t}(\rho\vec{V}) + \frac{\partial}{\partial x}(\rho u\vec{V}) + \frac{\partial}{\partial y}(\rho v\vec{V}) + \frac{\partial}{\partial z}(\rho w\vec{V})\right]$$

$$\sum d\vec{F} = dxdydz\left[\vec{V}\underbrace{\left[\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{V})\right]}_{=0} + \rho\underbrace{\left(\frac{\partial\vec{V}}{\partial t} + u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z}\right)}\right]$$

Total derivative of $\vec{V}$
Substantial " " $\vec{V}$
Material " " $\vec{V}$

rate of change in $\vec{V}$
following a particle

$$\frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt}$$

$$\Sigma d\vec{F} = \rho \frac{D\vec{V}}{Dt} dx\,dy\,dz$$

$$\frac{\Sigma d\vec{F}}{dx\,dy\,dz} = \rho \frac{D\vec{V}}{Dt}$$

forces in differential form per unit volume.

body: $\rho \vec{g}$

σ_{ij} stress tensor

Surface: $-\nabla p + \left(\frac{d\vec{F}}{d\,vol}\right)_{visc.}$

surface normal vector in i dir.

shear τ acting in j direction.

$$-\frac{\partial p}{\partial x}\vec{i} + \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)\vec{i}$$

$$-\frac{\partial p}{\partial y}\vec{j} + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right)\vec{j}$$

$$-\frac{\partial p}{\partial z}\vec{k} + \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right)\vec{k}$$

Newtonian fluid $\tau = \mu \cdot$ strain rate $\rightarrow \mu \cdot$ velocity gradient

for incompressible (ρ = constant) flows.

4 equations : mass
x-mom
y-mom
z-mom

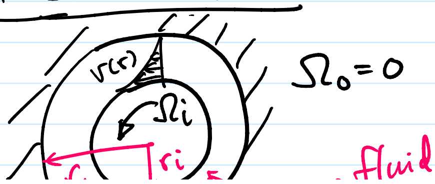
4 unknowns : p, u, v, w

Thermodynamics (1-D) mass + energy eqns

Fluid mechanics (1-D) incompressible $\rightarrow$ mass + mom eqn.
compressible $\rightarrow$ mass + mom + energy + state

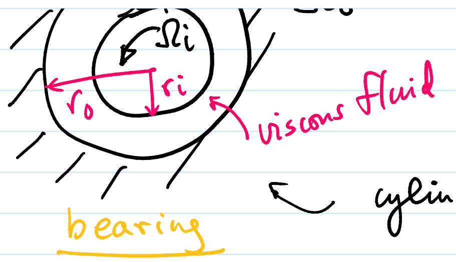
Some examples of solutions. $\rightarrow$ 1. Incompressible
2. Viscous

#1 Couette Flow.



$\Omega_o = 0$

e.g. washing machine
pump
rotating shaft



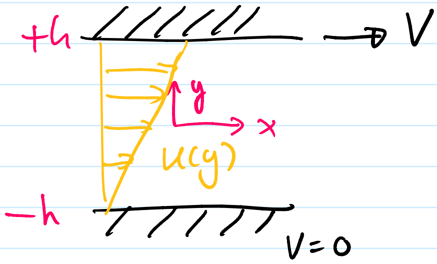
pump
rotating shaft

cylindrical coordinates. $(r\theta z)$

$\hookrightarrow v_\theta(r) = C_1 r + \frac{C_2}{r}$

↓ simplify.

2-D in Cartesian coordinate.



viscous flow between fixed & moving plates.

1. Incompressible
2. Viscous
3. steady $\left(\frac{\partial}{\partial t} = 0\right)$
4. neglect $\vec{g}$
5. $v = w = 0$
6. $\frac{\partial p}{\partial x} = 0$
7. $\frac{\partial}{\partial z} = 0$

mass: if incompressible (1) then $\nabla \cdot \vec{V} = 0$

continuity

$$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{5} + \cancel{\frac{\partial w}{\partial z}}^{5} = 0 \rightarrow u(y)$$

x-mom:
$$\rho\left(\cancel{\frac{\partial u}{\partial t}}^{3} + \cancel{u\frac{\partial u}{\partial x}}^{\text{continuity}} + \cancel{v}^{5}\frac{\partial u}{\partial y} + \cancel{w}^{5}\frac{\partial u}{\partial z}\right) = -\cancel{\frac{\partial p}{\partial x}}^{6} + \cancel{\rho g_x}^{4} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{\text{continuity}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}^{7}\right)$$

$$\mu \frac{d^2 u}{dy^2} = 0$$

↓

$$u(y) = C_1 y + C_2$$ linear velocity distribution

C_1? C_2?

Boundary conditions: at $y = +h$, $u(+h) = V$

$y = -h$, $u(-h) = 0$

$$C_1 = \frac{V}{2h}; \quad C_2 = \frac{V}{2}.$$

At the interface between a viscous fluid and a solid wall, NO-SLIP BOUNDARY CONDITION exists.

τ? $\quad \tau = \mu \frac{du}{dy}$

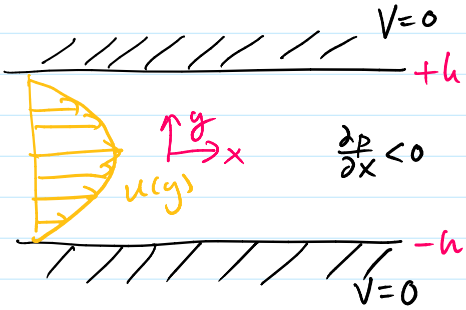
BOUNDARY CONDITION exists.

$\vec{V}_{solid} = \vec{V}_{fluid}$ at the interface

#2 Poiseuille flow

$\left(\frac{\partial p}{\partial x} \neq 0\right)$

1. Incompressible
2. Viscous
3. steady
4. neglect $\vec{g}$
5. $v = w = 0$
6. $\frac{\partial}{\partial z} = 0$



mass: $\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{5} + \cancel{\frac{\partial w}{\partial z}}^{5} = 0 \rightarrow u(y)$

X-mom: $0 = -\frac{\partial p}{\partial x} + \cancel{\rho g_x}^{4} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{\text{continuity}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}^{6.}\right)$

$$0 = -\frac{dp}{dx} + \mu\frac{d^2u}{dy^2}$$

$$u(y) = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2.$$

B.C. @ $y = \pm h$, $u(+h) = u(-h) = 0$

$$C_1 = 0$$

$$C_2 = -\frac{dp}{dx}\frac{h^2}{2\mu}$$

$$\underline{u(y) = -\frac{dp}{dx}\frac{h^2}{2\mu}\left(1 - \frac{y^2}{h^2}\right)}$$

u_{max} at $y=0$ $\rightarrow$ $\underline{u_{max} = \left(-\frac{dp}{dx}\right)\frac{h^2}{2\mu}} > 0$ because $\frac{dp}{dx} < 0$

τ at the wall and/or in the fluid

$$\tau_{wall} = \mu\left(\frac{du}{dy}\right)_{y=\pm h}$$

$$\tau_{wall} = -\frac{2\mu U_{max}}{h} \quad \text{at } y = +h$$

$$+\frac{2\mu U_{max}}{h} \quad \text{at } y = -h.$$

211019

2021년 10월 19일 화요일 오전 10:47

Last lecture

Momentum eqn for Newtonian fluid in differential form

Navier-Stokes Eqn.

Viscous solutions

- Couette
- Poiseuille

Boundary conditions.

1. fluid & wall interface

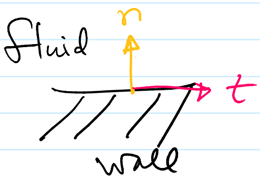
$\vec{V}_{fluid} = \vec{V}_{wall}$ (except in rarefied flow (space))

Viscous flow

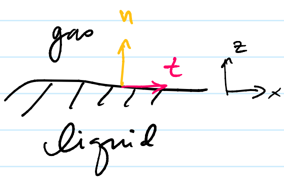
$V_{n\,fluid} = V_{n\,wall}$ (impermeable)

$V_{t\,fluid} = V_{t\,wall}$ (viscous) ; $V_{t\,fluid} \neq V_{t\,wall}$ (inviscid) (rarefied)

extremely low density



2. gas & liquid interface



1. $V_{n\,gas} = V_{n\,liquid}$

2. $\tau_{gas} = \tau_{liquid}$

τ_{zy}

τ_{zx}

3. $P_{liquid} \approx P_{gas}$ (neglecting surface tension) curvature

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Functions.

1. Stream function ψ

Assume 1. steady 2. incompressible 3. 2-D flow

continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ $\qquad \vec{V} = u\vec{i} + v\vec{j}$

introduce a stream function ψ (definition: $u = \frac{\partial \psi}{\partial y}$ & $v = -\frac{\partial \psi}{\partial x}$)

continuity equation is automatically satisfied

solve the momentum equation for ψ.

Continuity equation is automatically satisfied

solve the momentum equation for ψ.

Relationship between streamline and stream function ψ?

$u = \frac{\partial \psi}{\partial y}$ & $v = -\frac{\partial \psi}{\partial x}$

tangent to velocity

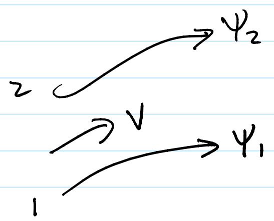
$$\left.\frac{dy}{dx}\right| = \frac{v}{u}$$

$$\frac{dx}{u} = \frac{dy}{v} = \cancel{\frac{dz}{w}} \quad \text{2-D}$$

$$u\,dy - v\,dx = 0$$

$$\frac{\partial \psi}{\partial y} dy + \frac{\partial \psi}{\partial x} dx = d\psi = 0$$

↳ ψ = constant on a streamline $\quad \left.\frac{dy}{dx}\right|_{\psi = const.} = \frac{v}{u}$.



$$\psi_2 - \psi_1 = \int_1^2 d\psi = \int_1^2 (\vec{V} \cdot \vec{n})\, dA = Q_{1-2}$$

volume flowrate between streamlines 1 & 2.

$\nabla \times \vec{V} \rightarrow$ $\nabla \times \vec{V} = 0$ irrotational flow (no rigid body motion)

$\nabla \times \vec{V} \neq 0$ rotational flow (rigid body motion)

Vorticity

↳ $\nabla \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \rightarrow$ for 2-D flow $\rightarrow \nabla \times \vec{V} = -\nabla^2 \psi \vec{k}$

LaPlace Equation $\quad \nabla^2 \psi = 0$ if irrotational flow

Poisson Equation $\quad \nabla^2 \psi = $ constant if rotational flow

Vorticity $\vec{\Omega}$ & Irrotationality.

↳ $\nabla \times \vec{V} = 0$ zero vorticity in irrotational flow

$\vec{\Omega} = \nabla \times \vec{V}$

Angular velocity $= \frac{1}{2} \vec{\Omega}$

for 2-D flow in xy plane

angular velocity $= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

2. Potential function ϕ

if flow is irrotational $\nabla \times \vec{V} = 0$, then potential function ϕ exists.

$$\vec{V} = \nabla \phi$$

$$\phi \rightarrow \quad u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = \frac{\partial \phi}{\partial z}$$

for 1. steady 2. incompressible 3. 2-D (ψ) 4. irrotational (ϕ) flow

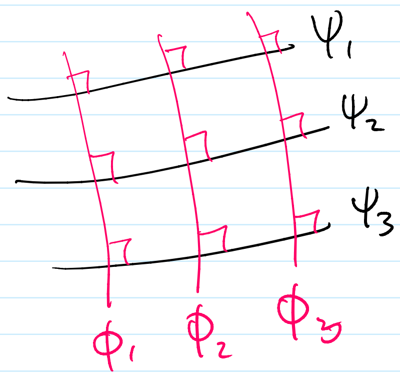
$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y}$$

on a constant ϕ line $\rightarrow$ $d\phi = 0 = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy$

$$d\phi = 0 = u\,dx + v\,dy$$

$$\left.\frac{dy}{dx}\right|_{\phi = \text{constant}} = -\frac{u}{v} \quad \text{vs.} \quad \left.\frac{dy}{dx}\right|_{\psi = \text{constant}} = \frac{v}{u}$$



if incompressible flow

$$\nabla \cdot \vec{V} = 0$$

if incompressible & irrotational flow

$$\nabla \cdot (\nabla \phi) = \nabla^2 \phi = 0$$

if incompressible & 2-D

$\downarrow$

ψ

$\downarrow$

$\nabla^2 \psi = 0$ irrotational

$\nabla^2 \psi = c$ rotational

$\nabla \psi = c$ [illegible]

LaPlace Eqn & Poisson Eqn $\rightarrow$ linear differential equations

superposition possible.

if a is a solution & b is a solution, so is a+b.