211021

2021년 10월 21일 목요일 오전 10:27

Last lecture

ψ, stream function & ϕ, potential function

$\left.\frac{dy}{dx}\right|_{\psi=c} \perp \left.\frac{dy}{dx}\right|_{\phi=c}$

$\nabla\times\vec{V}$ vorticity → rotationality → rigid body rotation

incompressible → $\nabla\cdot\vec{V}=0$ → $\nabla^2\phi=0$
irrotational → $\nabla\times\vec{V}=0$ → $\nabla^2\psi=0$ } linear d.e. → superposition

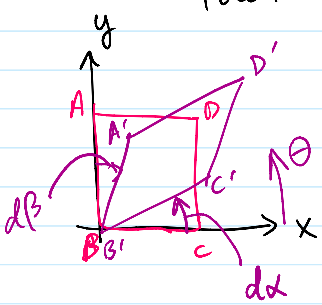
$$\boxed{\begin{array}{ll} \text{HW\#5} & \text{4-2, 11, 17} \\ & \text{-33, 35, 36} \quad \text{Due Oct. 26} \\ & \text{-56, 60} \\ & \text{-69, 71} \end{array}}$$

Revisit

vorticity $\nabla\times\vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\vec{i} - \left(\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}\right)\vec{j} + \left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\vec{k}$

if 2-D flow $\vec{V}=u\vec{i}+v\vec{j}$ & $\frac{\partial}{\partial z}=0$

then $\nabla\times\vec{V} = \left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\vec{k}$ ← physical meaning?



angular velocity of fluid element ABCD about z-axis

$\hookrightarrow = \frac{1}{2}\left(\frac{d\alpha}{dt}-\frac{d\beta}{dt}\right)$

angular velocity about z-axis $= \frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$

gradients of velocity components.

- if uniform flow then $\nabla\times\vec{V}=0$

Angular velocity fluid element $= \frac{1}{2}\nabla\times\vec{V}$

- for non-uniform flow, $\nabla\times\vec{V}\neq 0$

Revisit

Bernoulli Equation

For inviscid flow

$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \nabla p$$ Euler equation (momentum Navier-Stokes eqn for inviscid flow)

$$\rho \left[\frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V} \right] = \rho\vec{g} - \nabla p$$

Vector identity $(\vec{V}\cdot\nabla)\vec{V} = \nabla\left(\frac{V^2}{2}\right) + \underbrace{\nabla\times\vec{V}}_{\text{vorticity}}\times\vec{V}$

$$\left[\frac{\partial\vec{V}}{\partial t} + \nabla\left(\frac{V^2}{2}\right) + \nabla\times\vec{V}\times\vec{V} + \frac{\nabla p}{\rho} - \vec{g}\right]\cdot d\vec{r} = 0$$

for some cases, $\nabla\times\vec{V}\times\vec{V}\cdot d\vec{r} = 0$

1. $\vec{V} = 0$
2. $\nabla\times\vec{V} = 0 \rightarrow$ irrotational flow
3. $\nabla\times\vec{V}\times\vec{V} \perp d\vec{r}$ (Beltrami flow)
4. $d\vec{r} \parallel \vec{V}$ (along the streamline)

common

if inviscid & irrotational

invisid & along streamline (if rotational)

then, $$\frac{\partial\vec{V}}{\partial t}\cdot d\vec{r} + d\left(\frac{V^2}{2}\right) + \frac{dp}{\rho} + g\,dz = 0$$

Unsteady Bernoulli Equation

$$\int_1^2 \frac{\partial\vec{V}}{\partial t}\cdot d\vec{r} + \int_1^2 \frac{dp}{\rho} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0$$

if steady & incompressible, then recover the Bernoulli equation from Ch. 3.

if rotational → follow a streamline

irrotational → do not have to follow a single streamline

Revisit

Continuity Eqn.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad ; \quad \vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

if ρ = constant then $\rightarrow$ $\nabla \cdot \vec{V} = 0$

incompressible

water $\rightarrow$ liquid $\rightarrow$ incompressible

air $\rightarrow$ gas $\rightarrow$ incompressible?

compressible?

air is a gas which is compressible.

air $\rightarrow$ compressible fluid.

In some cases, air flow can be considered incompressible

for steady flow, continuity eqn

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

for simplicity, consider 1-D flow, $\vec{V} = u\vec{i}$

$$\frac{\partial}{\partial x}(\rho u) = u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} \approx \rho\frac{\partial u}{\partial x} \quad \text{if} \quad u\frac{\partial \rho}{\partial x} << \rho\frac{\partial u}{\partial x}$$

now, examine

$$u\frac{\partial \rho}{\partial x} << \rho\frac{\partial u}{\partial x}$$

$$\frac{\partial \rho}{\rho} << \frac{\partial u}{u}$$

($\partial \rho = \frac{\partial p}{a^2}$ where a = speed of sound in air)

$$\frac{\partial p}{\rho a^2} << \frac{\partial u}{u}$$

$$\frac{\partial p}{\rho a^2} << \frac{\partial u}{u}$$

(from Bernoulli equation where $z_1 = z_2$, $\partial p = -\rho u \partial u$)

$z + p + \rho \frac{u^2}{2} = \text{constant}$

thus, $-\frac{u \partial u}{a^2} << \frac{\partial u}{u}$

$$\left| \frac{u^2}{a^2} \right| << 1$$

$$Ma,\ M \equiv u/a$$

$$Ma^2 << 1 \quad \text{(low speed)}$$

So, if $Ma^2 << 1$, then even gas flow can be considered incompressible

e.g. $Ma^2 \sim 0.09$ then $Ma^2 << 1$ satisfied

$\downarrow$

$Ma \sim 0.3$

if $Ma < 0.3$ then flow incompressible

e.g. KTX train speed 300 km/h $\rightarrow$

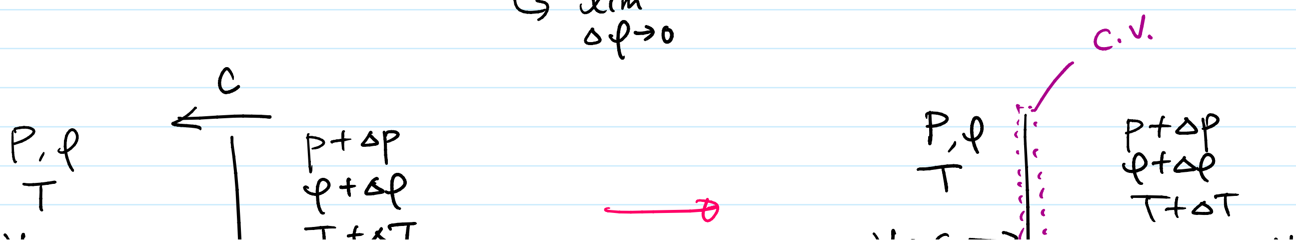
$$300 \frac{km}{h} \cdot \frac{hr}{3600\,s} \cdot \frac{1000\,m}{km} = 83\ m/s$$

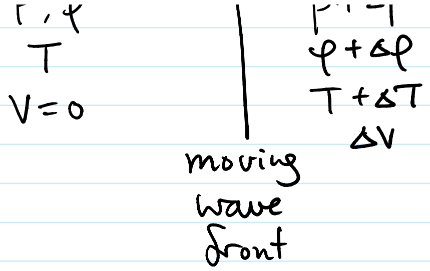
$a \approx 300$ m/s for air at STP

$\therefore$ air flow around KTX train $\rightarrow$ incompressible

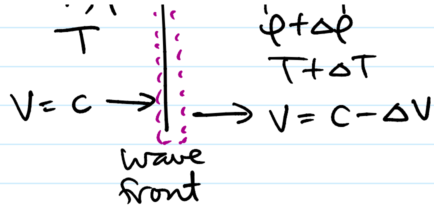
$a \rightarrow$ speed at which a sound wave travels in a stationary fluid

$\hookrightarrow \lim_{\Delta\rho \to 0}$





Absolute frame



Relative frame
flow is steady

For the C.V.

mass. $\rho A C = (\rho + \Delta\rho) A (C - \Delta V)$

$$\Delta V = C \frac{\Delta\rho}{\rho + \Delta\rho}$$

momentum $pA - (p + \Delta p)A = (\rho A c)(c - \Delta V - c)$

neglect τ, g

$$\Delta p = \rho c \Delta V$$

substituting for ΔV

$$c^2 = \frac{\Delta p}{\Delta\rho}\left(1 + \frac{\Delta\rho}{\rho}\right)$$

for sound wave $\Delta\rho \to 0$ and $c \to a$

$$a^2 = \frac{\partial p}{\partial \rho}$$

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2021년 10월 28일 목요일 오전 10:24

Last lecture

Vorticity & angular velocity

Unsteady Bernoulli Eqn.

Incompressible flow if $Ma^2 << 1$ or $Ma < 0.3$

Begin Ch. 5 Dimensional Analysis → Experiments.

To solve fluid dynamic probs.

1. Analytical approaches Ch. 3 → algebraic eqns ✓
   Ch. 4 → differential eqns ✓
2. Numerical approach → discretize differential eqns → difference eqns → use computer
3. Experimental approaches → Ch. 5 → dimensional analysis ✓

✓ → what we cover this semester.

Why do we need experiments? → require much time & expense. → careful planning is needed

1. real world problems very complex
2. need to verify and/or validate analytical/numerical solutions
   ALWAYS

Dimensions & Units

Dimensions → exist in nature

Units → invented by people

Dimensions		SI	British
mass	M	kg	lbm
length	L	m	ft
time	T	sec	sec
temperature	Θ	K	R

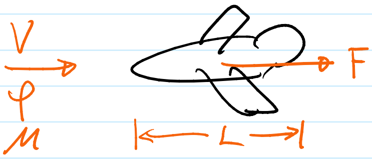
Conduct a dimensional analysis to identify nondimensional parameters (π's) which influence the fluid dynamic phenomenon of our interest

π's help us to

π's help us to

1. identify which parameters are important
2. figure out how to scale experimental results
3. save time & money for experiments

e.g.



$F(L, V, \rho, \mu) \rightarrow$ Dimensional Analysis $\rightarrow \dfrac{F}{\rho V^2 L^2}\left(\dfrac{\rho V L}{\mu}\right)$

$\underline{\pi_1 = \dfrac{F}{\rho V^2 L^2}}$ & $\underbrace{\pi_2 = \dfrac{\rho V L}{\mu}}_{Re\,\#}$

Similarity between $\overset{m}{\text{model}}$ & $\overset{p}{\text{prototype}}$

→ real size but not yet mass production (test devices)

(1/10, 1/100, etc.)
→ Scaled model for testing

→ In the example above,

if $\pi_{2_m} = \pi_{2_p}$ then $\pi_{1_m} = \pi_{1_p}$

Principle of Dimensional Homogeneity

If an equation expresses a <u>proper</u> relationship among variables in a physical process, it will be <u>dimensionally homogeneous</u> (i.e. all the terms in the equation will have the same dimension).

examples : $X(t) = X_0 + V_0 t + \frac{1}{2} a t^2 \rightarrow$ LENGTH, L

$p + \frac{1}{2}\rho V^2 + \rho g z = \text{constant} \rightarrow \dfrac{M}{LT^2}$

pressure $\rightarrow \dfrac{\text{Force}}{\text{Area}} \rightarrow \dfrac{ML}{T^2} \cdot \dfrac{1}{L^2} = \dfrac{M}{LT^2}$

pressure $\rightarrow \frac{\text{Force}}{\text{Area}} \rightarrow \frac{ML}{T^2} \cdot \frac{1}{L^2} = \frac{M}{LT^2}$

helps to check if answer is correct.

example of a wrong (not general) answer → empirical eqn from curve-fitting some data

e.g.

$$Q = 61.9\, D^{2.63} \left(\frac{dp}{dx}\right)^{0.54}$$

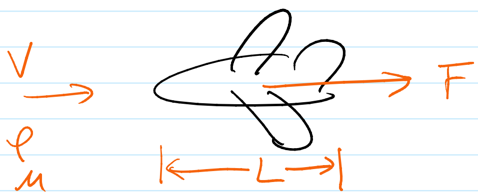
cannot be generalized.

volume flow rate (m^3/s)

Dimension: $\frac{L^3}{T} \neq L^{2.63} \left(\frac{M}{L^2T^2}\right)^{0.54}$

Buckingham Pi (π) Theorem → nondimensionalize variables

If there are n variables which are important in a physical phenomenon and those variables have j dimensions, then there will be $k = n - j$ nondimensional parameters.



$F, L, V, \rho, \mu \rightarrow n = 5$

$\frac{ML}{T^2}$, L, $\frac{L}{T}$, $\frac{M}{L^3}$, $\frac{M}{LT} \rightarrow j = 3$

$k = 5 - 3 = 2$

$\pi_1 = \frac{F}{\rho V^2 L^2}$ & $\pi_2 = \frac{\rho V L}{\mu}$

For the example above,

1. Find $n = 5$
   $j = 3$

2. Select variables which cannot form a nondimensional parameter (π) $\rightarrow$ $\underline{\rho, V, L}$

   $\uparrow$

   $\frac{M}{L^3}$ $\frac{L}{T}$ L

3. Expect $k = 5 - 3 = 2$ π's.

$$\pi_1 = L^a V^b \rho^c F = M^0 L^0 T^0$$

$$L^a (LT^{-1})^b (ML^{-3})^c (MLT^{-2}) = M^0 L^0 T^0$$

$$\left.\begin{array}{l} a = -2 \\ b = -2 \\ c = -1 \end{array}\right\} \rightarrow \pi_1 = \frac{F}{\rho V^2 L^2} = C_F$$

$$\pi_2 = L^a V^b \rho^c \mu = L^a (LT^{-1})^b (ML^{-3})^c (ML^{-1}T^{-1}) = M^0 L^0 T^0$$

$$\left.\begin{array}{l} a = -1 \\ b = -1 \\ c = -1 \end{array}\right\} \rightarrow \pi_2 = \frac{\mu}{\rho V L} = \frac{1}{Re}$$

Dimensionally, Nondimensionally

$\underbrace{F(\rho, V, L, \mu)}_{\text{4 independent variables}}$ $\rightarrow$ Dimensional Analysis $\rightarrow$ $C_F(Re)$

$\uparrow$ 1 independent variable

Similarity between model (m) & prototype (p)

(have identified k π's)

If $\pi_{i_m} = \pi_{i_p}$ for $i = 2, \dots, n$

then $\pi_{1_m} = \pi_{1_p}$

In reality, difficult to match all π's.

Another way to obtain π's $\rightarrow$ nondimensionalize equations

$$\nabla \cdot \vec{V} = 0 \qquad \text{incompressible flow}$$

$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \nabla p + \mu \nabla^2 \vec{V}$$