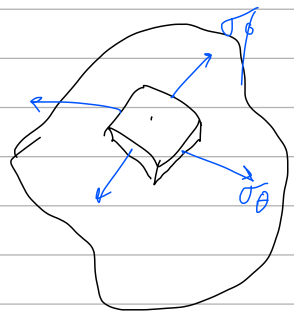
3.6. Thin-walled pressure vessels



$$\frac{p}{t} = \frac{\sigma_\theta}{r_\theta} + \frac{\sigma_\phi}{r_\phi}$$

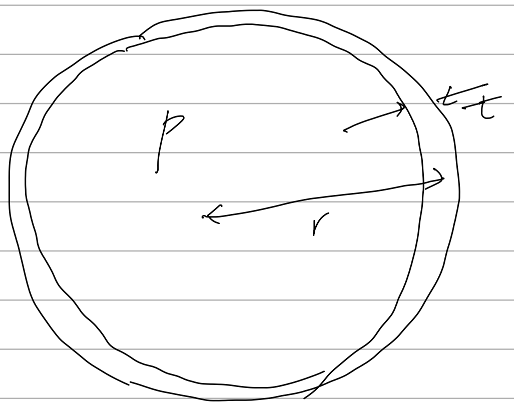
→ Membrane equation

$N_\theta = \sigma_\theta \cdot t$

$N_\phi = \sigma_\phi \cdot t$

$$p = \frac{N_\theta}{r_\theta} + \frac{N_\phi}{r_\phi}$$

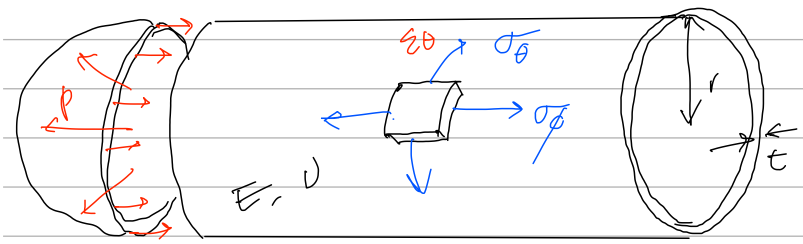
Spherical membrane



$$\frac{p}{t} = \frac{2\sigma}{r}$$

$$\rightarrow \sigma = \frac{pr}{2t}$$

Circular cylinder



σ_θ : hoop stress

σ_ϕ : axial stress

$$\frac{p}{t} = \frac{\sigma_\theta}{r_\theta} + \frac{\sigma_\phi}{r_\phi}$$

r_θ: r r_ϕ: ∞

$$\sigma_\theta = \frac{pr}{t}$$

$$2\pi r \cdot t \cdot \sigma_\phi = \pi r^2 \cdot p$$

$$\sigma_\phi = \frac{pr}{2t}$$

What would be the change of radius when $p : 0 \to p$

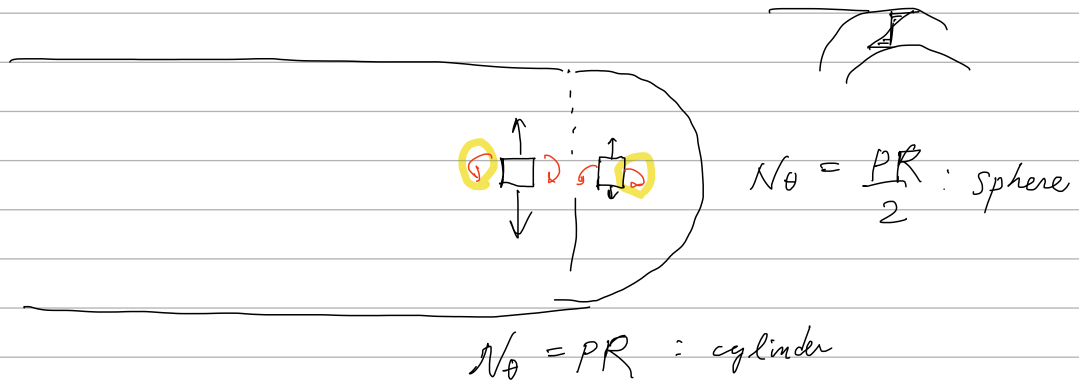
$$\varepsilon_\theta = \frac{\sigma_\theta}{E} - \nu \frac{\sigma_\phi}{E}$$

$$= \frac{1}{E}\left(\frac{pr}{t} - \frac{\nu pr}{2t}\right)$$

$$= \frac{pr}{Et}\left(1 - \frac{\nu}{2}\right)$$

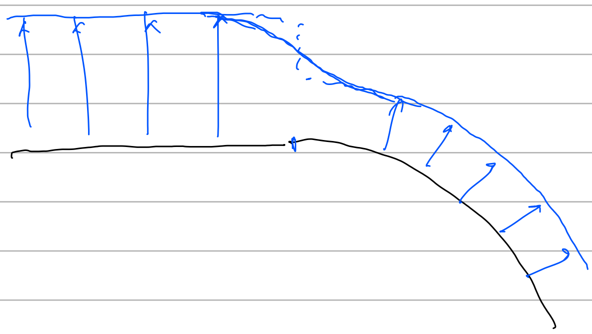
$$\frac{\Delta r}{r} = \varepsilon_\theta$$

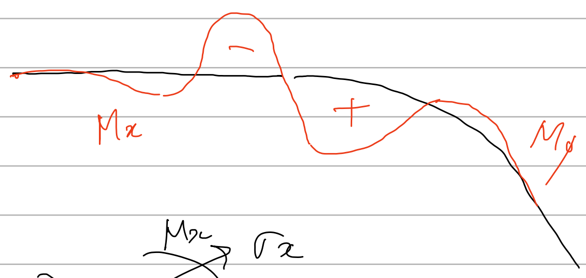
5-4.2. Boiler end

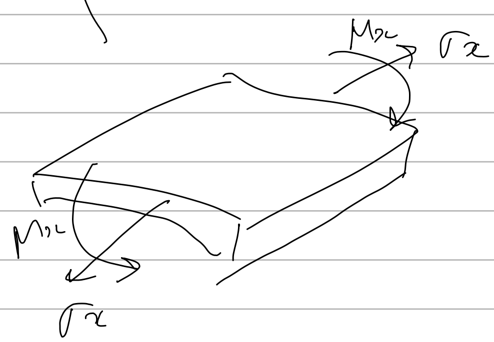


Sudden discontinuity in hoop stress

→ induces an additional distribution of bending moments

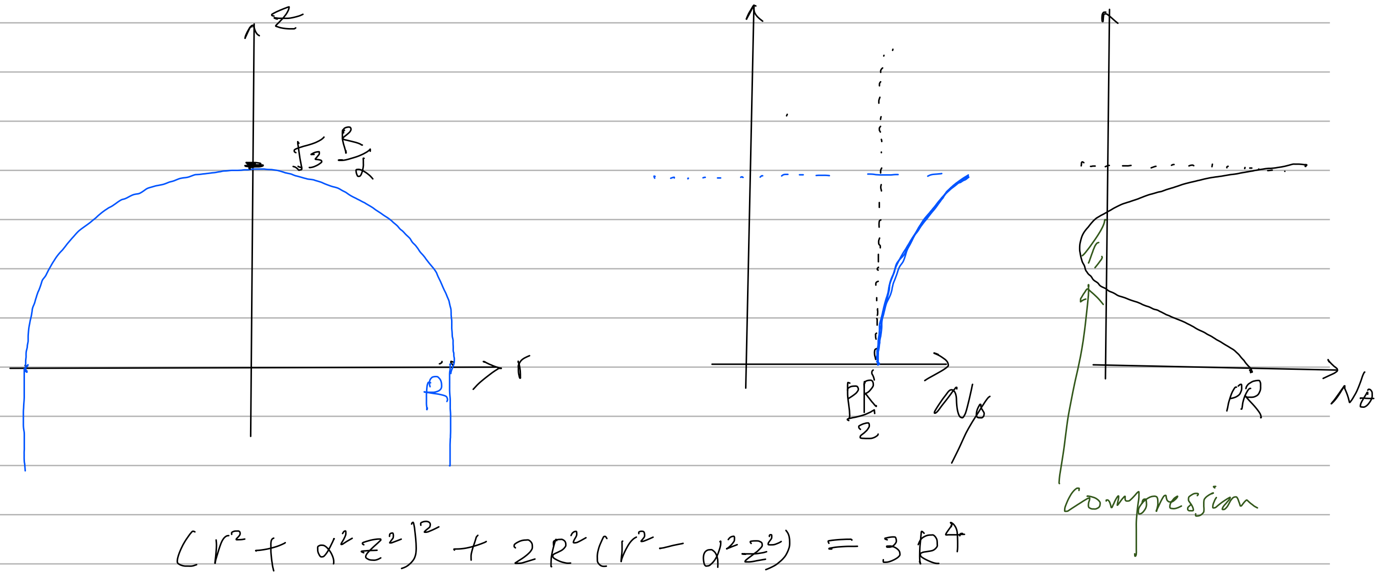






Discontinuity in the hoop forces is caused by the sudden change in the curvature from $0 \to 1/R$

→ Second order continuous profile



$$(r^2+\alpha^2 z^2)^2 + 2R^2(r^2-\alpha^2 z^2) = 3R^4$$

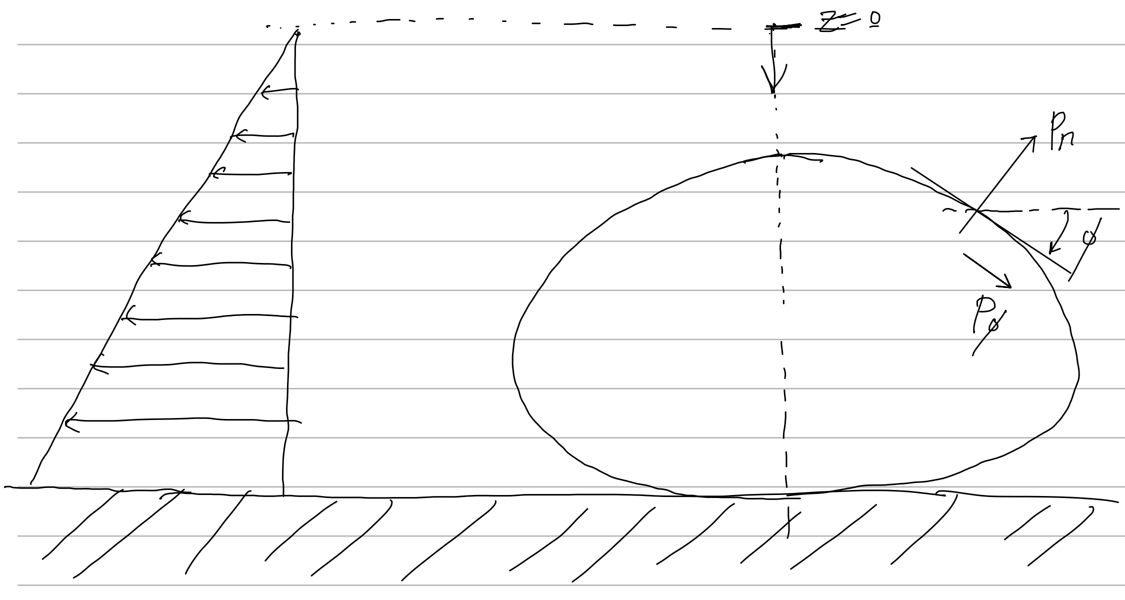
$r=0$: $\alpha^4 z^4 - 2R^2\alpha^2 z^2 - 3R^4 = 0$

$$(\alpha^2 z^2 - 3R^2)(\alpha^2 z^2 + R^2) = 0$$

$$z = \sqrt{3}\,\frac{R}{\alpha}$$

Compression can be avoided by $\alpha < 1.9$

5.4.3. Drop-shaped tank



$N_\phi = N_\theta = A$

Fully stressed axisymmetric tank (echinodomes)

: Shape identical to that of a drop of liquid resting on a plane surface

$p_\phi = 0, \quad p_n = \rho g z$

$$\frac{1}{R_\phi} + \frac{1}{R_\theta} = \frac{\rho g z}{A}$$

For axisymmetric profile $\frac{1}{R_\phi} = \frac{d \sin\phi}{dr}, \quad \frac{1}{R_\theta} = \frac{\sin\phi}{r}$

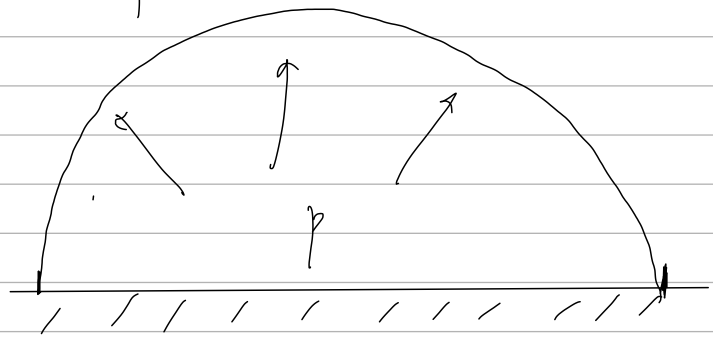
(From A. ~~38, 39~~)

$$\frac{d \sin\phi}{dr} + \frac{\sin\phi}{r} = \frac{\rho g}{A} \cdot z, \qquad \tan\phi = \frac{dz}{dr}$$

3.6. Pneumatic domes

: what is the best shape for an axisymmetric pneumatic dome with a vertical profile at the base?

— Hemisphere

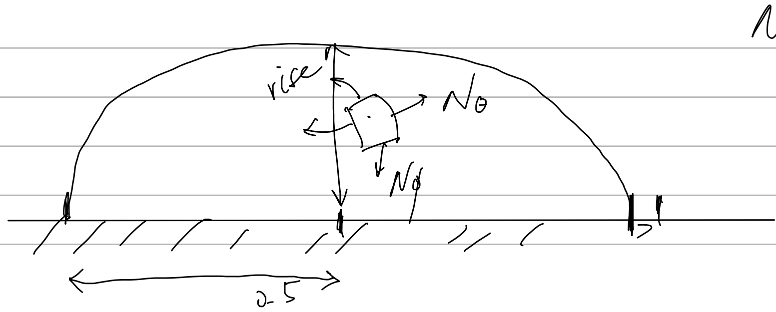


$N_\phi = N_\theta = \frac{pR}{2}$

→ Dead space at the top of the dome

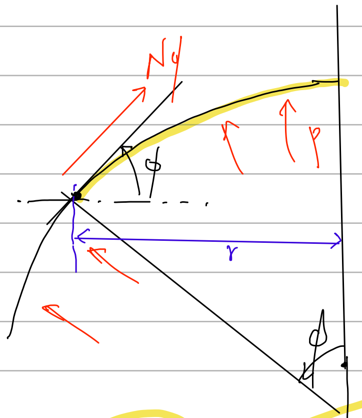
Q. Shallowest possible shape that avoids the formation of wrinkles (compression)?

— Ellipsoid



$N_\phi > 0$ for all ellipsoid

N_θ can be negative if rise is less than $\frac{1}{2\sqrt{2}} = 0.35$



$$\pi r^2 p = 2\pi r N_\phi \sin\phi$$

$$N_\phi = \frac{pr}{2\sin\phi} = \frac{p r_\theta}{2}$$

$$r_\theta = \frac{r}{\sin\phi}$$

$$\frac{N_\theta}{r_\theta} + \frac{N_\phi}{r_\phi} = p$$

$$\frac{N_\theta}{r_\theta} = p - \frac{N_\phi}{r_\phi} = p - \frac{p r_\theta}{2 r_\phi}$$

$$= p\left(1 - \frac{r_\theta}{2 r_\phi}\right)$$

$$= 0$$

$$\boxed{r_\phi = \frac{r_\theta}{2}}$$

(2)

$$\frac{1}{r_\phi} = \frac{d\sin\phi}{dr} = \cos\phi \frac{d\phi}{dr}$$

$$\frac{1}{r_\theta} = \frac{\sin\phi}{r}$$

$$r_\phi = \boxed{\frac{dr}{\cos\phi\, d\phi} = \frac{1}{2}\cdot\frac{r}{\sin\phi}}$$

$$\frac{dr}{r} = \frac{d\phi}{2\tan\phi}$$

Integration

$$\ln c\cdot r = \frac{1}{2}\ln\sin\phi$$

$$c\cdot r = \sqrt{\sin\phi} \rightarrow c^2 r^2 = \sin\phi$$

B.C. $r = r_0$ at $\phi = 90^\circ$

$$c^2 r_0^2 = \sin 90^\circ = 1, \quad c^2 = \frac{1}{r_0^2}$$

$$\frac{r^2}{r_0^2} = \sin\phi$$

Parachutes

- tightly packed for transportation
- once deployed, form a stable structure with high drag

projected area

$$D = C_D \cdot q \cdot A$$

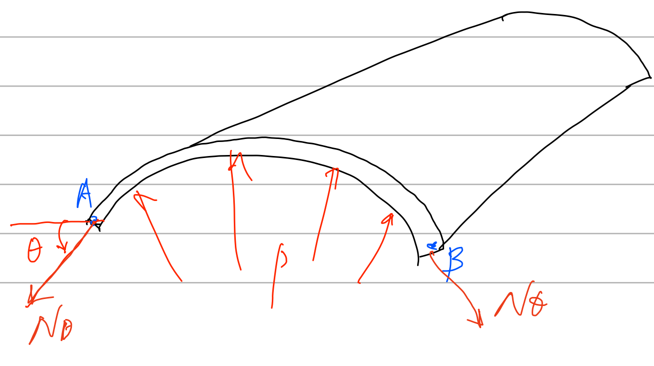
drag coefficient

$$q = \frac{1}{2}\rho v^2$$

A : increases structural mass

C_D : 0.5 ~ 1

→ compromise between A and C_D



$$p\,\overline{AB} = 2 N_\theta \sin\theta$$

$n \uparrow$, $\theta \uparrow$ → reduce N_θ