211102

2021년 11월 2일 화요일 오전 10:33

Last lecture
Dimensional Analysis
Dimensions & Units
Principle of Dimensional Homogeneity
Model vs. prototype
Buckingham π Theorem

HW # 6 5-4, 10, 14, 23, 25, 45, 47 Due Nov 7
Test #2 : 11-12:15 Nov 18, 2021
Final Exam : 11:00-13:30 Dec 9. 2021

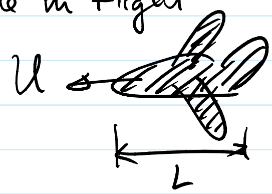
I. Dimensional Analysis $\rightarrow$ π's a nondimensional parameters

II. Nondimensionalization of Equations $\rightarrow$ π's

Incompressible flows.

$$\nabla \cdot \vec{V} = 0$$
$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \nabla p + \mu \nabla^2 \vec{V}$$

e.g. plane in flight



U = reference velocity
L = reference length

$\vec{V}^* \equiv \frac{\vec{V}}{U}$ nondimensional velocity

$x^* = \frac{x}{L}$; y^*, z^* nondimensional length

define $\tau \rightarrow$ characteristic time $\tau \equiv \frac{L}{U}$

$t^* \equiv \frac{t}{\tau} = \frac{tU}{L}$ nondimensional time

nondimensional equations,

$* \rightarrow$ nondimensional

$$\nabla^* \cdot \vec{V}^* = 0 \rightarrow \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0$$

$$\frac{D\vec{V}^*}{Dt} = -\nabla^* p^* + \frac{\mu}{\rho U L} \nabla^{*2}(V^*)$$

$\frac{1}{\dots}$ $\rightarrow$ for incompressible flow, Re is ,

$\frac{D}{Dt}$ ... $\frac{\mu}{\rho UL}$ ...

$\frac{1}{Re}$ → for incompressible flow, Re is important

can obtain π's from nondimensionalizing equations.

Table 5.2 → Nondimensional parameters

$St \equiv \frac{\omega L}{U_0}$ when $u(t) = U_0 \sin \omega t$

Strouhal → German physicist.

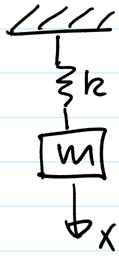
e.g. vortex shedding at ω

(for example, behind a cylinder)

if vortex shedding frequency ω = resonant frequency of structure

natural frequency

$\omega_{natural}^2 = \frac{k}{m}$



then, the structure can break.

Similarity

Similarity between model (m) and prototype means

If $\pi_{im} = \pi_{ip}$ for $i = 2, \dots k$

then $\pi_{1m} = \pi_{1p}$

the basis for model testing

Similarity types

1. geometric → same angles but lengths scaled. → same shape
2. Kinematic → length scale & velocity scales → important for 2-phase flows (air & water)
3. dynamic → force scale

... π ... Re → dynamic similarity ...

(air & water)

3. dynamic → force scale

in single-phase incompressible flows → Re → dynamic similarity achieved

in single-phase compressible flows → Re & Ma → to achieve dynamic similarity.

Difficulties in Achieving Similarity between model & prototype

e.g. compressible flows.

model	prototype
Re_m	Re_p
Ma_m	Ma_p

for dynamic similarity, $Re_m = Re_p \rightarrow \dfrac{V_m L_m}{\nu_m} = \dfrac{V_p L_p}{\nu_p}$ $\qquad \nu \equiv \mu/\rho$

$Ma_m = Ma_p \rightarrow \dfrac{V_m}{a_m} = \dfrac{V_p}{a_p}$

$$\frac{\nu_m}{\nu_p} = \frac{L_m}{L_p}\frac{V_m}{V_p} = \underbrace{\frac{L_m}{L_p}}_{\ll 1}\cdot\frac{a_m}{a_p}$$

then you need $\dfrac{\nu_m}{\nu_p} < 1$ and/or $\dfrac{a_m}{a_p} > 1$

usually prototype (p) fluid → air, water

model fluid? → difficult to find fluid to satisfy make

END of Ch. 5.

From now on

↳ flows contained by structures (rigid flexible)

internal flows and external flows

↳ a body submerged in fluid → aircraft, car submarine, etc.

flows contained by (rig... flexible

internal flows and external flows

(flows in pipes → oil, gas)

blood flow in blood vessels

air flows in lungs

heating/cooling in building

Ch. 6.

Ch. 7. (beginning).

211104

2021년 11월 4일 목요일 오전 10:21

Last lecture

Nondimensionalization of equations

Table of π's

Similarity – geometric ✓

– kinematic ✓

– dynamic ✓ → Re, Ma

Difficulties of achieving similarity

Ch. 6 → internal flows

heating/ventilating flows → buildings, cars

pipelines → oil, natural gas, hydrogen → $\boxed{\text{Newtonian fluid, rigid walls}}$

blood flow → non-Newtonian fluid, → flexible walls } fluid-structure interaction

$$Re_d \equiv \frac{\bar{U} d}{\nu}$$

$\bar{U}$ = avg velocity in pipe

d = pipe diameter

$\nu \equiv \mu/\rho$ kinematic viscosity of fluid

$Re_d < 1{,}000$ laminar

$1{,}000 < Re_d < 10{,}000$ transitional

$Re_d > 10{,}000$ turbulent

Revisit steady flow energy equation (SFEE) in Ch. 3

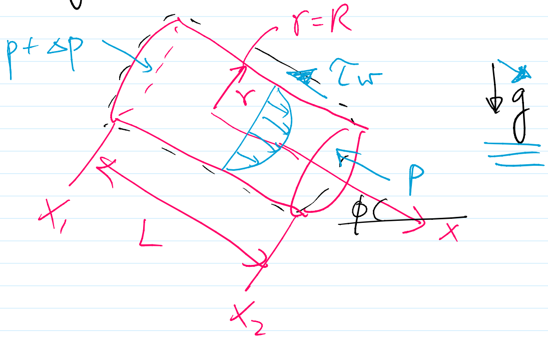
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$ ← pipe head loss

$$h_f = (z_1 - z_2) + \frac{(P_1 - P_2)}{\rho g}$$ if $V_1 = V_2$ (if fully developed) $\frac{\partial u}{\partial x} = 0$

$$h_f = \Delta z + \frac{\Delta P}{\rho g}$$

$$h_f = \Delta z + \frac{\Delta p}{\rho g}$$

Fig 6.3 Control volume analysis.



1. 1-D
2. steady
3. incompressible
4. $A_1 = A_2$
5. fully developed $V_1 = V_2$

$Q_1 = Q_2$

momentum eqn in x-dir.

average shear stress by the wall

$$\Delta p \pi R^2 + \rho g \pi R^2 \underbrace{L \sin\phi}_{\Delta z} - \tau_w 2\pi R L = \dot{m}(V_2 - V_1) = 0$$

$$h_f = \Delta z + \frac{\Delta p}{\rho g} = \frac{2\tau_w}{\rho g}\frac{L}{R}$$

→ relationship between h_f & τ_w

true for { laminar, transition, turbulent } flows

Weissbach in 1850

$$h_f\left(Re_d, \frac{\varepsilon}{d}, \text{duct shape}\right) = f\,\frac{L}{d}\frac{V^2}{2g}$$

ε = surface roughness

d = pipe diameter

V = average velocity.

Darcy friction factor

from •• & •••

$$\boxed{f = \frac{8\tau_w}{\rho V^2}}$$

nondimensional

$f = \frac{64}{Re_d}$ laminar $Re_d < 2{,}300$

Moody Chart

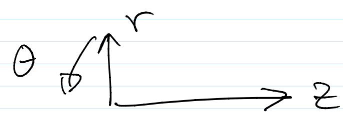
Fig. 6.13

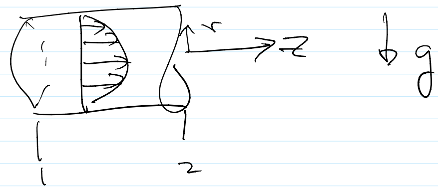
$$f = \frac{64}{Re_d} \quad \text{laminar} \quad Re_d < 2{,}300$$

$$f = \sim 0.316\, Re_d^{-1/4} \quad \text{turbulent} \quad 4{,}000 < Re_d < 10^5$$

Laminar pipe flow (Hagen-Poiseuille flow)

1. steady
2. 1-D $\longrightarrow$ $v_r = v_\theta = 0$
3. incompressible
4. horizontal pipe ($\phi = 0$) $\quad z_1 = z_2$
5. axisymmetric $\left(\frac{\partial}{\partial\theta} = 0\right)$
6. laminar





continuity eqn. $\longrightarrow$ $v_r = 0$, $v_\theta = 0$ $\Rightarrow$ $\frac{\partial}{\partial z}(v_z) = 0 \rightarrow v_z(r)$

momentum eqn

$r, \theta \rightarrow 0$

$$z: \quad \rho v_z \cancel{\frac{\partial v_z}{\partial z}}^{\,0 \text{ continuity}} = -\frac{\partial p}{\partial z} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right) \rightarrow \quad \frac{\partial p}{\partial z} \rightarrow \frac{dp}{dz}$$

$$v_z = \left(\frac{dp}{dz}\right)\frac{r^2}{4\mu} + C_1 \ln r + C_2$$

Boundary conditions
1. at $r = R$ $\quad v_z(R) = 0$
2. $r = 0$ $\quad v_z$ finite

$$C_1 = 0 \quad \& \quad C_2 = \left(-\frac{dp}{dz}\right)\left(\frac{R^2}{4\mu}\right)$$

$$v_z(r) = \left(-\frac{dp}{dz}\right)\frac{1}{4\mu}(R^2 - r^2) \qquad \left(\frac{dp}{dz} < 0\right) \therefore \left(-\frac{dp}{dz}\right) > 0$$

$$v_{z\,max} \text{ at } r = 0 = \left(-\frac{dp}{dz}\right)\frac{R^2}{4\mu}$$

$$\bar{U} = V = V_{z\,avg} = \frac{1}{A}\int_0^R V_z\, dA = \frac{V_{z\,max}}{2} = \left(-\frac{dp}{dz}\right)\frac{R^2}{8\mu}$$

$$Q = \int_0^R V_z\, dA = \frac{\pi R^4}{8\mu}\left(-\frac{dp}{dz}\right)$$

$$\tau_w = \mu\left(\frac{\partial V_z}{\partial r}\right)_{r=R} = \frac{R}{2}\left(+\frac{dp}{dz}\right) = \frac{2\, V_{z\,max}\,\mu}{R}$$

$$f = \frac{8\,\tau_w}{\rho V_{avg}^2} = \frac{64\,\mu}{\rho V_{avg}\, 2R} = \frac{64\,\mu}{\rho V_{avg} d} = \frac{64}{Re_d}$$

Have looked at qualitative differences between laminar & turbulent flows

$\bar{u}$ = time average or mean

$u'(t)$ = perturbation or fluctuation

$u(t)$ = instantaneous

$$u(t) = \bar{u} + u'(t)$$

time average

$$\bar{u} = \frac{1}{T}\int_0^T u(t)\, dt$$

$$u'(t) = u(t) - \bar{u}$$

$$\overline{u'(t)} = \frac{1}{T}\int_0^T u'(t)\, dt = 0$$

$$\overline{u'(t)^2} = \frac{1}{T}\int_0^T u'(t)^2\, dt \neq 0$$

tells us how turbulent flow is.
quantifies turbulent flow

turbulence intensity.