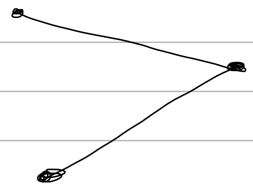
Folding geometric objects

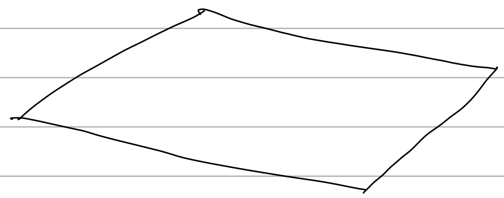
Geometric objects

① Linkage (1D)



e.g., bars and hinges

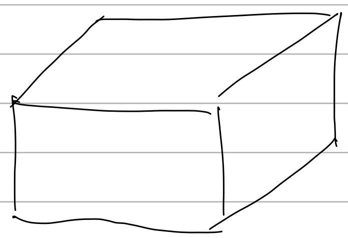
② Planes (2D)



e.g. polygons

origami

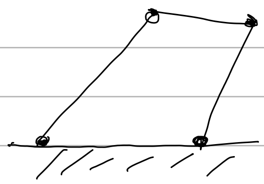
③ Volume (3D)



e.g., polyhedra

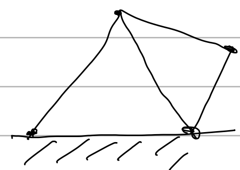
Q: Foldability

1D



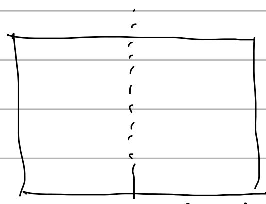
foldable
(mechanism)

vs.



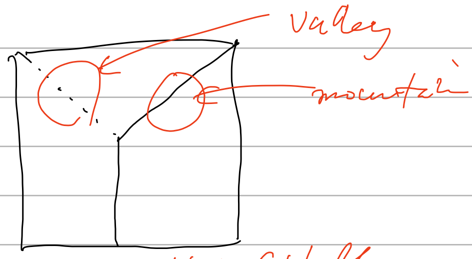
Not foldable
(structure)

2D



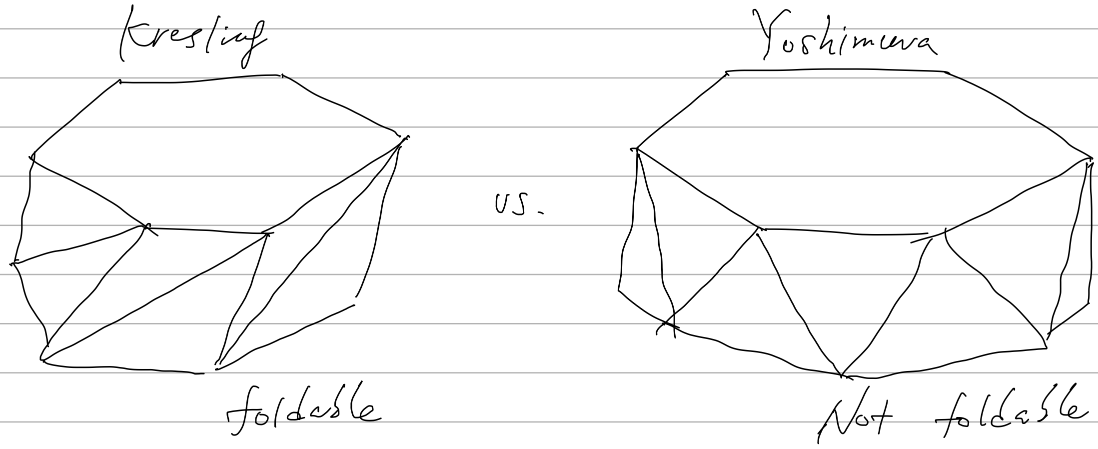
foldable

vs.



Not foldable

3D



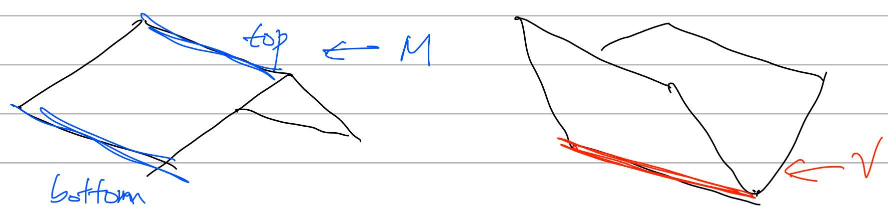
→ Transition of foldable structures' properties

; density, Poisson's ratio, stiffness, stability, ...

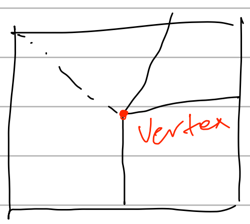
Applications : robotics, aerospace, biomedical, architecture, environments, manufacturing, biology (protein folding) ....

Origami terminology

① crease : lines about which the folding occurs
② crease pattern : bunch of creases
③ folded state : plane → finished origami
④ flat folding : folded state lying in the plane
(call crease pattern flat foldable)
⑤ mountain creeses : bottom sides touch
valley " : ~~top~~ " "

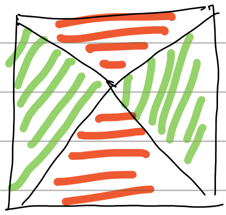


⑥ Vertex : a point where multiple creeses meet

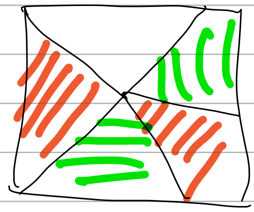


Properties of flat foldable crease patterns

- 2-colorability : crease patterns can be colored using two colors without the same color meeting

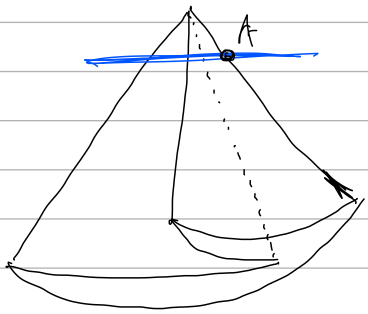


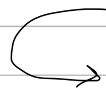
vs.

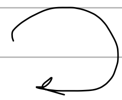


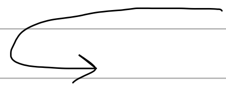
- Maekawa's theorem

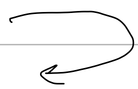
  # of mountains (M) − # of valley (V) = ±2

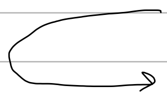


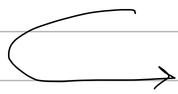
Mountain  180°

Valley  −180°

①  M

②  V

③  M

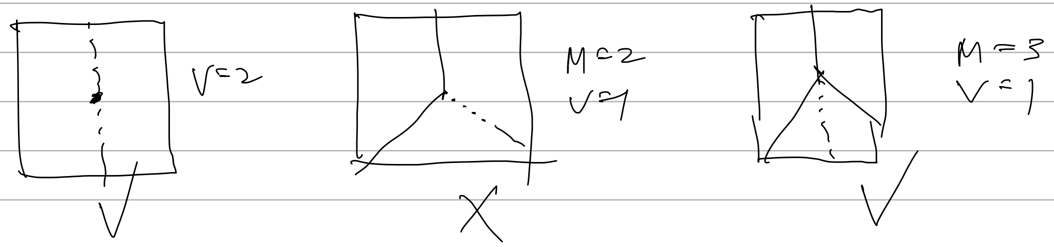
④  M

Σ turn angles

$= 180° \times M - 180° \times V = \pm 360°$

$M - V = \pm 2$

e.g.,

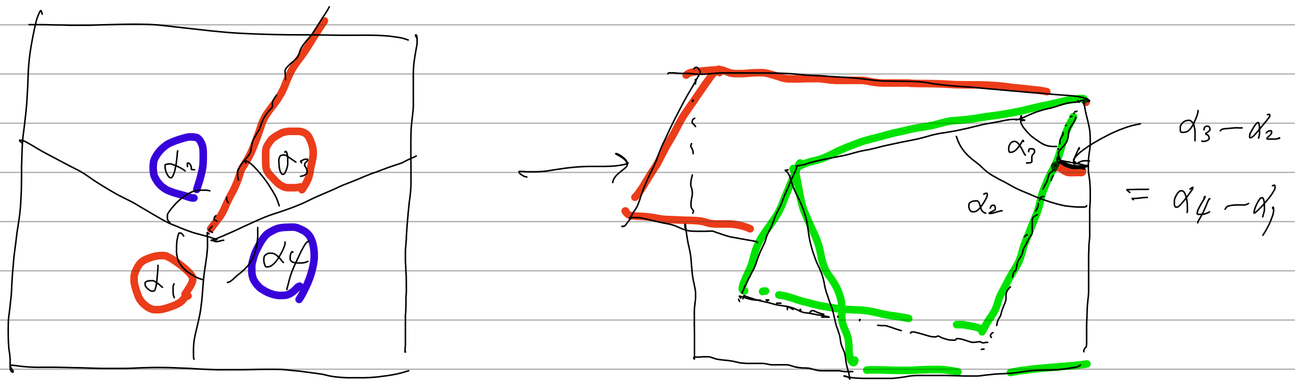


- Even degree theorem : Every flat vertex fold has even degree

$$n = M + V = (V \pm 2) + V$$
$$= 2(V \pm 1) \quad : \text{even number}$$

→ consistent with 2-colorability nature

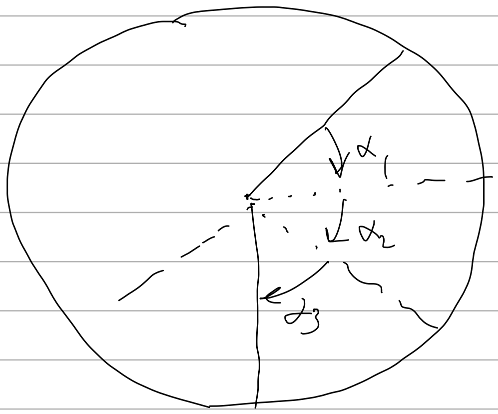
- Husimi's theorem



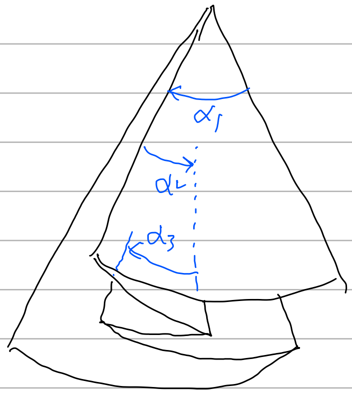
$$\alpha_4 - \alpha_1 = \alpha_3 - \alpha_2$$

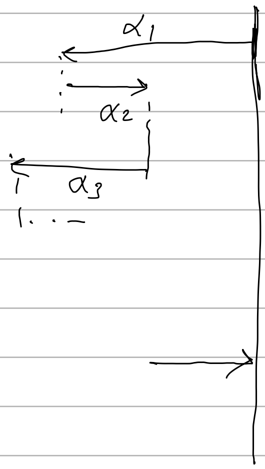
$$\rightarrow \alpha_1 + \alpha_3 = \alpha_2 + \alpha_4 = 180^\circ$$

- Kawasaki's theorem



$$\alpha_1 + \alpha_3 + \alpha_5 + \cdots + \alpha_{n-1}$$
$$= \alpha_2 + \alpha_4 + \alpha_6 + \cdots + \alpha_n$$
$$= 180^\circ$$





$$\alpha_1 - \alpha_2 + \alpha_3 - \cdots + \alpha_{n-1} - \alpha_n = 0$$

$$\alpha_1 + \alpha_3 + \cdots = \alpha_2 + \alpha_4 + \cdots = 180^\circ$$

Packaging of Membranes

inextensional plates of zero thickness

Ways of folding

① crumpling by random creeses

: not answering

- how densely can the membrane be packed
- maximum force required to pull it out
- residual deformation after unpacking

② packaging

- Packaging efficiency : ratio between actual volume of material and the volume of the package (< 50%)

Recall

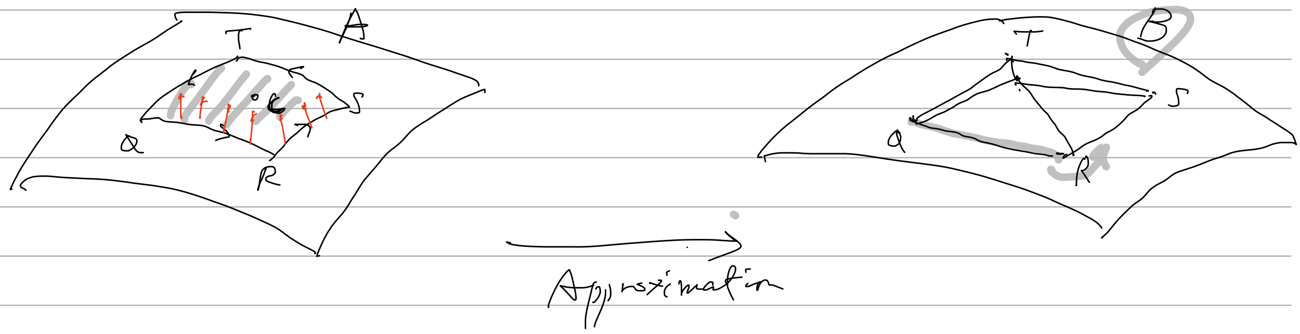
Gaussian curvature

$$K = K_1 K_2 = \frac{1}{R_1} \cdot \frac{1}{R_2}$$

$K = 0$: developable

$$\delta K = -\frac{\partial^2 \varepsilon_y}{\partial x^2} + \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} - \frac{\partial^2 \varepsilon_x}{\partial y^2}$$

Angular defect (A. K.S)



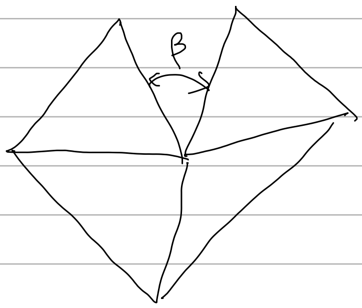
Mean Gaussian curvature of a curved patch lying on the original, smooth surface

$$\bar{K}_c = \frac{\alpha}{A}$$

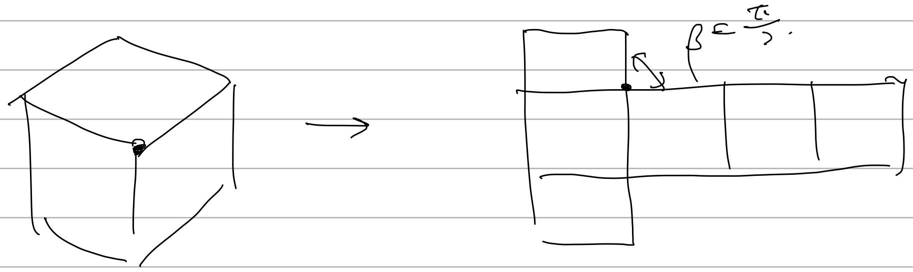
For these triangles

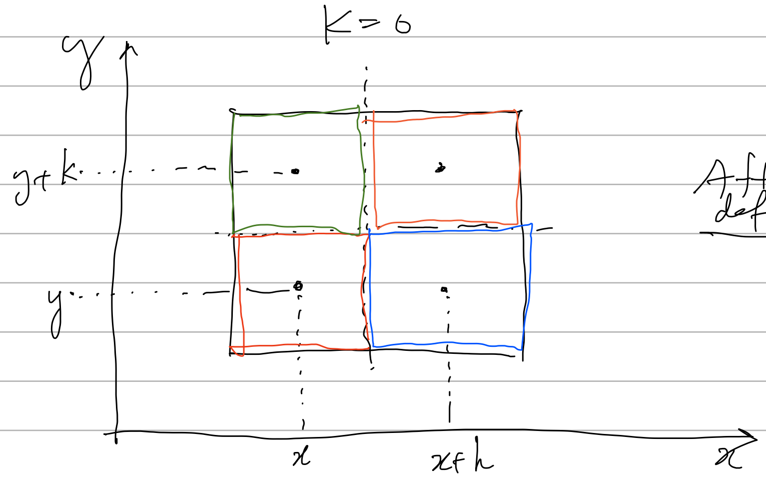
$$\bar{K}_f = \frac{\beta}{B}$$

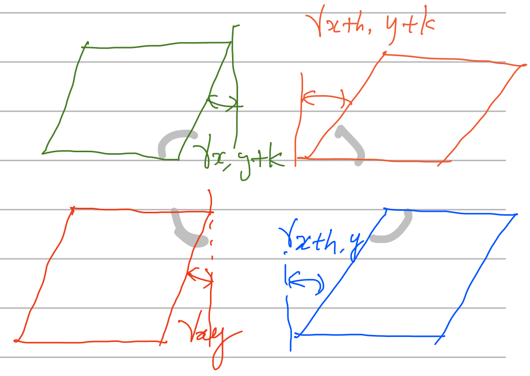
$$\bar{K}_c = \bar{K}_f$$



β : angular defect







Angular defect

$$360^\circ - \left[(90^\circ - \gamma_{x,y}) + (90^\circ + \gamma_{x+h,y}) + (90^\circ + \gamma_{x,y+k}) + (90^\circ - \gamma_{x+h,y+k}) \right]$$

$$= \gamma_{x,y} - \gamma_{x+h,y} - \gamma_{x,y+k} + \gamma_{x+h,y+k}$$

$$K = \frac{\gamma_{x+h,y+k} - \gamma_{x,y+k} - (\gamma_{x+h,y} - \gamma_{x,y})}{h \cdot k}$$

$$= \frac{1}{k}\left[\frac{\partial \gamma_{x,y+k}}{\partial x} - \frac{\partial \gamma_{x,y}}{\partial x} \right]$$

$$= \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

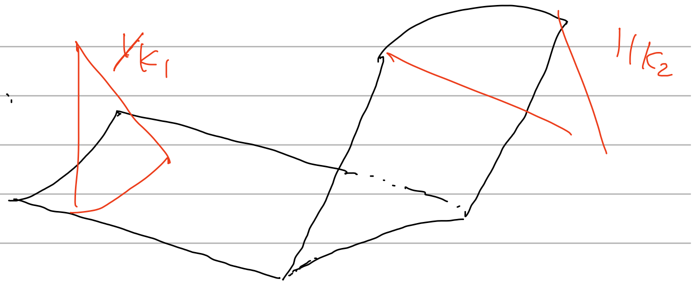
$$K = -\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} - \frac{\partial^2 \varepsilon_y}{\partial x^2}$$

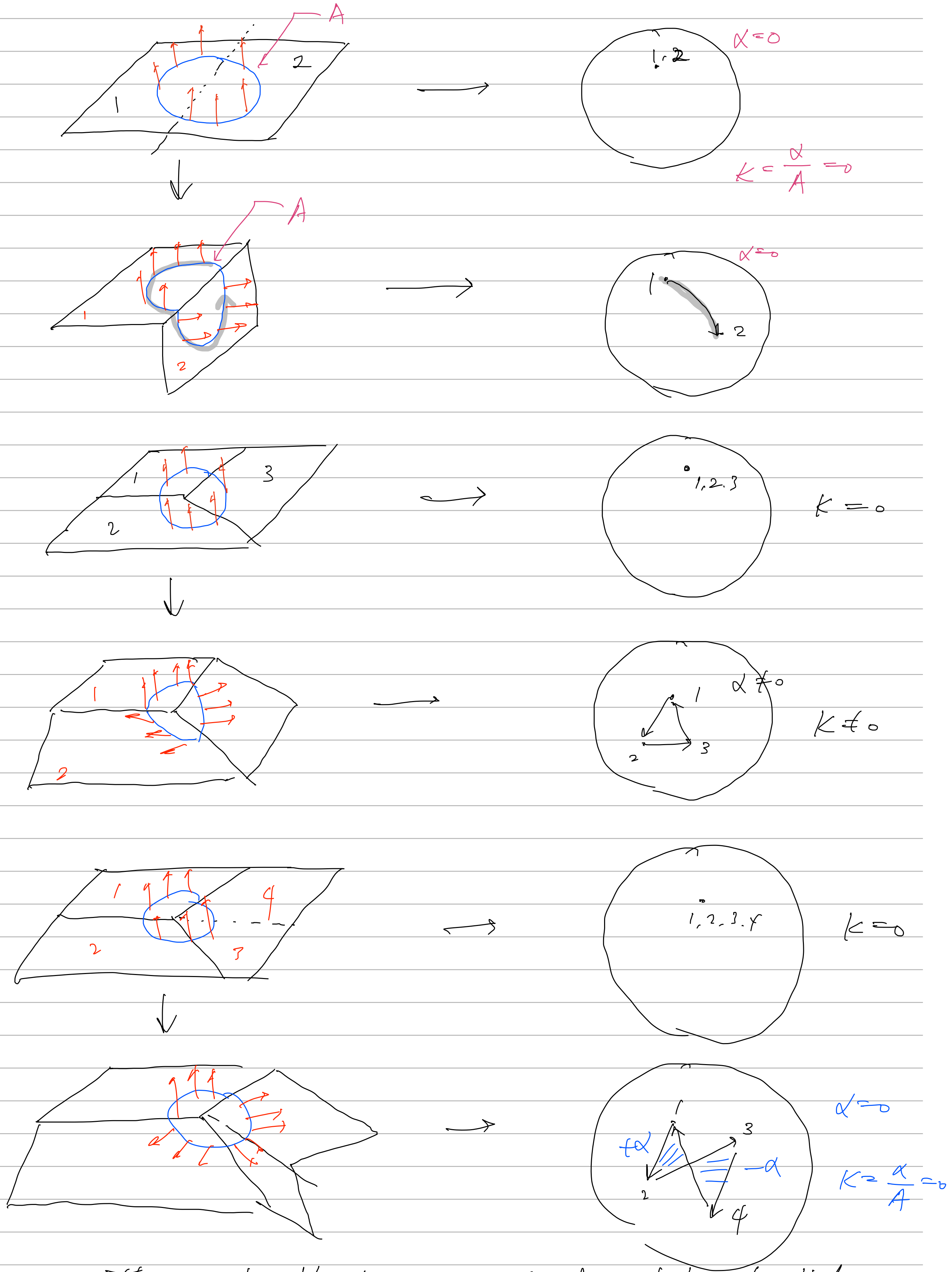
→ For the deformation inextensional, $\varepsilon_x = \varepsilon_y = \gamma_{xy} = 0$

$$dK = 0$$

Therefore, initially flat membrane ($K=0$ everywhere) can only be folded to deform in such a way that its Gaussian curvature remains zero everywhere.

→ Use straight folds





There should be at least four folds, of which three have one sign, and one fold has the opposite sign