211109

2021년 11월 9일 화요일 오전 10:36

Last lecture

internal flows

fully developed flow $\left(\frac{\partial u}{\partial x} = 0\right)$

$Re_d = \rho V d / \mu$

laminar, transition, turbulent flows

h_f pipe head loss

f Darcy's friction factor

τ_w wall shear stress

Moody Chart $\rightarrow f(Re_d)$

Laminar flow example $\rightarrow$ Hagen-Poiseuille pipe flow

$f = 64 / Re_d$

$v_z(r)$, v_{max}, V, τ_w, f

(V: average velocity)

$u(t) = \bar{u} + u'(t)$

$\bar{u} = \frac{1}{T}\int_0^T u(t)\,dt$ mean or time average

$u'(t) = u(t) - \bar{u}$ perturbation or fluctuation

$\overline{u'(t)} = 0 \leftarrow$

$\overline{u'(t)^2} \neq 0$ turbulence intensity $\rightarrow$ quantifies turbulence

HW #6: 6-10, 12, 16, 42, 62 Due Nov 16

Test #2: 11:00-12:15 Nov 18 (cover upto Ch. 5)

Final Exam: 11:00-13:30 Dec 9 (cover everything)

course will cover upto laminar boundary layer in Ch. 7.

Turbulence

$u(t) = \bar{u} + u'(t)$, $v(t)$, $w(t)$

$p(t) = \bar{p} + p'(t)$

1. substitute into [mass conservation eqn
   x, y, z momentum conservation eqns

2. take time average

mass conservation $\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$

viscous shear force

x-mom : $\rho \frac{D\bar{u}}{Dt} = -\frac{\partial \bar{p}}{\partial x} + \rho g_x + \frac{\partial}{\partial x}\left(\mu \frac{\partial \bar{u}}{\partial x} - \rho \overline{u'^2}\right)$
$+ \frac{\partial}{\partial y}\left(\mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'}\right)$
$+ \frac{\partial}{\partial z}\left(\underbrace{\mu \frac{\partial \bar{u}}{\partial z}}_{\tau_{laminar}} - \underbrace{\rho \overline{u'w'}}_{\tau_{turbulent}}\right)$

per unit volume

pressure force

gravity force

for Newtonian fluids → $\tau_{laminar}$

$\tau_{turbulent}$

y-mom :
z-mom :

Turbulent Boundary Layer (TBL) consists of

1. Viscous wall layer $\tau_{laminar} \gg \tau_{turbulent}$
2. Overlap layer $\tau_{laminar} \sim \tau_{turbulent}$
3. Outer layer $\tau_{laminar} \ll \tau_{turbulent}$

δ boundary layer thickness $\equiv$ y at which $\frac{u}{U_\infty} = 0.99$

Define

$u^+ = \frac{u}{u^*}\left(\frac{y u^*}{\nu}\right)$ where $u^* = \sqrt{\frac{\tau_w}{\rho}}$ has dimensions of velocity

frictional velocity

$y^+ = \frac{y u^*}{\nu}$

1. In the viscous wall layer

$$u^+ = y^+$$ Law of the Wall

2. In the overlap layer (logarithmic)

$$u^+ = \frac{1}{K} \ln\left(\frac{y u^*}{\nu}\right) + B$$ where $\underbrace{K = 0.41 \text{ \& } B = 5.0}_{\text{measured}}$

3. In the outer layer

$$\frac{U_\infty - u}{u^*} = G\left(\frac{y}{\delta}\right)$$ Velocity Defect Law

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Engineering approach → complex problem

↓

1. make simplifying assumptions

↓

2. obtain solution to the simplified problem

↓

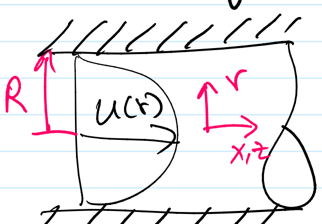
3. verify solution (check if assumptions are valid)

- TBL consists of 3 regions

BUT

we will assume that the entire TBL is composed of the logarithmic overlap layer

therefore → $$\frac{u(r)}{u^*} = \frac{1}{K} \ln \frac{(R-r)u^*}{\nu} + B$$ (velocity)



turbulent fully developed

average velocity V

$$V = \frac{Q}{A} = \frac{1}{\pi R^2} \int_0^R u^* \left[\frac{1}{K} \ln \frac{(R-r)u^*}{\nu} + B\right] 2\pi r\, dr$$

$K = 0.41$ & $B = 5.0$

fully developed flow

$\kappa = 0.41$ & $B = 5.0$

$$\frac{V}{u^*} = \frac{1}{2}\left(\frac{2}{\kappa}\ln\frac{Ru^*}{\nu} + 2B - \frac{3}{\kappa}\right)$$

$$\frac{V}{u^*} \approx 2.44 \ln\frac{Ru^*}{\nu} + 1.34 \qquad (1)$$

now. because $u^* = \tau_w/\rho$

$$\frac{V}{u^*} = \left(\frac{\rho V^2}{\tau_w}\right)^{1/2} = \left(\frac{8}{f}\right)^{1/2}$$

in addition

$$\frac{Ru^*}{\nu} = \frac{\frac{1}{2}Vd}{\nu}\,\frac{u^*}{V} = \frac{1}{2}Re_d\left(\frac{f}{8}\right)^{1/2}$$

Substitute into Eqn (1),

$$\left(\frac{8}{f}\right)^{1/2} \approx 2.44\ln\left[\frac{1}{2}Re_d\left(\frac{f}{8}\right)^{1/2}\right] + 1.34$$ implicit for f

Prandtl suggested $\frac{1}{f^{1/2}} \approx 1.99 \log\left(Re_d f^{1/2}\right) - 1.02$

still implicit for f

$\therefore$ yet another curve-fit of $\boxed{f \approx 0.316\, Re_d^{-1/4}}$

for turbulent smooth internal flow.

suggested by Blasius (student of Prandtl)

for rough (ε/d) surfaces,

for rough (ε/d) surfaces,

another curve-fit can be used

$$\frac{1}{f^{1/2}} \simeq -1.8 \log \left[\frac{6.9}{Re_d} + \left(\frac{\varepsilon/d}{3.7} \right)^{1.11} \right]$$

When is a surface rough? smooth?

$\frac{\varepsilon u^*}{\nu} < 5$ hydraulically SMOOTH

$5 < \frac{\varepsilon u^*}{\nu} < 70$ transitionally rough

$\frac{\varepsilon u^*}{\nu} > 70$ fully rough

$f(Re_d)$

$$f(Re_d) \begin{cases} = \dfrac{64}{Re_d} & \text{laminar} \\ \approx 0.316\, Re_d^{-1/4} & \text{turbulent (smooth)} \end{cases}$$

Moody Chart

Can analyze / design various networks, pipelines, etc.

211111

2021년 11월 11일 목요일 오전 10:26

Last lecture

turbulent boundary layer

inner (viscous wall) layer $\tau_{lam} \gg \tau_{turb}$

overlap layer $\tau_{lam} \sim \tau_{turb}$

outer layer $\tau_{lam} \ll \tau_{turb}$

Law of wall → inner

Logarithmic velocity profile → overlap * → u^+, y^+

Velocity defect law → outer

approximation of f for turbulent flows [smooth / rough → $\frac{\varepsilon u^*}{\nu}$

Today: How do we use $f(Re_d)$ and/or the Moody chart.

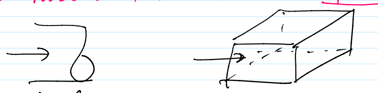
3 types of problems

1. Given: d, L, V, Q, ρ, μ, g → calculate h_f [head loss / pressure drop
2. Given: d, L, h_f, ρ, μ, g → calculate V, Q → average velocity & volume flowrate
3. Given: Q, L, h_f, ρ, μ, g → calculate d → pipe sizing

So far, our discussion has been for circular pipes only

but not all pipes/ducts are circular in cross section.

in these cases, what to do? → Hydraulic diameter D_h



for circular pipes

mom: $\Delta p \pi R^2 + \rho g \pi R^2 L \sin\phi - \tau_w 2\pi R L = \dot{m}(V_2 - V_1) = 0$

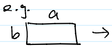
for non-circular ducts (NCD)

A = cross sectional area

P = perimeter of cross section

mom: $\Delta p A + \rho g A L \sin\phi - \overline{\tau_w} P L = 0$

e.g.



→ $A = ab$, $P = 2(a+b)$

$h_f = \frac{\Delta p}{\rho g} + \Delta z = \frac{\overline{\tau_w}}{\rho g}\left(\frac{L}{A/P}\right)$

$f_{NCD} = \frac{8\,\overline{\tau_w}}{\rho V^2}$

then $h_f = f \cdot \frac{L}{4\left(\frac{A}{P}\right)} \frac{V^2}{2g}$

if circular, $A = \pi R^2$, $P = 2\pi R$, $4\left(\frac{A}{P}\right) = d = 2R$

∴ for non-circular ducts, L is divided by $4\left(\frac{A}{P}\right)$ instead of d

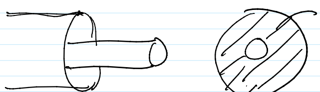
Hydraulic Diameter $\boxed{D_h \equiv \frac{4A}{P}}$

→ $Re_{D_h} \equiv \frac{V D_h}{\nu}$ → $f(Re_{D_h})$

Solutions accurate to within $\sim \pm 40\%$ (laminar) } not good

$\pm 15\%$ (turbulent) } not bad

now, can do cases like



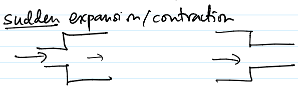
So far,

1. h_f, f in fully developed constant area [circular → d / non-circular → D_h] duct flows

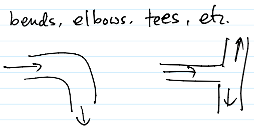
but there are other types of losses.

a) entrance or exit effects

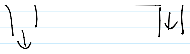
b) sudden expansion/contraction



c) bends, elbows, tees, etc.

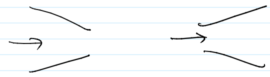


d) valves → open & close



d) valves → open & close

e) gradual expansion/contraction



f) more

Define $\boxed{K \equiv \frac{\Delta p}{\frac{1}{2}\rho V^2}}$ → given in $\boxed{\text{Figs \& Tables (data)}}$

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then, finally

$$h_{total} = h_f + \Sigma h_m$$

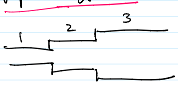
h_f: pipe flow

Σh_m: other types of minor losses — bends, valves, ΔA's, etc. etc.

from Tables & Figures in Ch. 6.

Multiple Pipe Systems

Pipes in series

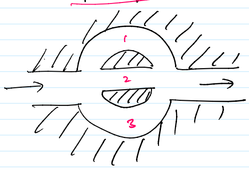


$V_1 d_1^2 = V_2 d_2^2 = V_3 d_3^2$

$Q_1 = Q_2 = Q_3$

$h_{total} = h_1 + h_2 + h_3$ including area change etc.

Pipes in parallel

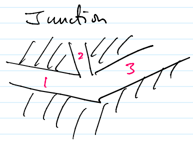


$Q = Q_1 + Q_2 + Q_3$

$h_1 = h_2 = h_3$

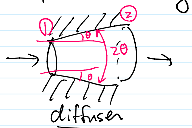
Check examples.

Junction



Still to be covered in Ch. 6.

Flow area change → Diffusers



diffuser

$A_1 < A_2$

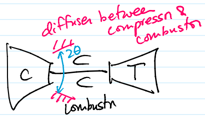
$V_1 > V_2$

$p_1 < p_2$

2θ is an important parameter

e.g. gas turbine engines (aircraft engines)

diffuser between compressor & combustor



C = compressor

T = turbine

gas turbine

would like 2θ to be large to

minimize friction

minimize engine length & weight ∝ length