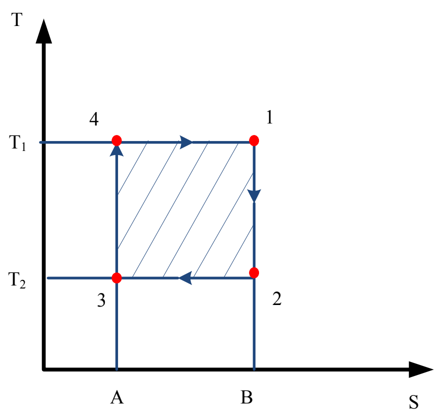
Heat Engines

Carnot Cycle

Carnot, a French engineer, showed in 1824 that the most efficient possible cycle is one in which all the heat supplied is supplied at one fixed temperature, and all the heat rejected is rejected at a lower fixed temperature.



1-2: **(S)** expansion from T_1 to T_2

2-3: **(T)** heat rejection $q_{2\text{-}3}$

3-4: **(S)** compression from T_2 to T_1

4-1: **(T)** heat supply $q_{4\text{-}1}$

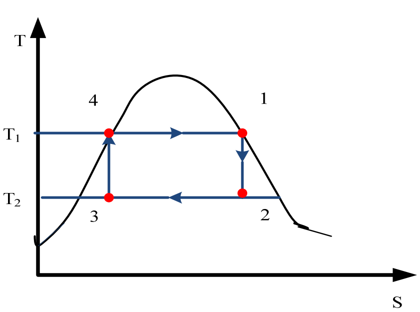
The cycle is completely independent of working fluid.

Thermal Efficiency

$$\eta = \frac{q_{4\text{-}1} - q_{2\text{-}3}}{q_{4\text{-}1}} = 1 - \frac{q_{2\text{-}3}}{q_{4\text{-}1}} = 1 - \frac{T_2\left(S_B - S_A\right)}{T_1\left(S_B - S_A\right)}$$

Given T_2 for heat sink, $\eta_{carnot} = 1 - \frac{T_2}{T_1}$, $T_1 \uparrow \rightarrow \eta_{carnot} \uparrow$

$$\sum q = \sum W, \quad W = q_{4\text{-}1} - q_{2\text{-}3} = \left(T_1 - T_2\right)\left(S_B - S_A\right)$$



HR furnace gases at 2000 ^{0}C, CR water at 10 ^{0}C

$$\eta_{carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{10+273}{2000+273}$$

$$= 0.8754 = 87.54\%$$

$\eta_{practice} \cong 30\%$ due to irreversibility & deviations.

Absolute Temperature Scale

independent of working fluid

$$\eta = 1 - \frac{q_2}{q_1} = \phi(X_1, X_2)$$

$$\frac{q_2}{q_1} = F(X_1, X_2)$$

$$\frac{q_2}{q_1} = \frac{X_1}{X_2}$$

$$\eta_{carnot} = 1 - \frac{q_2}{q_1} = 1 - \frac{T_2}{T_1} \qquad \therefore \frac{q_2}{q_1} = \frac{T_2}{T_1}$$

X: temperature

ϕ: function

F: new function

Carnot Cycle for PG

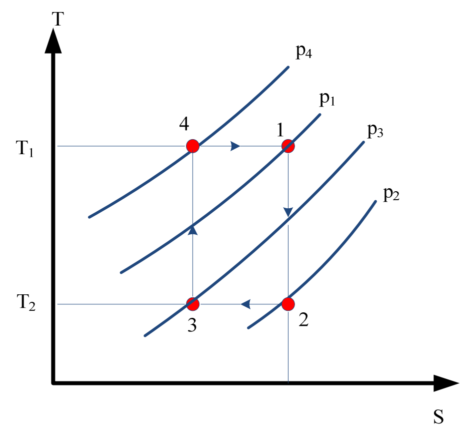
4-1: **(T)** heat supply $p_4 \rightarrow p_1$

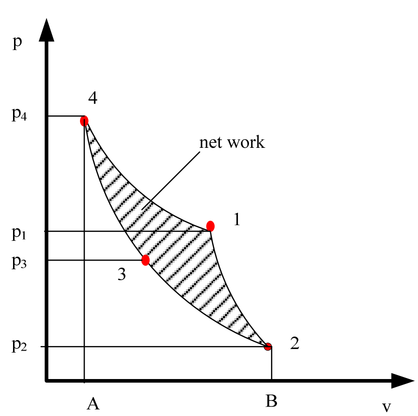
2-3: **(T)** heat rejection $p_2 \rightarrow p_3$

In practice it is more convenient

To heat a gas **(V)** or **(P)**

→ difficult to operate an actual heat engine in Carnot cycle.





Net Work: 12341=412BA4 – 234AB2

Expansion Work: 412BA4 Area

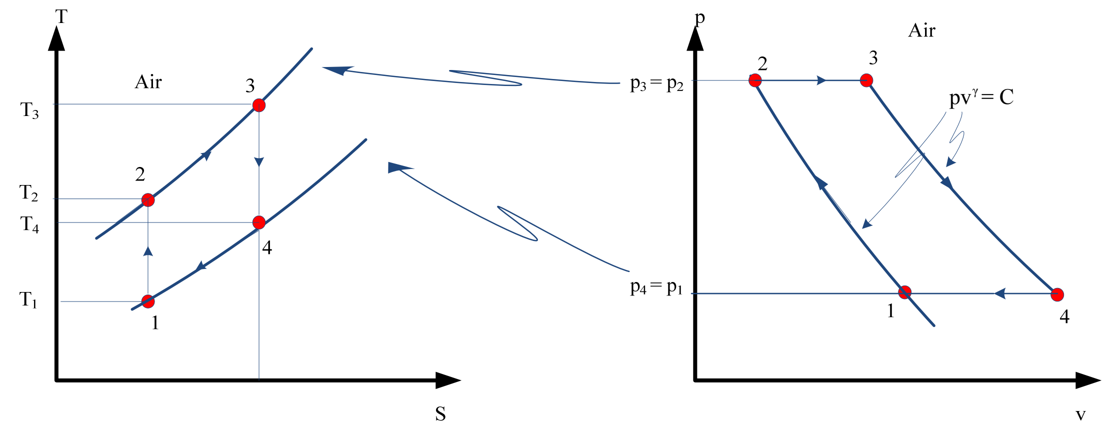
Compression Work: 234AB2 Area

$$\text{Work ratio} = \frac{\text{Net work output}}{\text{Gross work output}}$$

$$WR = \frac{(12341)}{(412BA4)}$$

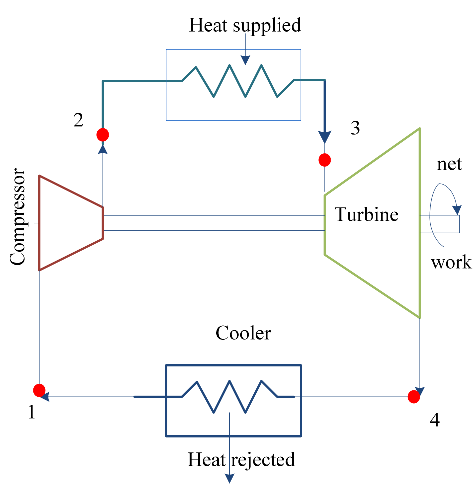
WR is small despite its high η => Carnot cycle is not practical.

<u>Constant Pressure Cycle</u> (Joule or Brayton)



Rev. heat supply + rejection **(p)**; expansion+ compression **(S)**

$\Rightarrow$ ideal for closed cycle gas turbine



Work input to CP = $h_2 - h_1 = C_p\,(T_2 - T_1)$

Work output from TB = $h_3 - h_4 = C_p\,(T_3 - T_4)$

Heat supplied = $h_3 - h_2 = C_p\,(T_3 - T_2)$

Heat rejected = $h_4 - h_1 = C_p\,(T_4 - T_1)$

$$\eta = 1 - \frac{q_{CR}}{q_{HR}} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$WR = \frac{\text{net work}}{\text{gross work}} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

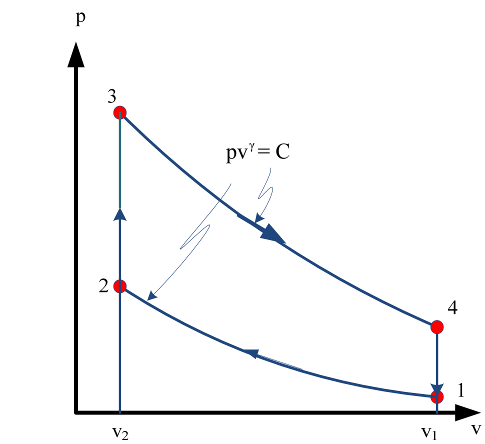
$$= \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_4)}$$

$$\text{Pressure ratio: } r_p = \frac{p_2}{p_1} > 1$$

$$\eta = 1 - r_p^{\frac{1-\gamma}{\gamma}}, \qquad WR = 1 - \frac{T_1}{T_3}\, r_p^{\frac{\gamma-1}{\gamma}}$$

The maximium temperature T_3 must be as high as possible for a high WR given the inlet temperature T_1.

Air Standard Cycle for Petrol Engine (Otto):



1-2: **(S)** compression

2-3: rev **(V)** heating = $C_V (T_3 - T_2)$

3-4: **(S)** expansion

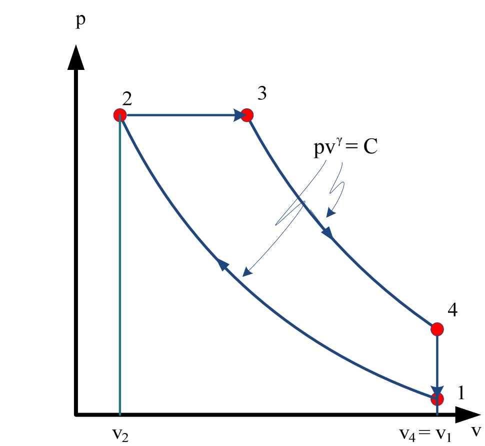
4-1: rev **(V)** cooling = $C_V (T_4 - T_1)$

Compression ratio:

$$r_V = \frac{V_1}{V_2}$$

$$\eta = 1 - \frac{Q_{CR}}{Q_{HR}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{r_V^{\gamma-1}}$$

Diesel Cycle



1-2: **(S)** compression

2-3: rev **(p)** heating = $C_p (T_3 - T_2)$

3-4: **(S)** expansion

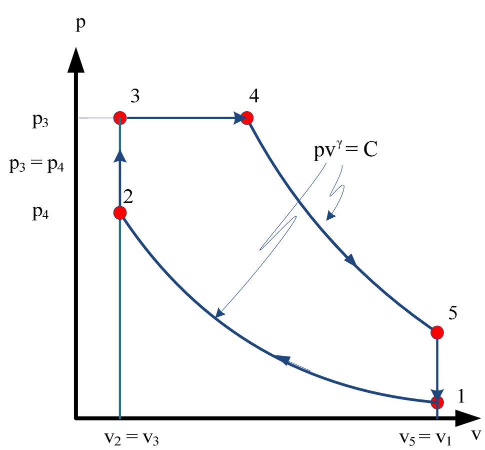
4-1: rev **(V)** cooling = $C_V (T_4 - T_1)$

Cutoff ratio:

$$\beta = \frac{V_3}{V_2}$$

$$\eta = 1 - \frac{C_V \left(T_4 - T_1\right)}{C_p \left(T_3 - T_2\right)} = 1 - \frac{\beta^{\gamma} - 1}{\left(\beta - 1\right) r_V^{\gamma-1} \gamma}$$

Duel Combustion Cycle (Mixed):



1-2: **(S)** compression

2-3: rev **(V)** heating = $C_v (T_3 - T_2)$

3-4: rev **(p)** heating = $C_p (T_4 - T_3)$

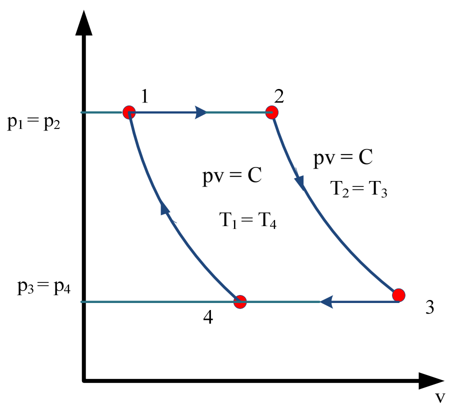
4-5: **(S)** expansion

5-1: rev **(V)** cooling = $C_V (T_5 - T_1)$

$$r_V = \frac{V_1}{V_2}, \quad K = \frac{p_3}{p_2}, \quad \beta = \frac{V_4}{V_3}$$

$$\eta = 1 - \frac{C_V\left(T_5 - T_1\right)}{C_V\left(T_3 - T_2\right) + C_p\left(T_4 - T_3\right)}$$
$$= 1 - \frac{K\beta^{\gamma} - 1}{\left[\left(K - 1\right) + \gamma K\left(\beta - 1\right)\right] r_V^{\gamma-1}}$$

Ericsson Cycle



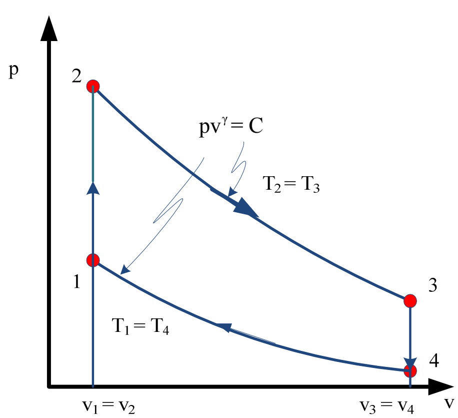
1-2: rev **(p)** heating = C_p ($T_2 - T_1$)

2-3: **(T)** heating

3-4: rev **(p)** Cooling = C_p ($T_2 - T_1$)

4-1: **(T)** cooling

Sterling Cycle



2-3: **(T)** heating as gas expands

3-4: rev **(V)** Cooling = C_V ($T_2 - T_1$)

4-1: **(T)** cooling as gas is compressed

1-2: rev **(V)** heating = C_V ($T_2 - T_1$)

$q_{3\text{-}4}$ is used for $q_{1\text{-}2}$ in a regenerator.

$$\left.\begin{aligned} q_{2\text{-}3} &= W_{2\text{-}3} = RT_2 \ln\frac{p_2}{p_3} \\ q_{4\text{-}1} &= W_{4\text{-}1} = RT_1 \ln\frac{p_1}{p_4} \end{aligned}\right\}$$

Net work done:

$$W = W_{2\text{-}3} - W_{4\text{-}1}$$
$$= q_{2\text{-}3} - q_{4-1}$$

$$\eta = \frac{W}{q_{2\text{-}3}} = \frac{q_{2\text{-}3} - q_{4\text{-}1}}{q_{2\text{-}3}} = 1 - \frac{q_{4\text{-}1}}{q_{2\text{-}3}}$$

$$= 1 - \frac{RT_1 \ln\frac{p_1}{p_4}}{RT_2 \ln\frac{p_2}{p_3}}$$

1-2: rev. const. V process $\frac{p_2}{p_1} = \frac{T_2}{T_1}$

3-4: $\frac{p_3}{p_4} = \frac{T_3}{T_4} = \frac{T_2}{T_1}$

$$\frac{p_2}{p_1} = \frac{p_3}{p_4} \qquad \therefore \frac{p_1}{p_4} = \frac{p_2}{p_3}$$

$\eta = 1 - \frac{T_1}{T_2}$ = the carnot efficiency

This result may have been deduced w/o formal proof as the heat supply and rejection took place at const T's.

$$WR = \frac{W_{2\text{-}3} - W_{4\text{-}1}}{W_{2\text{-}3}} = 1 - \frac{W_{4\text{-}1}}{W_{2\text{-}3}} = 1 - \frac{q_{4\text{-}1}}{q_{2\text{-}3}} = 1 - \frac{T_1}{T_2}$$

$$\therefore\ WR = \eta$$

The Sterling & Ericsson cycles are superior to the Carnot cycle in that they have higher work ratios with an efficiency equal to that of the Carnot cycle.