211116

2021년 11월 16일 화요일 오전 10:48

Last lecture

3 types of internal flow problems (Ch. 6) → $f(Re_d)$

Noncircular ducts

hydraulic diameter $D_h \equiv 4A/P$

Minor losses → K

Multiple pipes → series vs. parallel

HW #8: 6-64, 78, 109, 113 Due Nov 23

Test #2 (upto Ch. 5) 11:00-12:15 Thurs Nov 18

Final Exam 11:00-13:30 Thurs Dec 9

Diffusers & flow meters (Ch. 6) → we will come back to this

Boundary layer, external flows [ laminar, turbulent (Ch. 7) } → next.

Internal (Ch. 6) vs. external flow (Ch. 7)

e.g. pipe flow

↳ flow can expand freely

body is submerged in the flow

e.g. flow around an airplane

a car

etc.

today, we begin external flow discussion.

e.g. flows over a flat plate.

boundary layer → a region where $\tau = \mu \frac{du}{dy} \neq 0$ or $\frac{du}{dy} \neq 0$

very important

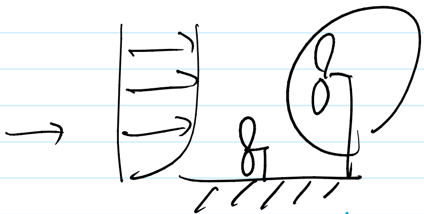
aerodynamics

hydrodynamics

transportation

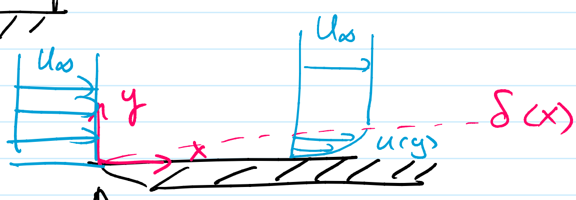
wind engineering

ocean engineering




external flows → bodies are submerged in fluid.

$$Re_x = \frac{\rho U_\infty x}{\mu}$$




x=0 leading edge

$Re_x > 10^6$ turbulent.

$Re_x > 10^6$ turbulent.
$Re_x < 10^6$ laminar

boundary layer thickness $\delta \equiv y$ at which $\frac{u(y)}{U_\infty} = 0.99$

bodies
- streamlined (thickness << length) e.g. wing
  vs.
- bluff (thickness ~ length) e.g. sphere

flow boundary layer → attached
vs.
separated. → need adverse pressure gradient $\left(\frac{dp}{dx} > 0\right)$

- increased drag & loss → usually bad for aerodynamics
- sometimes good → for heat transfer

pressure gradient $\frac{dp}{ds}$
- adverse if $\frac{dp}{ds} > 0$ → needed for flow separation
- favorable if $\frac{dp}{ds} < 0$

$s$ → flow direction

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boundary layer over a flat plate (Example 3.11)

Integral Analysis of Flat Plate Boundary Layer

$$D = \rho b \int_0^\delta u(U_0 - u)\, dy \Big|_{x=L}$$

$$D = \rho b U_0^2 \Theta \quad \text{where } \Theta \equiv \int_0^\delta \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right) dy$$

$$\boxed{D \propto \Theta} \qquad \Theta = \int_0^\infty \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right) dy$$

where $D$ = drag → force exerted in $-x$ direction by the plate on the fluid

$$D = b\int_0^x \tau_w(x)\,dx = \rho b U_o^2 \underline{\underline{\theta}}$$

momentum thickness

$$\boxed{\tau_w = \rho U_o^2 \frac{d\theta}{dx}}$$

Momentum Integral Relation

valid for [laminar / turbulent] boundary layers.

∴ von Karman analysis of b.l. is powerful because (integral)

one can estimate $D$ (force) from velocity $\left(\frac{u(y)}{U_o}\right)$

e.g. laminar boundary layer

$$\boxed{\frac{u}{U_o} \approx \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)} \rightarrow \quad u(0)=0, \quad u(\delta)=U_o$$

assumed

measured possible

$$D = \rho b U_o^2 \theta$$

$$D = \frac{2}{15}\rho U_o^2 b\delta$$

$$D(x) \propto \delta(x)$$

How does $\delta(x)$ vary?

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0} = \rho U_o^2 \frac{d\theta}{dx}$$

$$\tau_w = 2\mu \frac{U_o}{\delta} = \rho U_o^2 \frac{2}{15}\frac{d\delta}{dx}$$

$$\delta\, d\delta \approx 15 \frac{\mu}{\rho}\frac{dx}{U_o}$$

Boundary condition : at $x=0$, $\delta=0$

Boundary Condition : at $x=0$, $\delta=0$

$$\frac{\delta^2}{2} \simeq \frac{15\nu x}{U_0} \qquad \nu \equiv \mu/\rho$$

$$\frac{\delta}{x} \approx 5.5\left(\frac{\nu}{U_0 x}\right)^{1/2}$$

$$\boxed{\frac{\delta}{x} \approx \frac{5.5}{\sqrt{Re_x}}}$$

$\downarrow$

$\underline{\tau_w}$, $\underline{D}$, etc.

<u>friction coefficient</u> $\quad \underline{C_f \equiv \frac{\tau_w}{\left(\frac{\rho U_0^2}{2}\right)}} \approx \left(\frac{8/15}{Re_x}\right)^{1/2} = \underline{\frac{0.73}{\sqrt{Re_x}}}$

$\uparrow$

for $\frac{u}{U_0}\left(2\frac{y}{\delta}-\frac{y^2}{\delta^2}\right)$

211125
2021년 11월 25일 목요일 오전 10:24

Last lecture

integral analysis of boundary layer (external flow)

momentum integral relation

laminar b.l. example

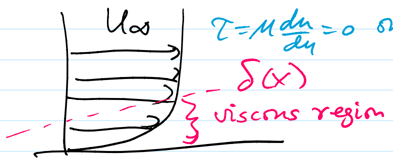
for the rest of the course

1. continue discussion of b.l.
2. revisit latter part of Ch. 6 → diffusers, flow meters.
3. integral analysis of turbulent boundary layer
4. selected topics

## Thicknesses related to boundary layers

a) boundary layer thickness $\delta$ → marks the boundary of the region where viscous effects exist.

$y$ at which $\frac{u(y)}{U_\infty} = 0.99$




$\tau = \mu \frac{du}{dy} = 0$ outside b.l.

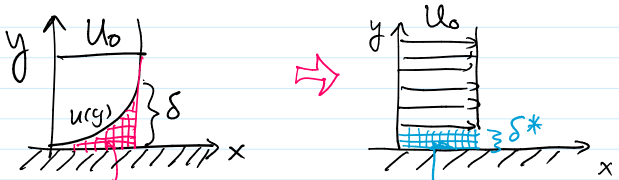
b) momentum thickness

$$\theta = \int_0^\infty \frac{u}{U_0}\left(1-\frac{u}{U_0}\right)dy$$

$$\theta \equiv \int_0^\delta \frac{u}{U_0}\left(1-\frac{u}{U_0}\right)dy$$

quantifies momentum defect due to the boundary layer

velocity mass defect in boundary layer




(velocity) mass flow defect due to boundary layer → red shaded region

area = area

$$U_0\delta^* = \int_0^\delta U_0\,dy - \int_0^\delta u(y)\,dy$$

area

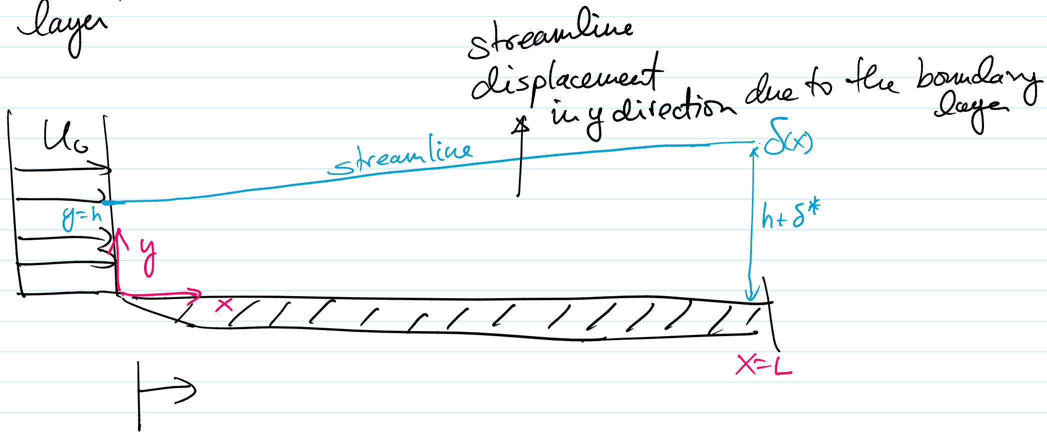
c) displacement thickness

$$\delta^* \equiv \int_0^\infty \left(1-\frac{u}{U_0}\right)dy = \int_0^\delta \left(1-\frac{u}{U_0}\right)dy$$

Flow is displaced (pushed away from the wall) due to

Flow is displaced (pushed away from the wall) due to the boundary layer

Example




going back to laminar b.l. example

if $\quad \dfrac{u}{u_0} = \left(2\dfrac{y}{\delta} - \dfrac{y^2}{\delta^2}\right)$

$$\Downarrow$$

$$\frac{\delta^*}{\delta} = \frac{1}{3} \qquad \text{where} \qquad \frac{\delta}{x} \approx \frac{5.5}{\sqrt{Re_x}}$$

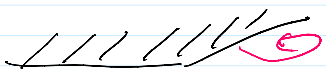
$$\therefore \frac{\delta^*}{x} \approx \frac{1.83}{\sqrt{Re_x}}$$

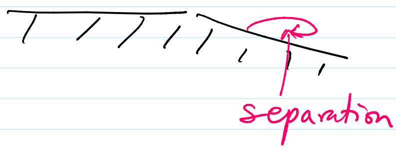
Shape Factor H → indicates the condition (shape) of the boundary layer.

$$H \equiv \frac{\delta^*}{\theta}$$

higher the value of H → boundary layer is more likely to separate

lower the value of H → boundary layer less likely to separate






Summarizing integral analysis of boundary layer

laminar boundary layer $\dfrac{u}{u_0} \approx \left(2\dfrac{y}{\delta} - \dfrac{y^2}{\delta^2}\right)$

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boundary layer thickness $\frac{\delta(x)}{x} \approx \frac{5.5}{\sqrt{Re_x}}$

displacement thickness $\frac{\delta^*(x)}{x} \approx \frac{1.83}{\sqrt{Re_x}}$

friction coefficient $C_f \equiv \frac{\tau_w}{\left(\frac{\rho u_0^2}{2}\right)} \approx \frac{0.73}{\sqrt{Re_x}}$

momentum thickness $\theta = \frac{2}{15}\delta$ where $\frac{\delta}{x} \approx \frac{5.5}{\sqrt{Re_x}} \Rightarrow \frac{\theta}{x} \approx \frac{0.73}{\sqrt{Re_x}}$

drag $D = \rho b u_0^2 \theta$

Drag Coefficient $C_D \equiv \frac{D}{\left(\frac{\rho u_0^2}{2}\right) bL} = \frac{\rho b u_0^2 \theta}{\left(\frac{\rho u_0^2}{2}\right) bL} = \frac{2\theta}{L}$

$$C_D = \frac{1.46}{\sqrt{Re_x}}$$

Shape factor $H \equiv \frac{\delta^*}{\theta} \approx 2.59$ for $\frac{u}{u_0} = \left(2\frac{y}{\delta} - \frac{y^2}{\delta^2}\right)$

Question : Is $H = 2.59$ high or not (relative to what) ?

$H = 2.59$ (laminar b.l.) is higher than $H$ for turbulent boundary layers

$\therefore$ laminar b.l. more likely to separate than turbulent b.l.