Announcement

- To be updated

Progress

- ~~Chapter 1 Material properties~~
- ~~Chapter 2 Axially loaded member~~
- ~~Chapter 3 Torsion~~
- ~~Chapter 4 Shear and Bending moment~~
- ~~Chapter 5 Stress in beams~~
- Chapter 7 Stress and Strain
- Chapter 9 Deflection of beams
- Chapter 11 Columns

Chapter 7 Analysis of Stress and Strain

KEYWORDS:

Plane stress

Mohr's Circle

Uniaxial, Biaxial, Triaxial stress

Principal stress

Flexure formula

$$\sigma = My/I$$

Shear formula

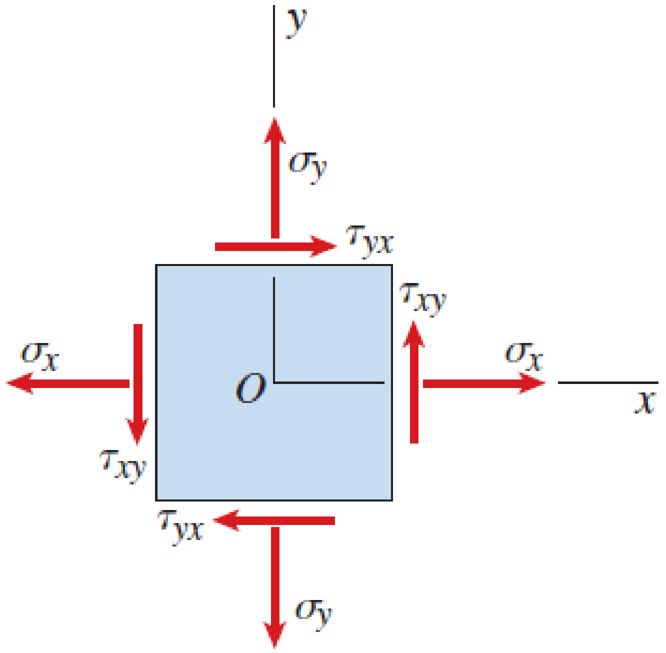
$$\tau = VQ/Ib$$

Torsion formula

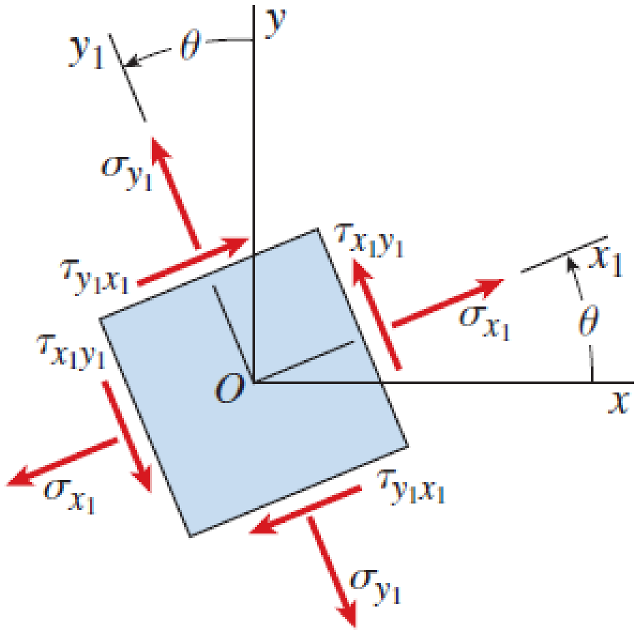
$$\tau = T\rho/I_P$$

Introduction

All we need to know..



(Sign convention)



σ_x =applied normal stress

σ_y =applied normal stress

τ_{xy} =applied shear stress

θ = angle rotation

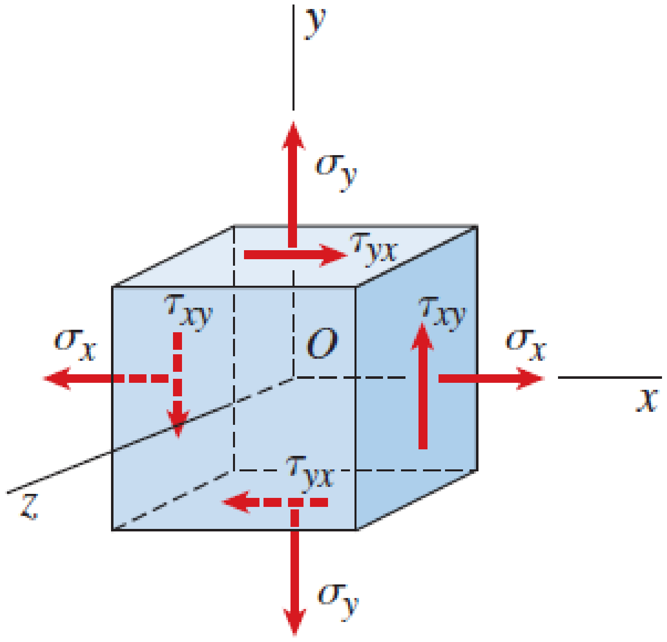
σ_{x_1} =new normal stress

σ_{y_1} =new normal stress

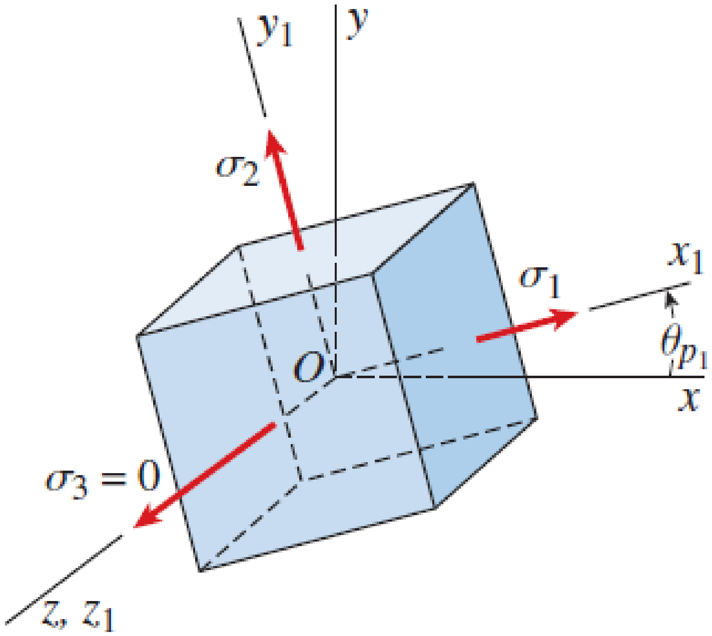
$\tau_{x_1y_1}$ =new shear stress

Introduction

At certain rotation angle, there will be **no shear stress**



(Sign convention)



θ_{p1} =Principal angle

σ_1 =Principal stress

σ_2 =Principal stress

τ_{12} =?

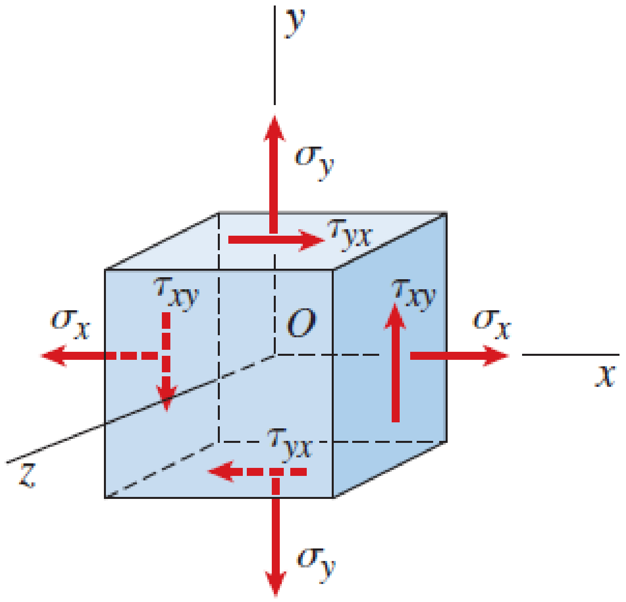
Introduction

Because x-y or x1-y1 axis is always **perpendicular,**

θ_{p1} =Principal angle

σ_1 =Principal stress

σ_2 =Principal stress



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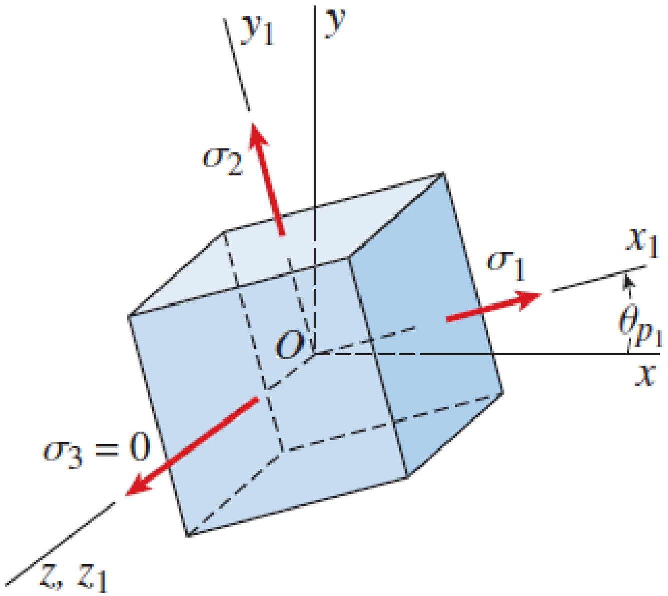


$+\mathbf{90}\ rotation, \theta_{p2} = \theta_{p1} + 90$

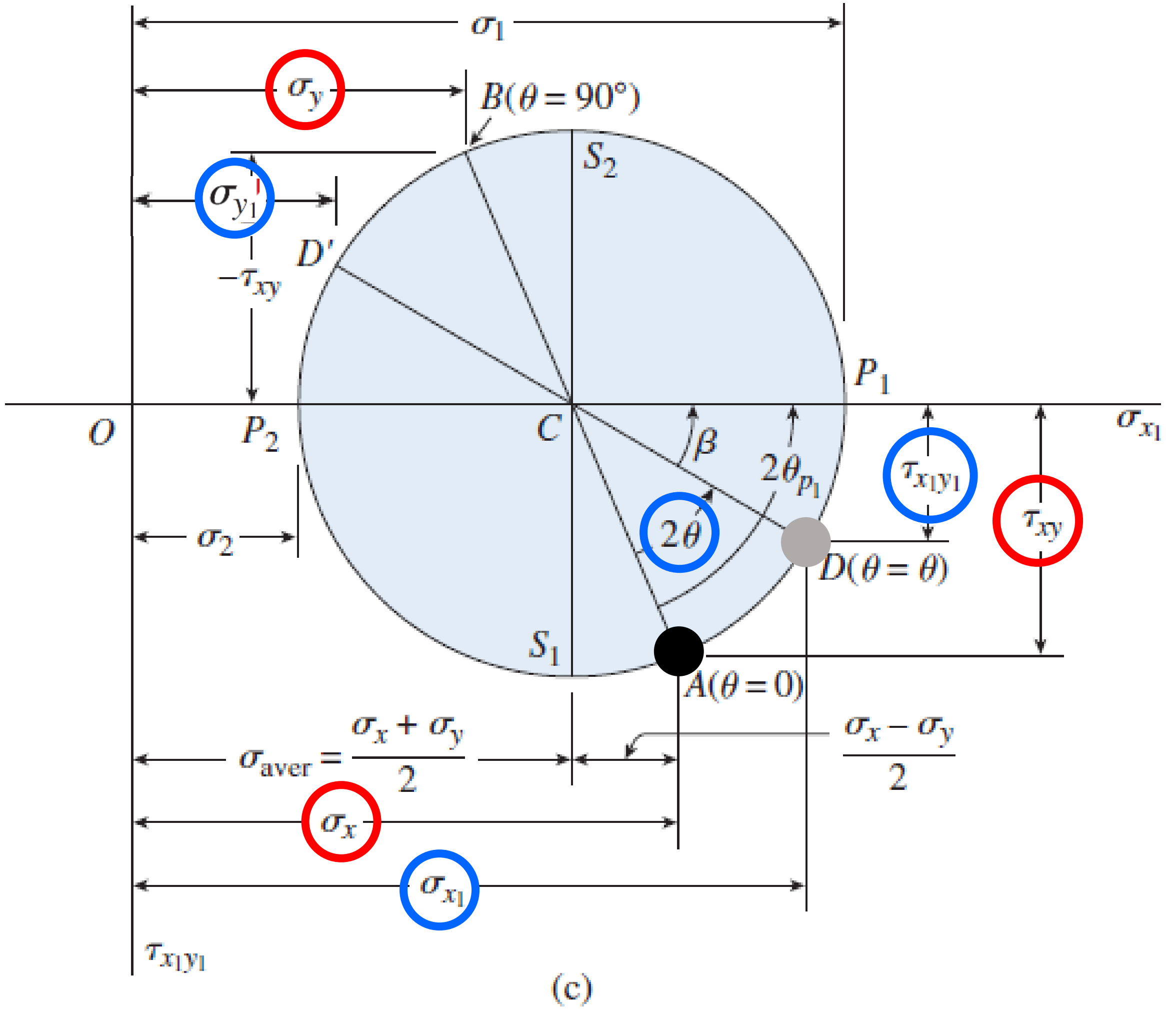
θ_{p2} =Principal angle

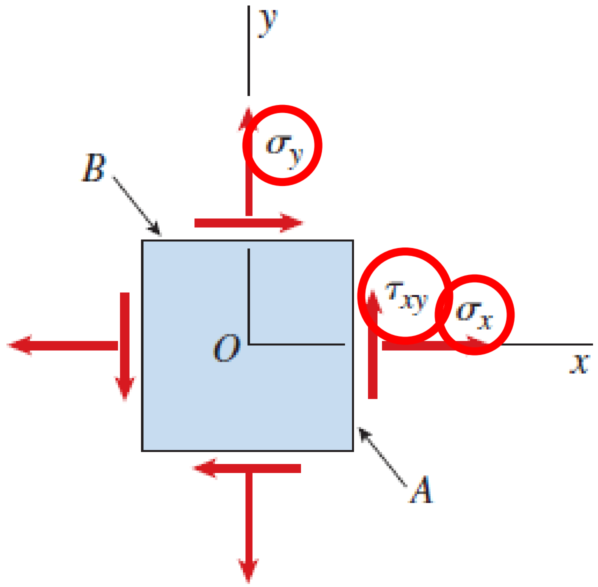
σ_2 =Principal stress

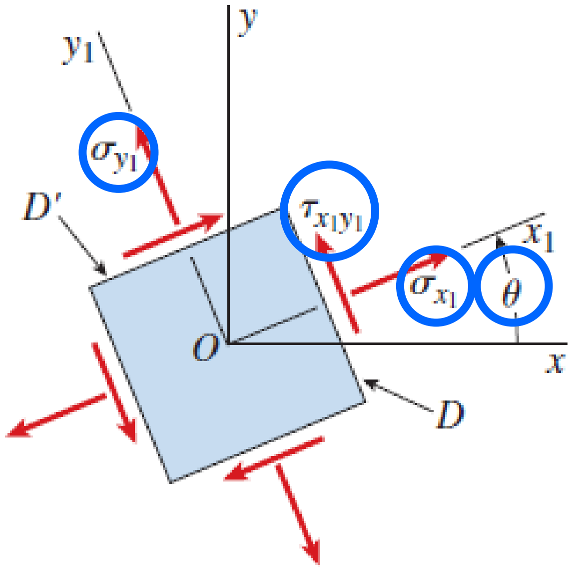
σ_1 =Principal stress

Mohr's Circle

Derivation can be tedious but final result is very useful!

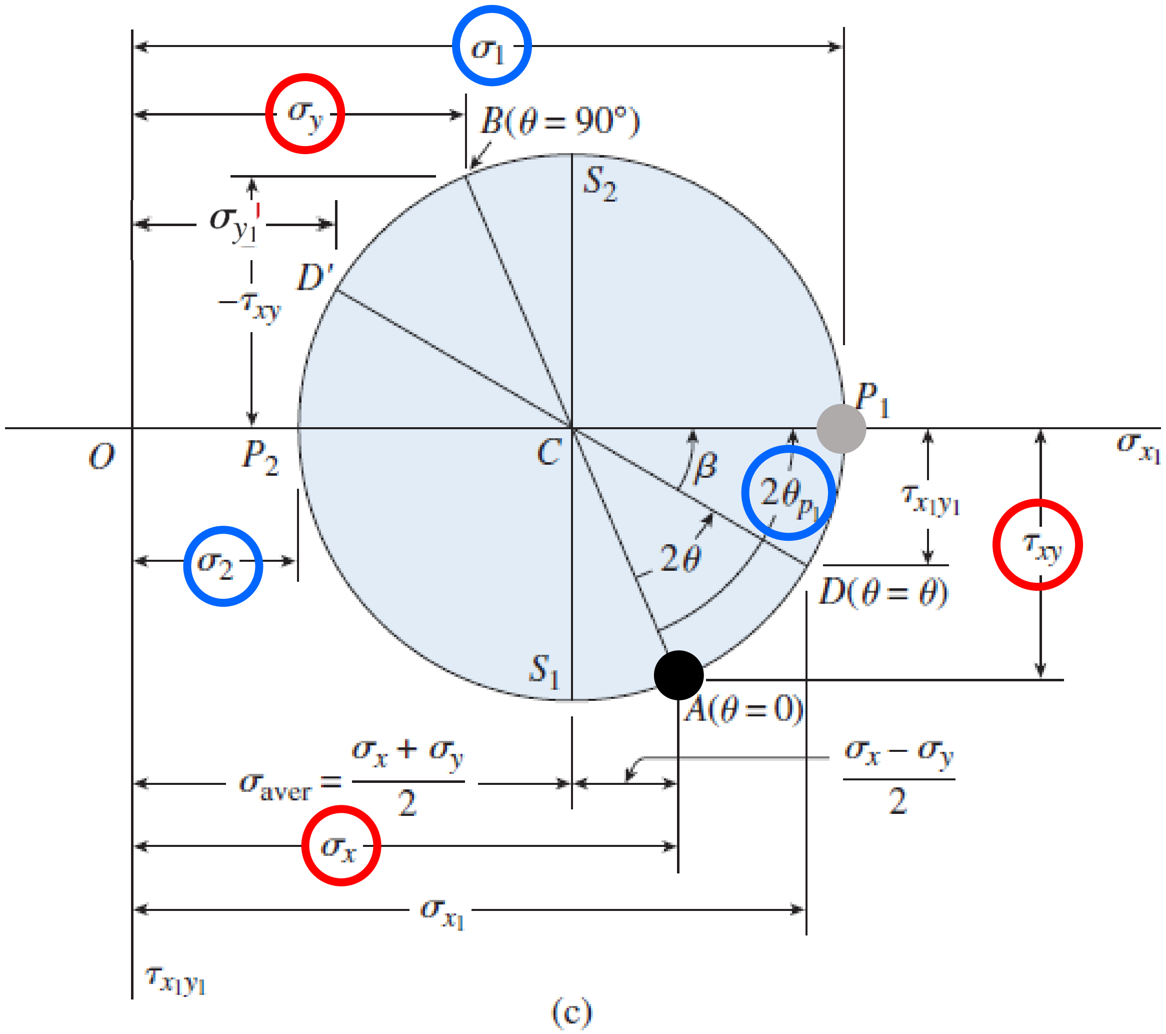


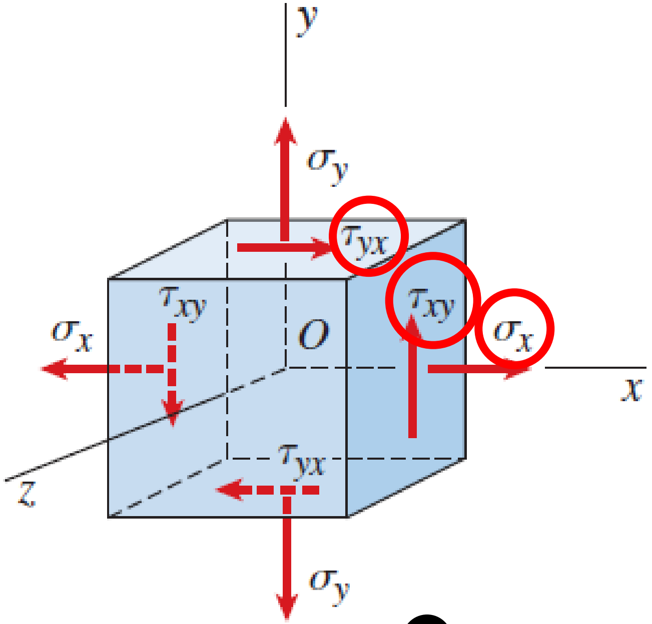




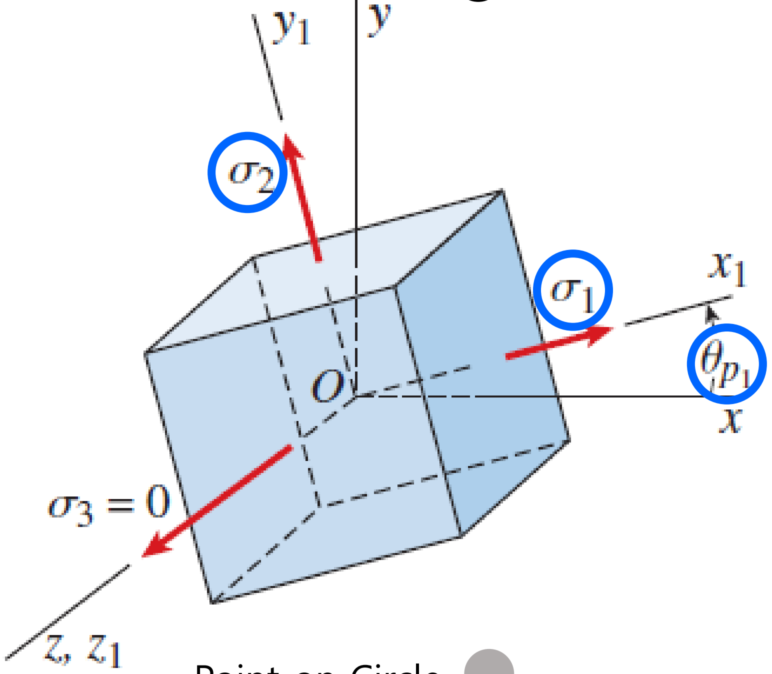
Mohr's Circle

Principal stress can be also found easily





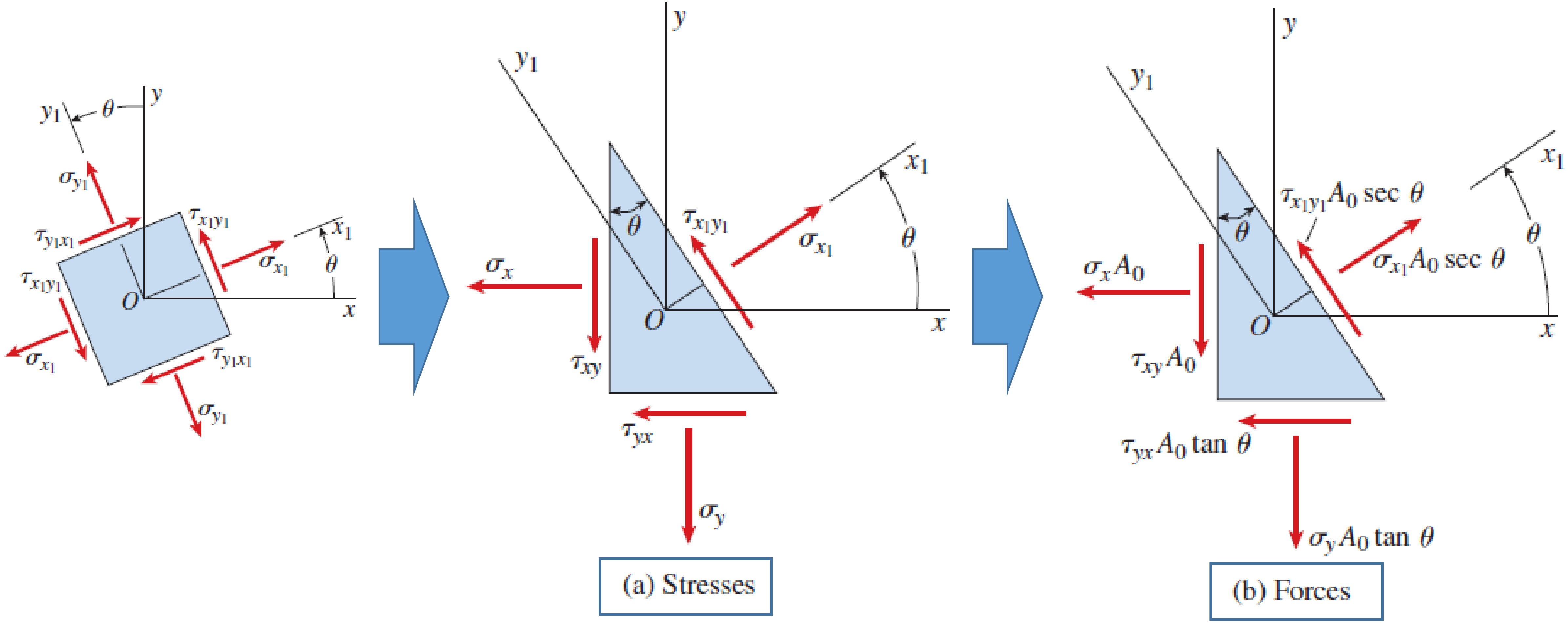
Point on Circle ●



Point on Circle ●

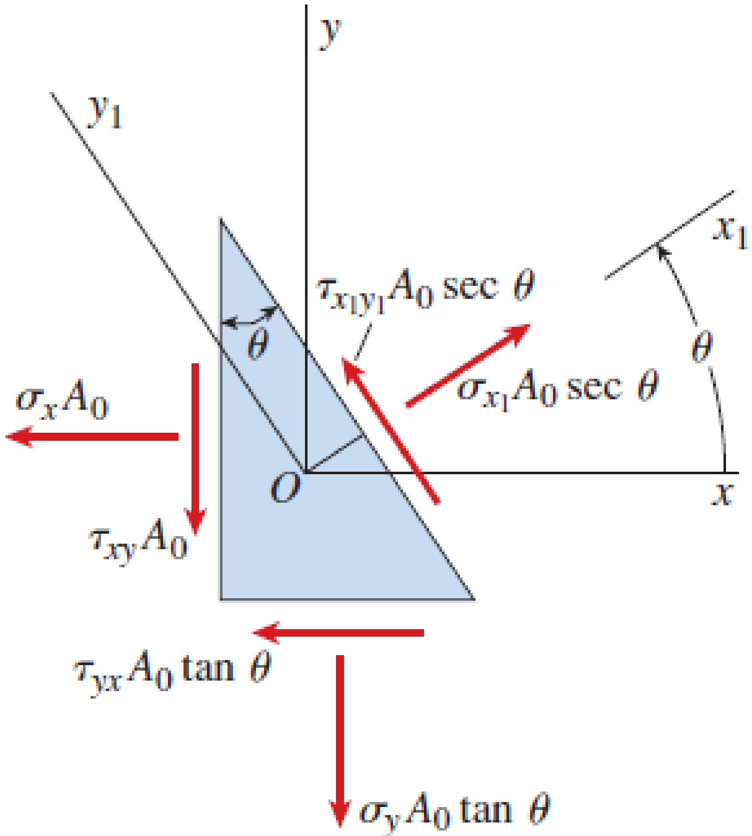
Transformation

Wedge-shape stress element



Transformation

Force equilibrium condition



(b) Forces

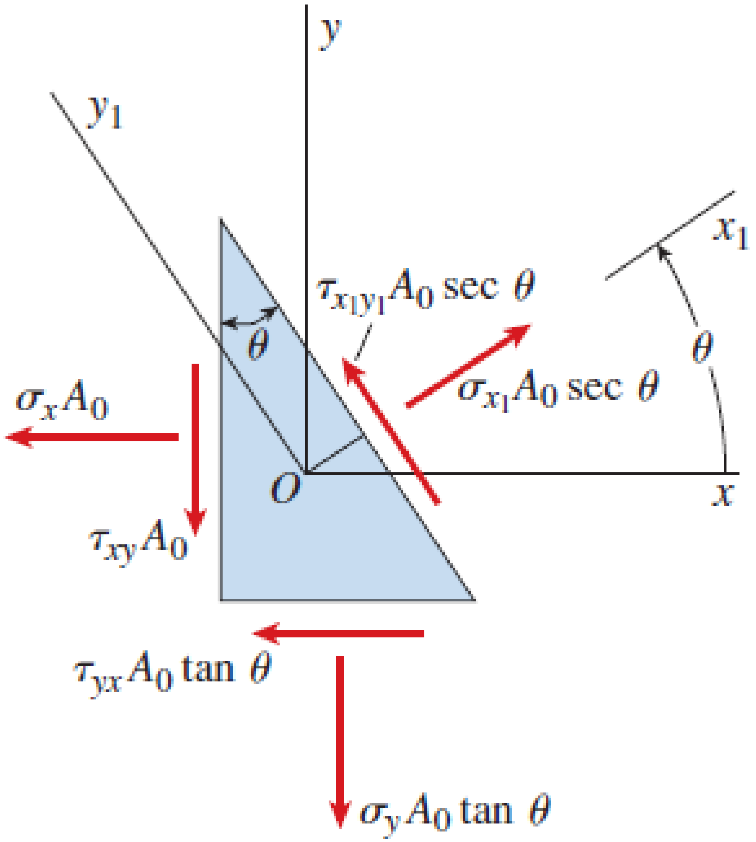
<x1 direction>

$$\sigma_{x_1}A_0 \sec\theta - \sigma_x A_0 \cos\theta - \tau_{xy}A_0 \sin\theta$$

$$- \sigma_y A_0 \tan\theta \sin\theta - \tau_{yx}A_0 \tan\theta \cos\theta = 0$$

Transformation

Force equilibrium condition



(b) Forces

<y1 direction>

$$\tau_{x_1y_1}A_0 \sec\theta + \sigma_x A_0 \sin\theta - \tau_{xy}A_0 \cos\theta$$

$$- \sigma_y A_0 \tan\theta \cos\theta + \tau_{yx}A_0 \tan\theta \sin\theta = 0$$

Transformation equations for plane stress

<x1 direction> - normal

$$\sigma_{x_1} A_0 \sec\theta - \sigma_x A_0 \cos\theta - \tau_{xy} A_0 \sin\theta$$
$$- \sigma_y A_0 \tan\theta \sin\theta - \tau_{yx} A_0 \tan\theta \cos\theta = 0$$

$$\sigma_{x_1} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

<y1 direction> - shear

$$\tau_{x_1 y_1} A_0 \sec\theta + \sigma_x A_0 \sin\theta - \tau_{xy} A_0 \cos\theta$$
$$- \sigma_y A_0 \tan\theta \cos\theta + \tau_{yx} A_0 \tan\theta \sin\theta = 0$$

$$\tau_{x_1 y_1} = -(\sigma_x - \sigma_y) \sin\theta \cos\theta + \tau_{xy} (\cos^2\theta - \sin^2\theta)$$

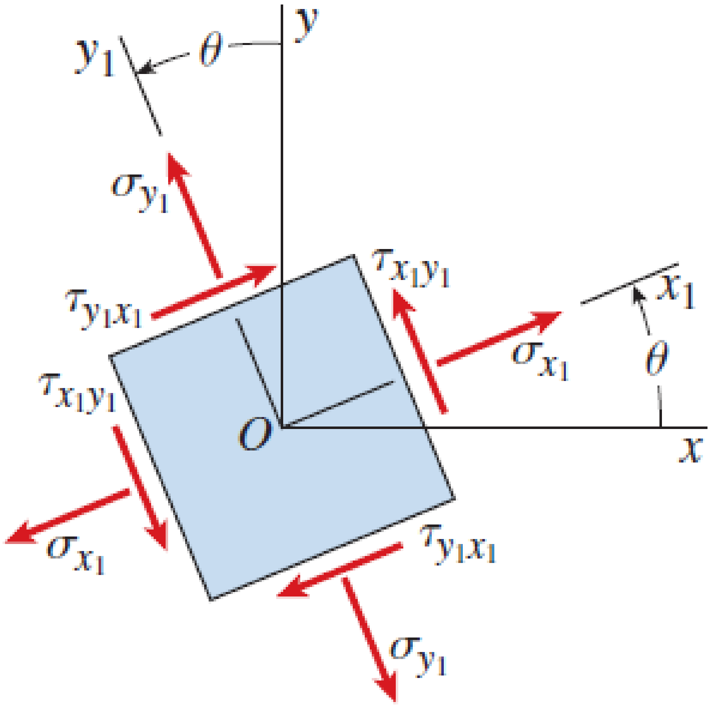
$$\tau_{x_1 y_1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{xy} = \tau_{yx}$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta) \qquad \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\sin\theta \cos\theta = \frac{1}{2} \sin 2\theta$$

Transformation equations for plane stress



<x1 direction> - normal

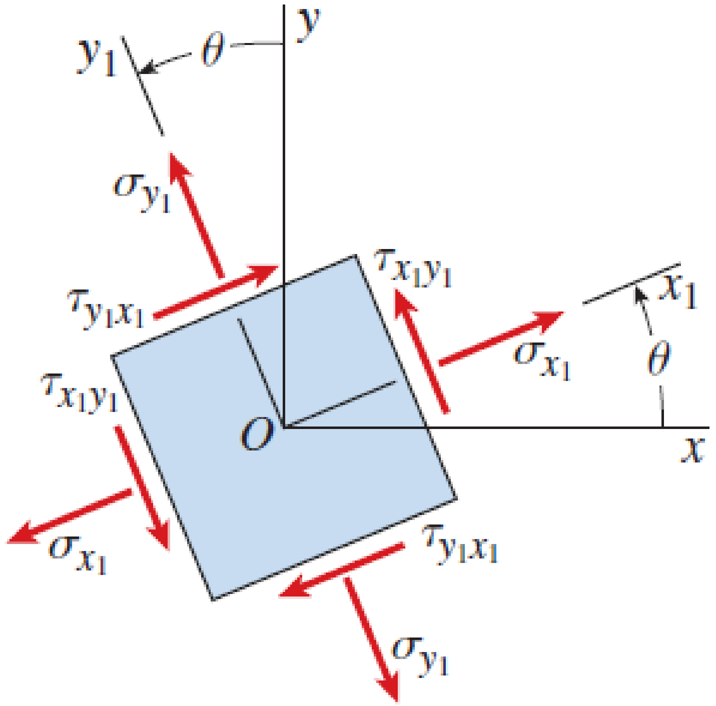
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\theta = \theta + 90°$$

<y1 direction> - normal

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

Observation



<x1 direction> - normal

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

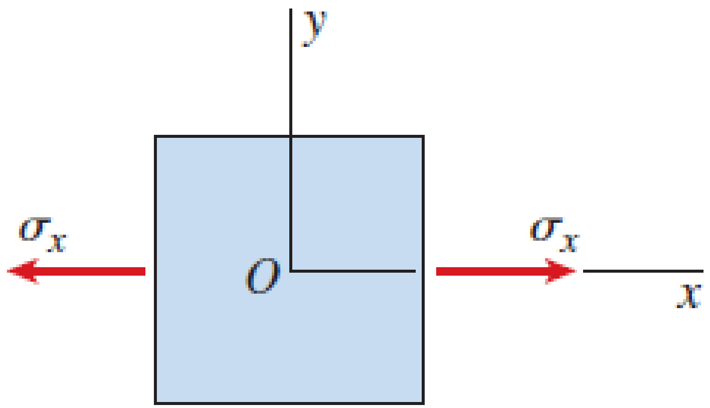
<y1 direction> - normal

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x_1} + \sigma_{y_1} = \sigma_x + \sigma_y$$

Observation

<uniaxial stress>



<x1 direction> - normal

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\Rightarrow \quad \sigma_{x_1} = \frac{\sigma_x}{2}(1 + \cos 2\theta)$$

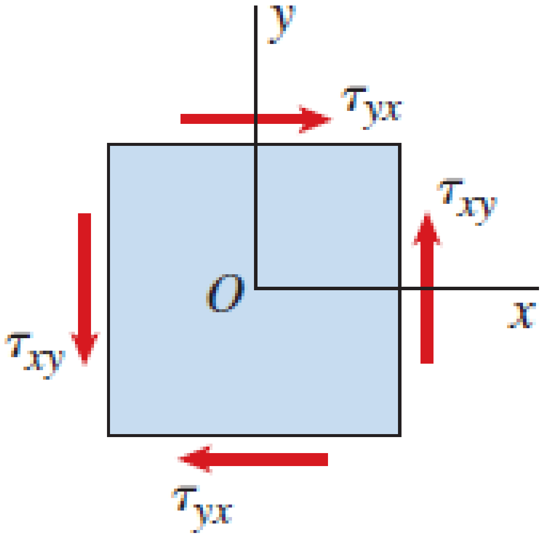
<y1 direction> - shear

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\Rightarrow \quad \tau_{x_1y_1} = -\frac{\sigma_x}{2}(\sin 2\theta)$$

Observation

<pure shear>



<x1 direction> - normal

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

➡ $$\sigma_{x_1} = \tau_{xy} \sin 2\theta$$

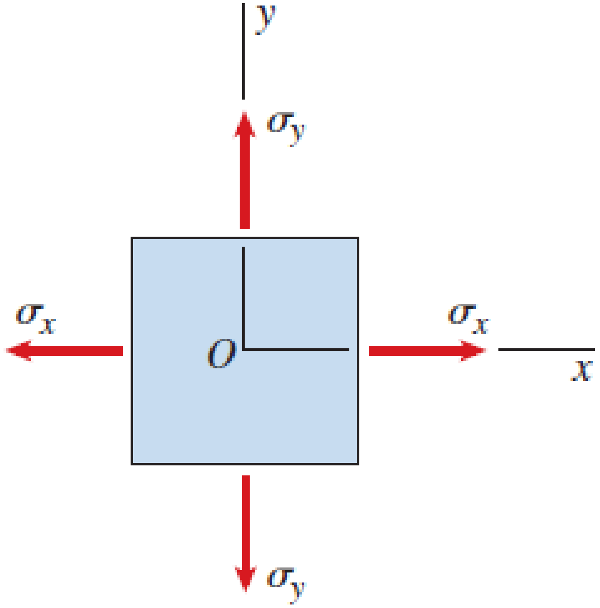
<y1 direction> - shear

$$\tau_{x_1 y_1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

➡ $$\tau_{x_1 y_1} = \tau_{xy} \cos 2\theta$$

Observation

<biaxial stress>



<x1 direction> - normal

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

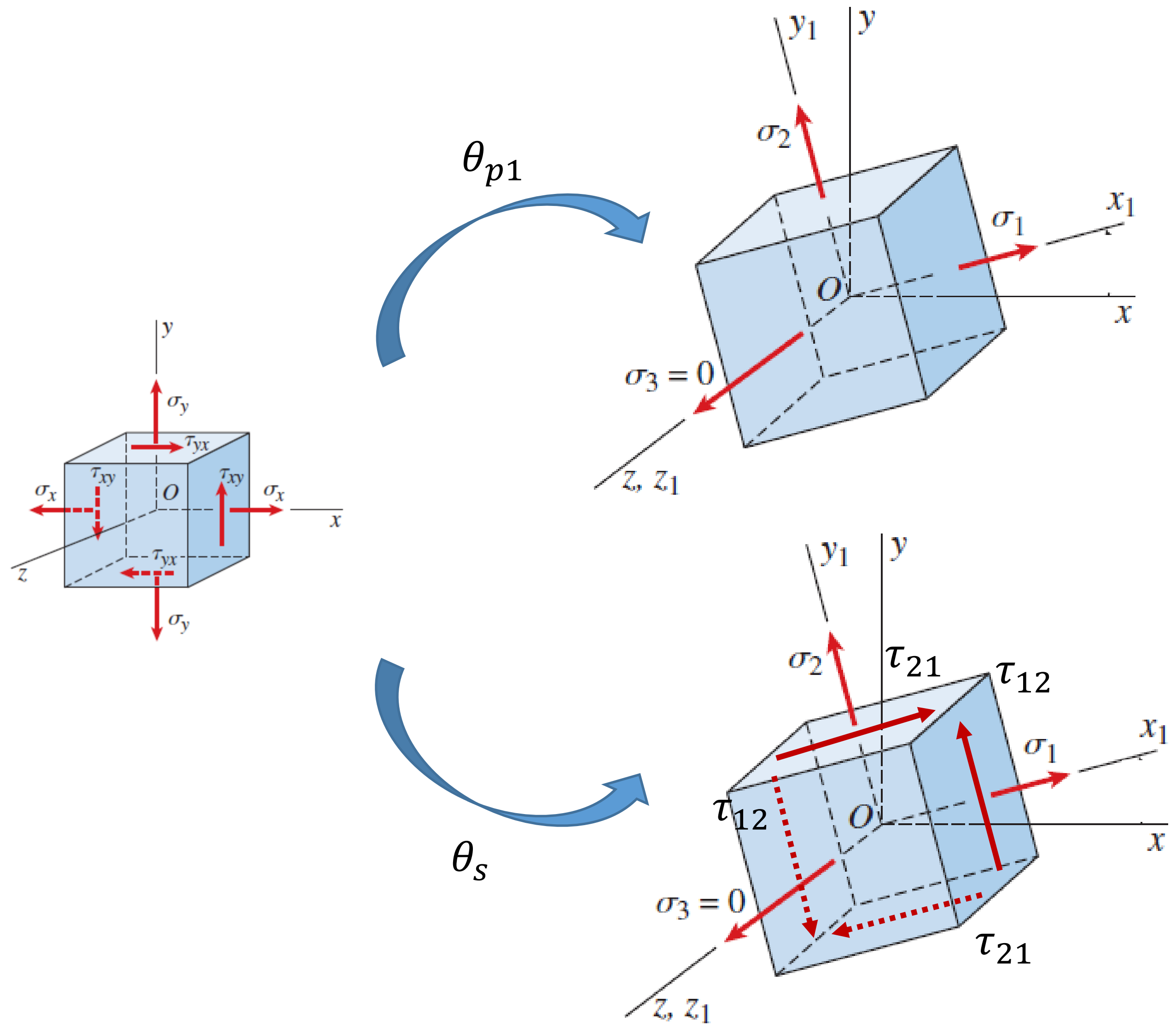
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

<y1 direction> - shear

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$

Principal stress and Maximum shear stress



Principal stress

θ_{p1} → $\sigma_1 = max$ $\tau_{12} = 0$

$\theta_{p2} = \theta_{p1} + 90$ → $\sigma_2 = min$ $\tau_{12} = 0$

Max. shear stress

θ_s → $\sigma_1 \neq 0$, $\sigma_2 \neq 0$

$\tau_{12} = max$

$\tau_{21} = max$

Principal stress

Principal stress

at θ_p

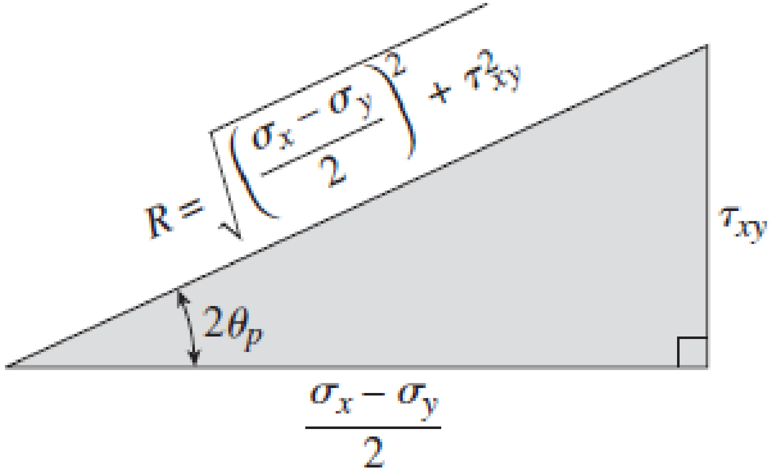
$$\sigma_1 = max$$

$$\sigma_2 = min$$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

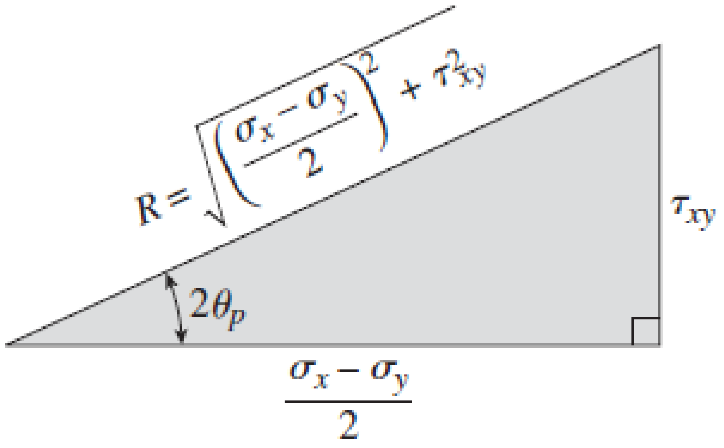
Differentiation for max or min of σ_{x1}

$$\frac{d\sigma_{x_1}}{d\theta} = -(\sigma_x - \sigma_y) \sin 2\theta + 2\tau_{xy} \cos 2\theta = 0$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Principal stress



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \cos 2\theta_{p_1} = \frac{\sigma_x - \sigma_y}{2R} \qquad \sin 2\theta_{p_1} = \frac{\tau_{xy}}{R}$$

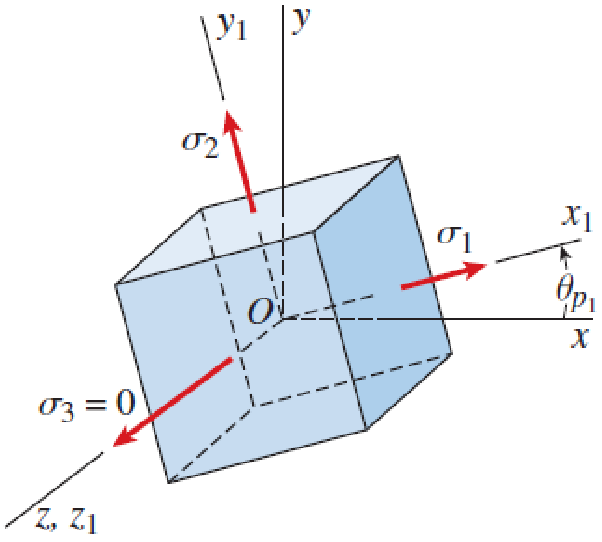
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Your homework

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Principal stress

Finally...



$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

using

$$\sigma_{x1} + \sigma_{y1} = \sigma_x + \sigma_y$$

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

Principal stress =

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Principal angle =

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \cos 2\theta_{p_1} = \frac{\sigma_x - \sigma_y}{2R} \qquad \sin 2\theta_{p_1} = \frac{\tau_{xy}}{R}$$

Maximum Shear Stress

Max. shear stress

$at\ \theta_s$

$$\tau_{12} = max$$

$$\tau_{21} = max$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Differentiation for max of $\tau_{x_1y_1}$

$$\frac{d\tau_{x_1y_1}}{d\theta} = -(\sigma_x - \sigma_y)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Maximum Shear Stress

Max. shear stress

at θ_s

$$\tau_{12} = max$$

$$\tau_{21} = max$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\cos 2\theta_{s_1} = \frac{\tau_{xy}}{R} \qquad \sin 2\theta_{s_1} = -\frac{\sigma_x - \sigma_y}{2R}$$

(where $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$)

Maximum shear stress = $$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Max shear angle = $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Max. shear stress angle vs Principal stress angle

Max. shear stress

$$\tau_{12} = max$$

$$\tau_{21} = max$$

$at\ \theta_s$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$at\ \theta_p$

Principal stress

$$\sigma_1 = max$$

$$\sigma_2 = min$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_s = -\frac{1}{\tan 2\theta_p} = -\cot 2\theta_p$$

$$\frac{\sin 2\theta_s}{\cos 2\theta_s} + \frac{\cos 2\theta_p}{\sin 2\theta_p} = 0$$

$$\sin 2\theta_s \sin 2\theta_p + \cos 2\theta_s \cos 2\theta_p = 0$$

$$\cos(2\theta_s - 2\theta_p) = 0$$

$$2\theta_s - 2\theta_p = \pm 90°$$

$$\theta_s = \theta_p \pm 45°$$

Max. shear stress angle vs Principal stress angle

Max. shear stress

$\tau_{12} = max$

$\tau_{21} = max$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$at\ \theta_s$

$$\theta_s = \theta_p \pm 45°$$

$at\ \theta_p$

Principal stress

$\sigma_1 = max$

$\sigma_2 = min$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Max. shear stress angle vs Principal stress angle

Max. shear stress

$$\tau_{12} = max$$
$$\tau_{21} = max$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$at\ \theta_s$

$$\theta_s = \theta_p \pm 45°$$

$at\ \theta_p$

Principal stress

$$\sigma_1 = max$$
$$\sigma_2 = min$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

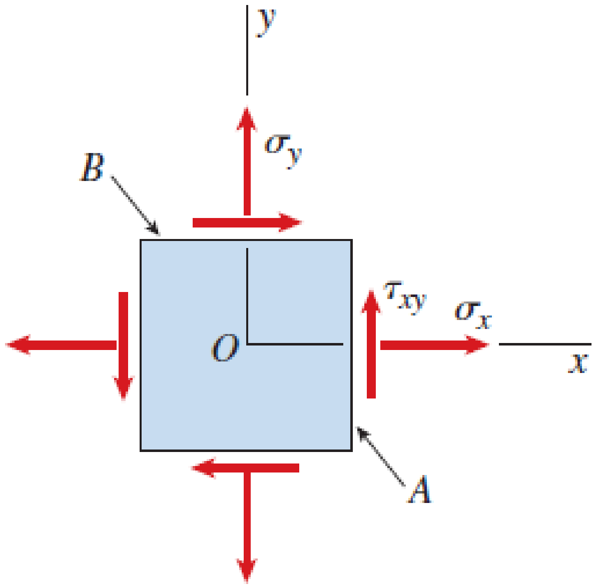
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

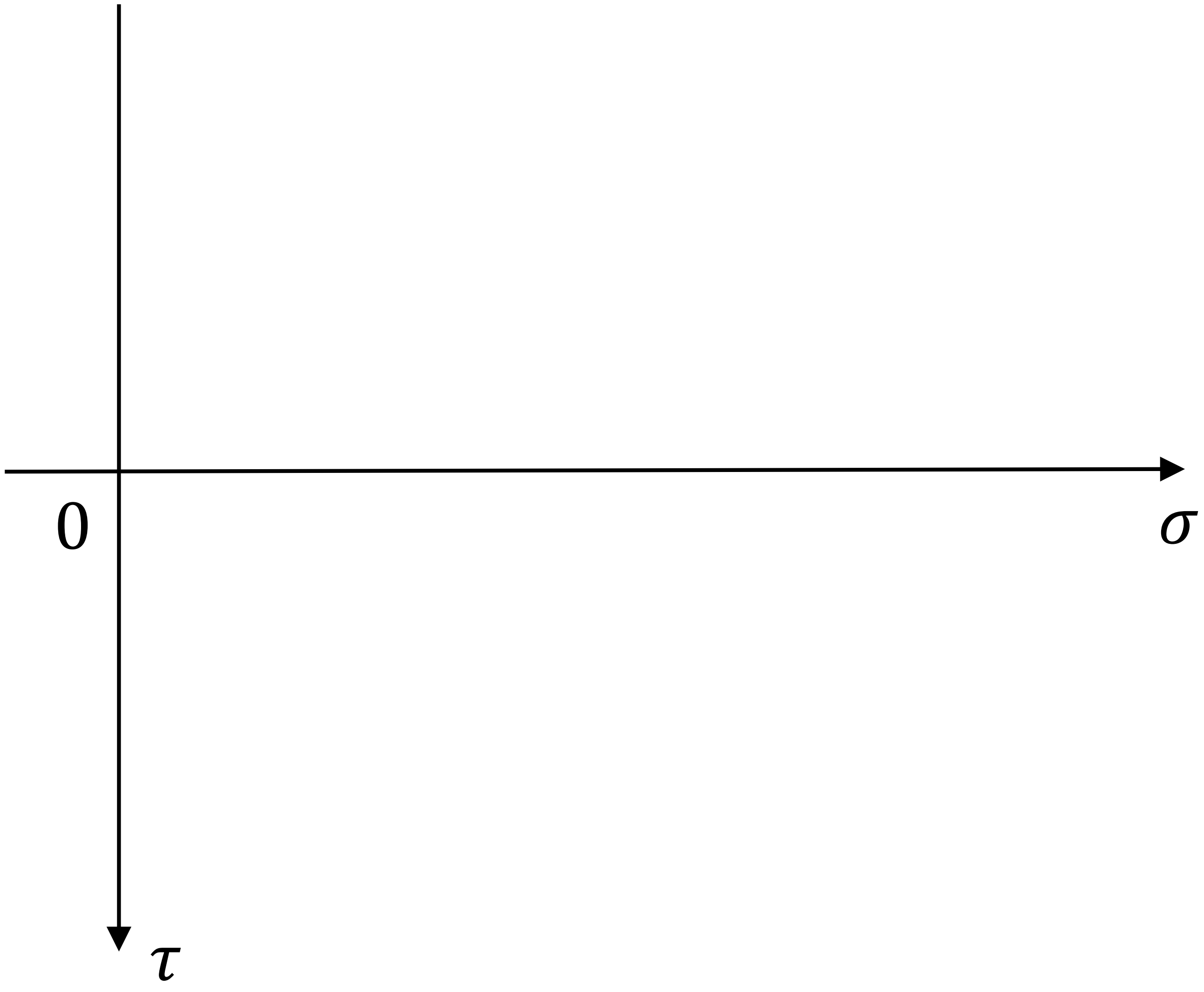
$$\tau_{x_1 y_1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

at arbitrary angle θ

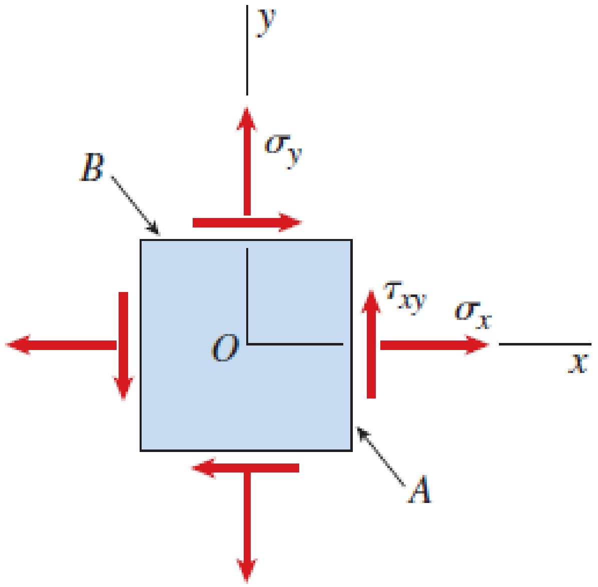
$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

Mohr's circle

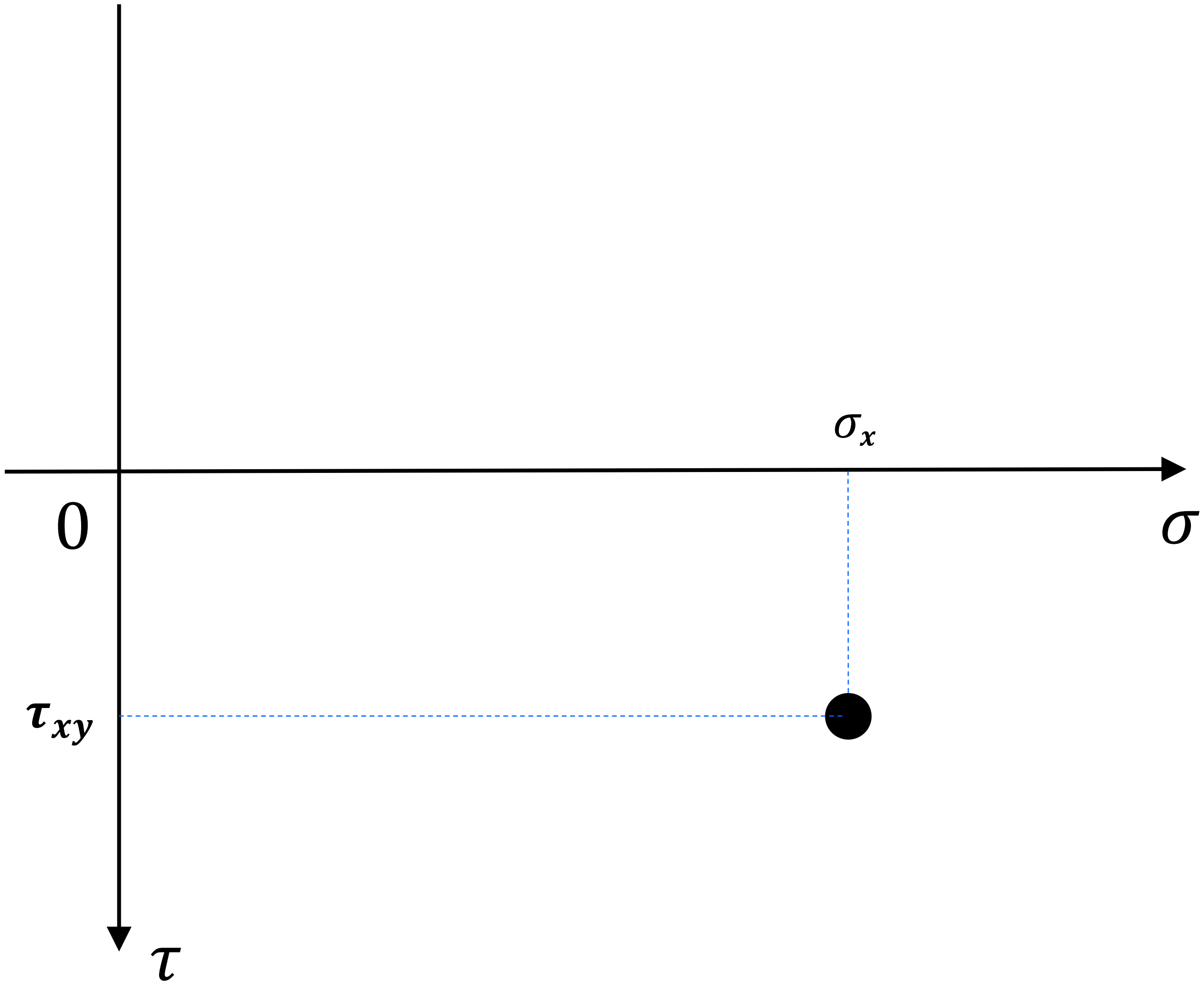




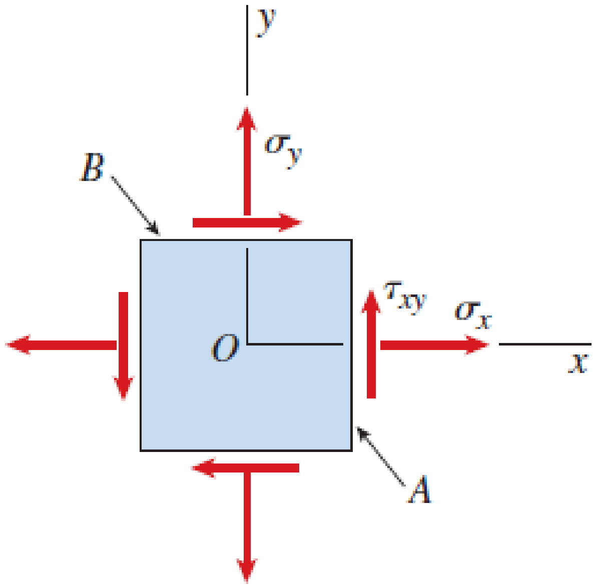
Mohr's circle



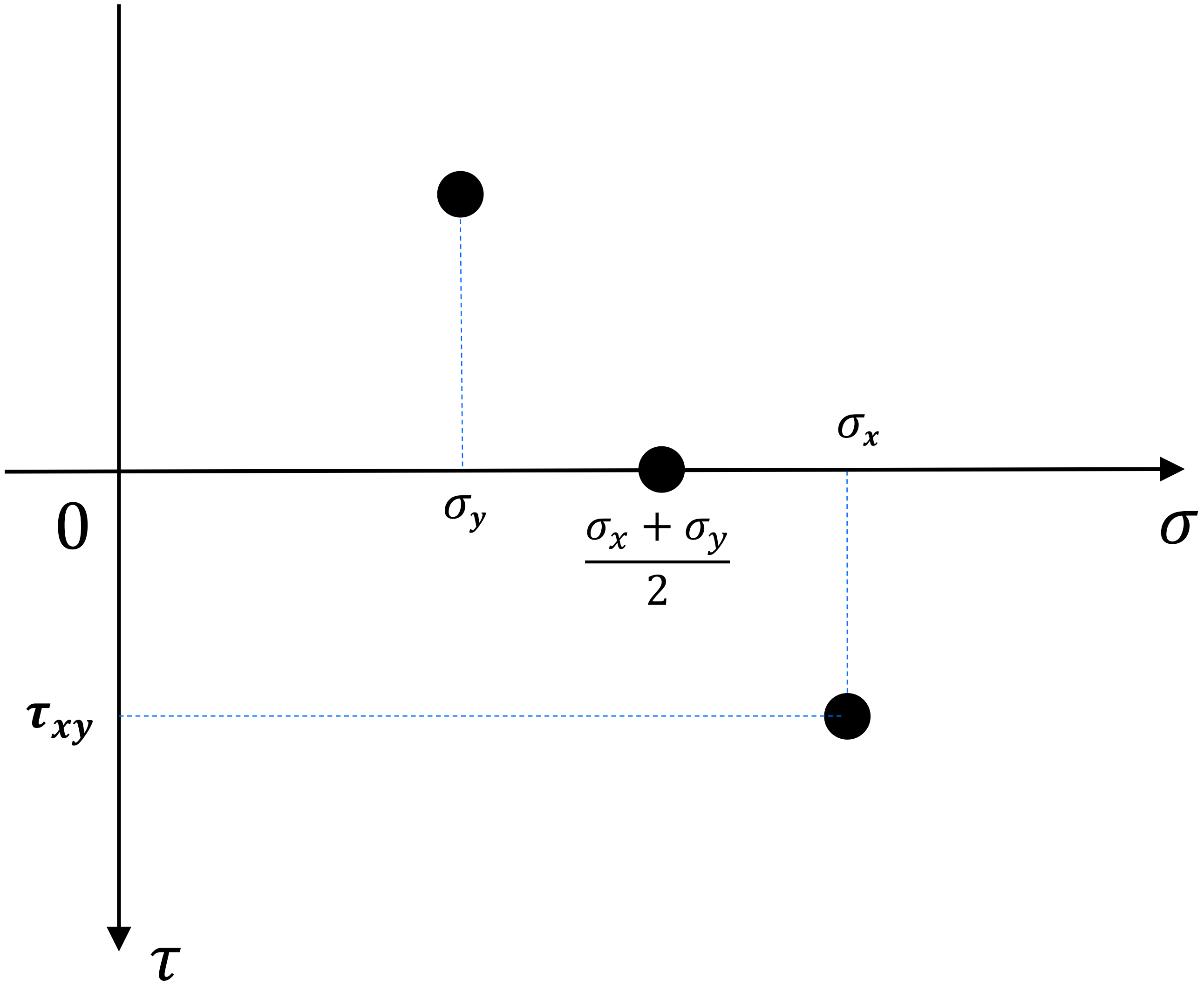
Point on Circle ●



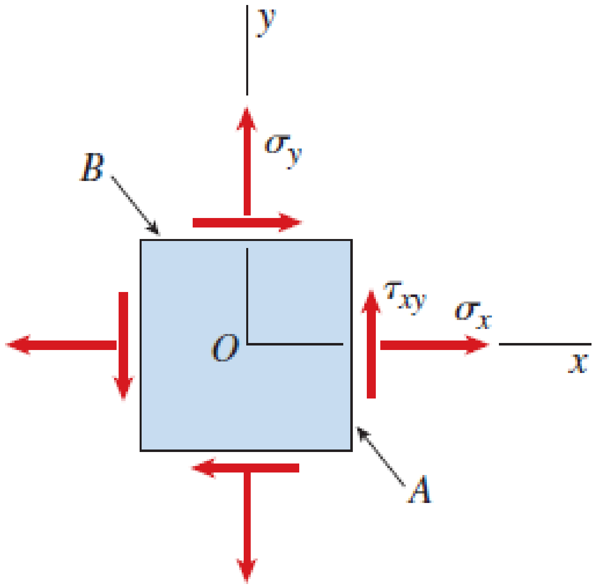
Mohr’s circle



Point on Circle ●

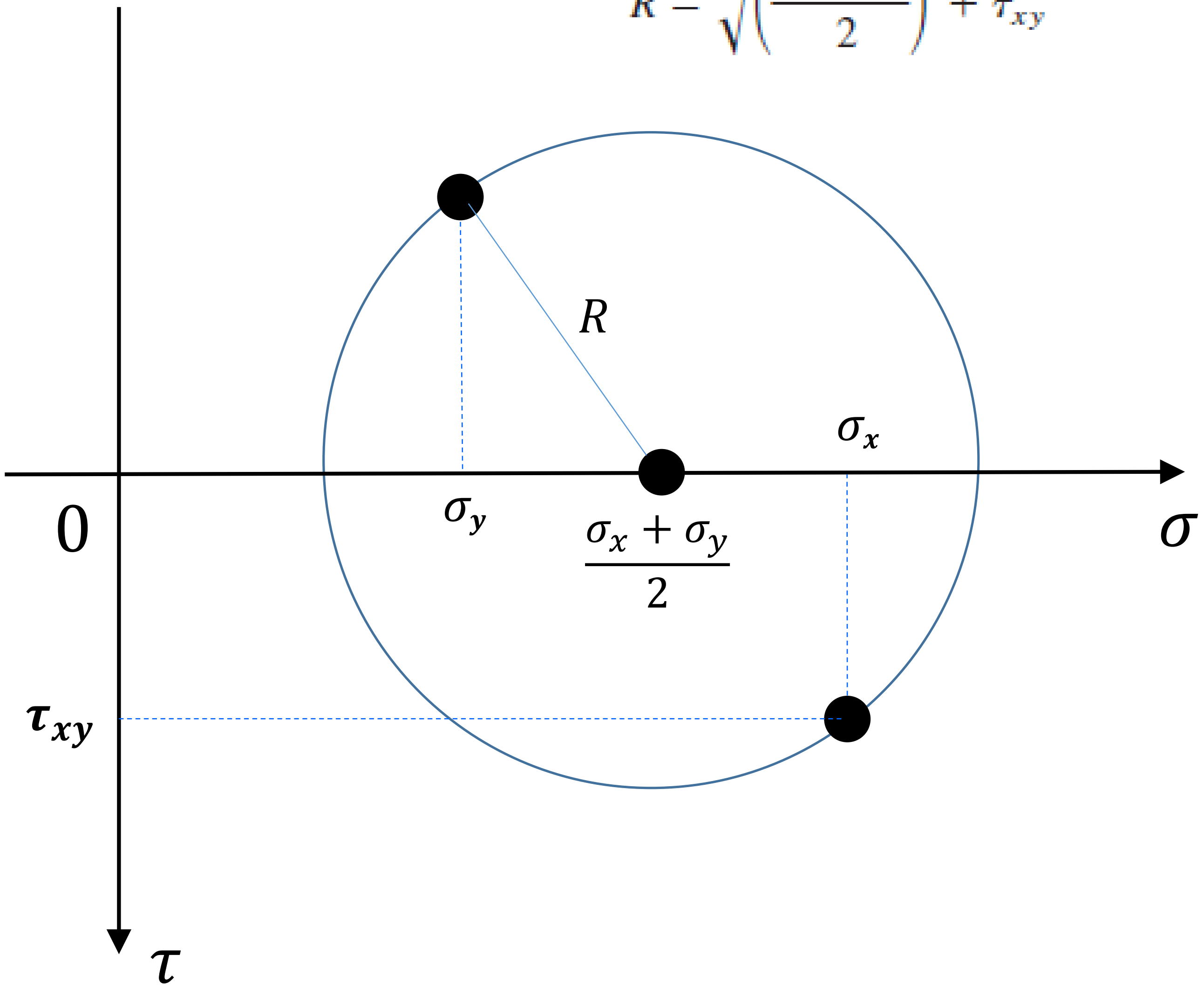


Mohr's circle



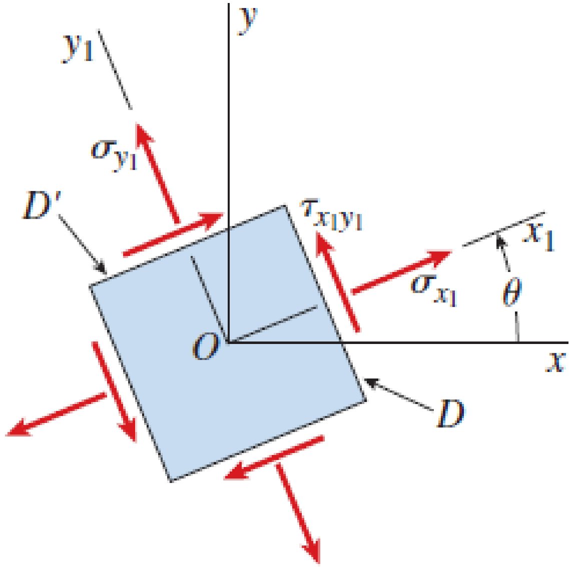
Point on Circle ●

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



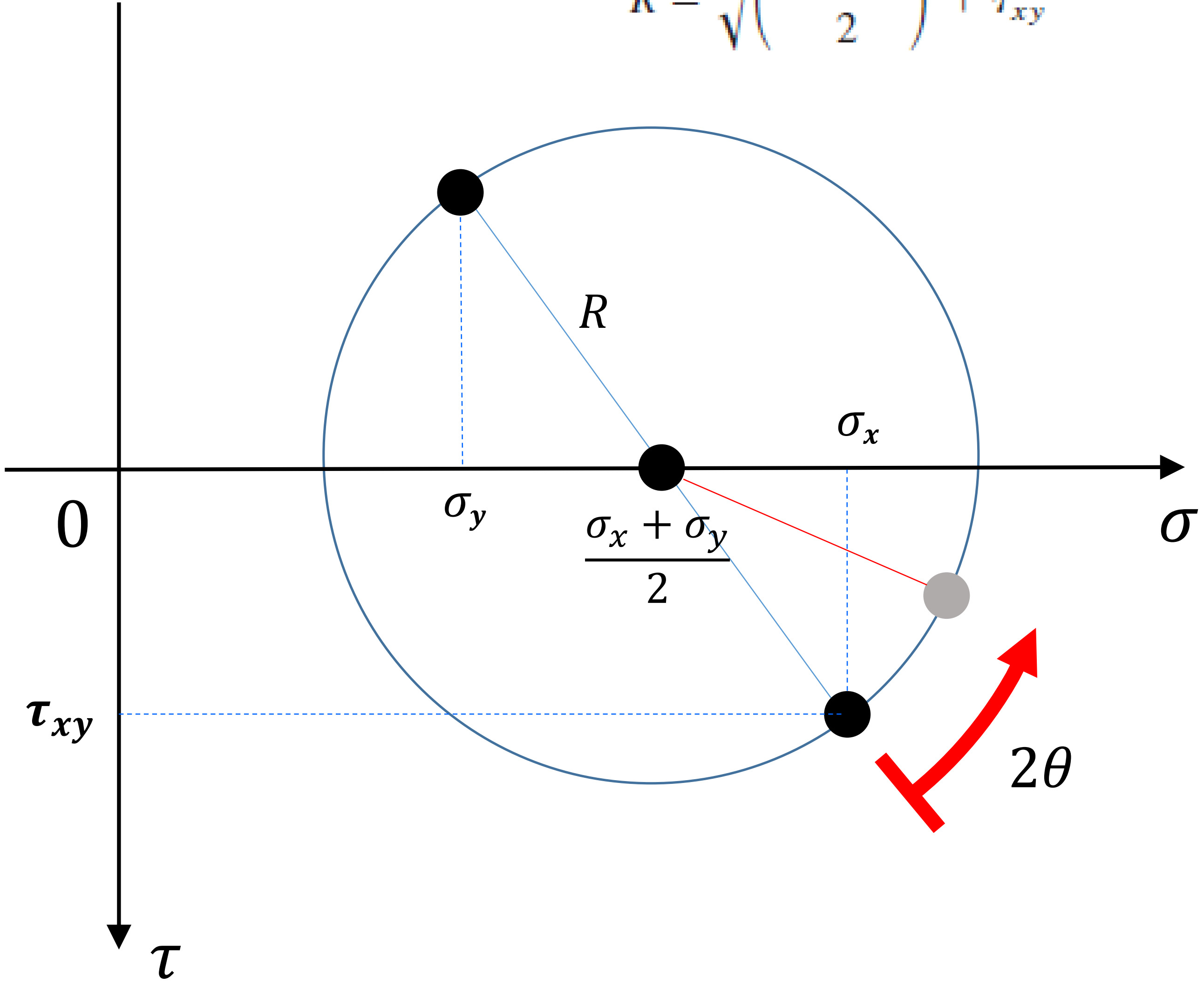
Mohr's circle application I

at arbitrary angle θ



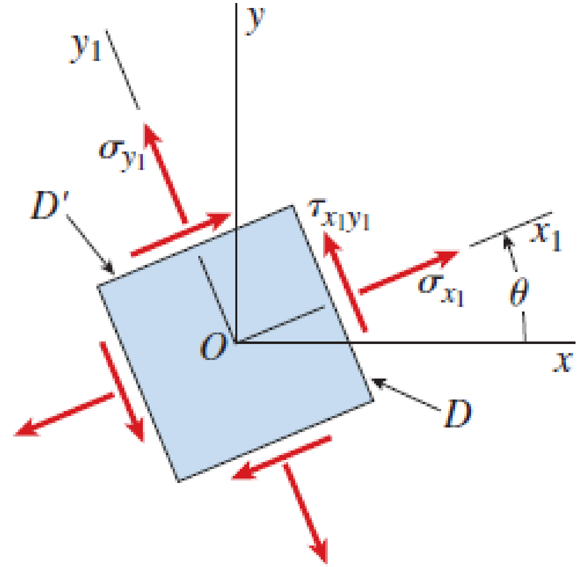
Point on Circle

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



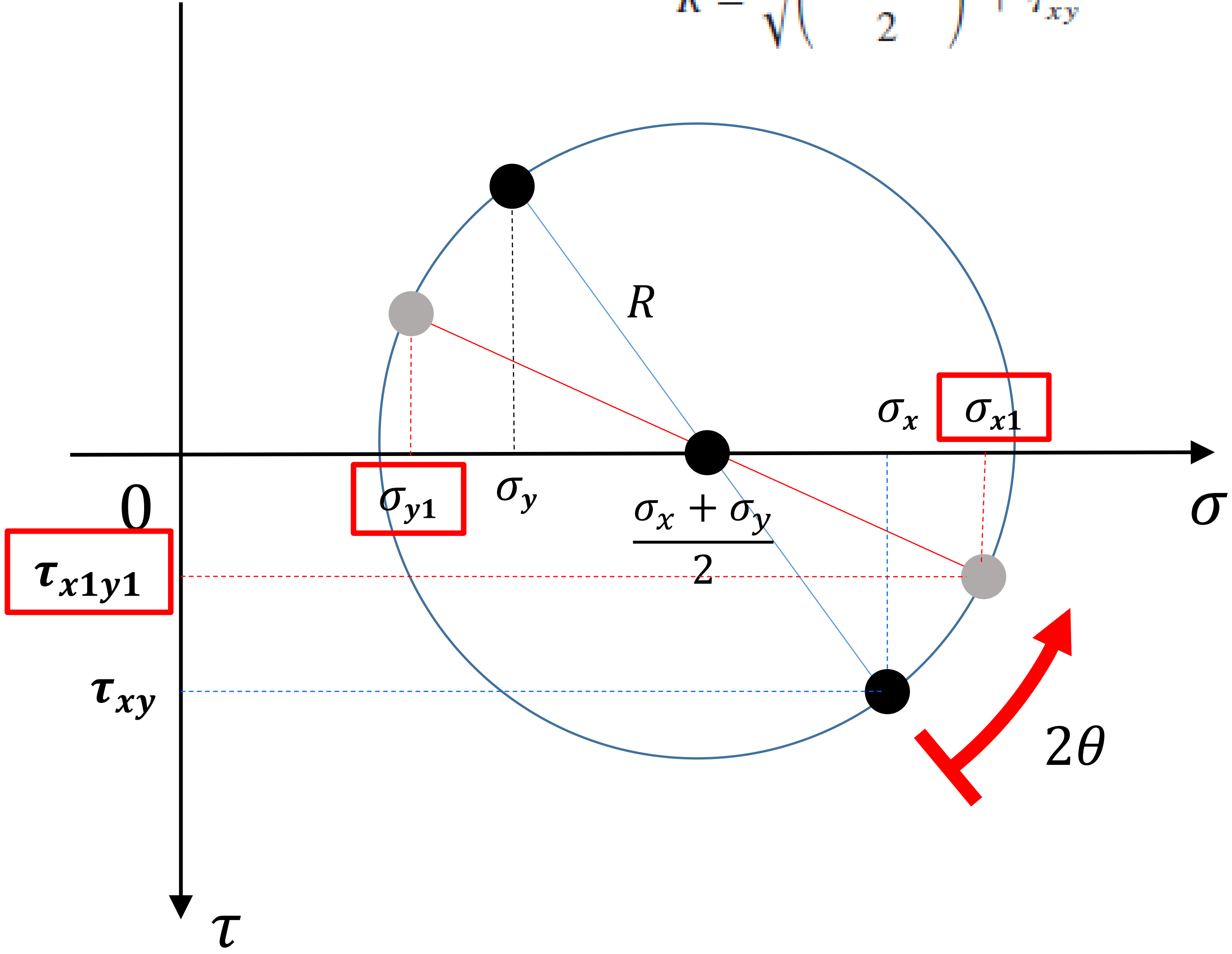
Mohr's circle application I

at arbitrary angle θ



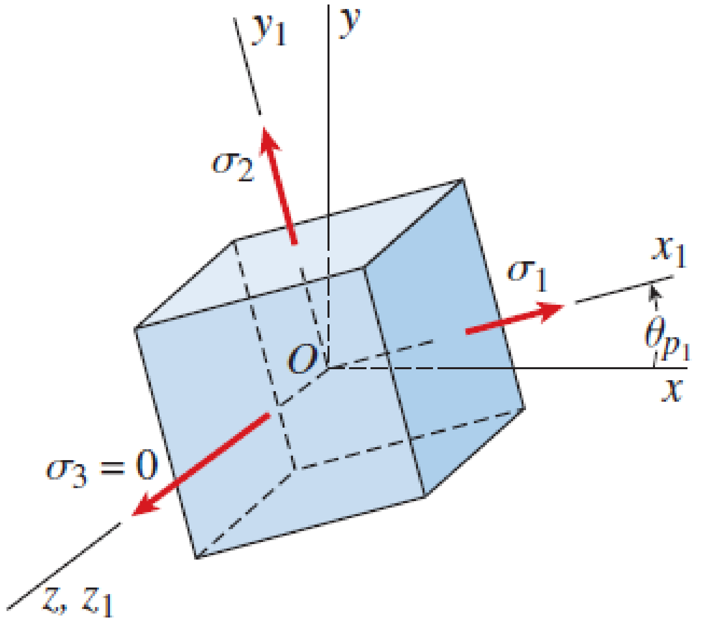
Point on Circle

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

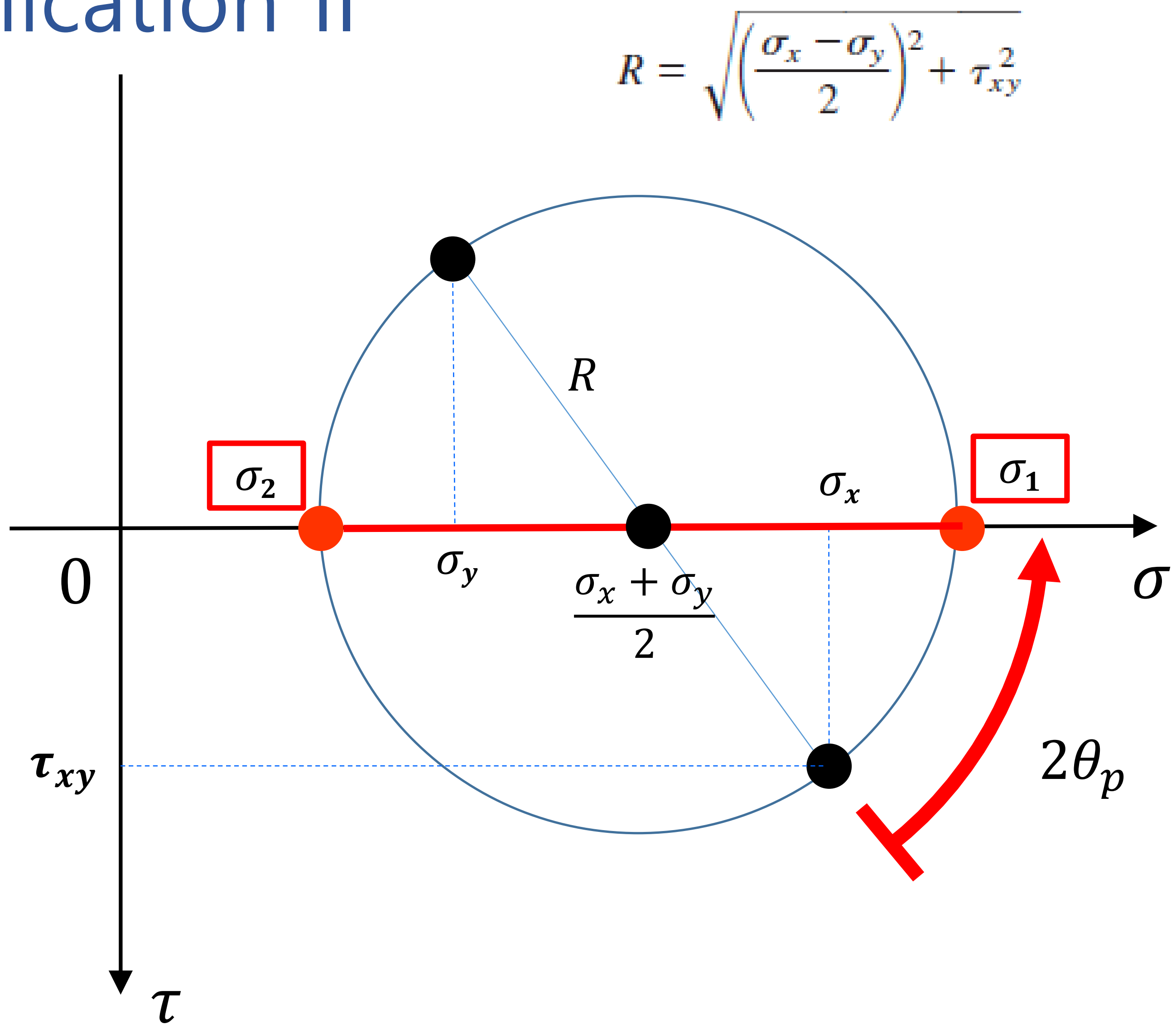


Mohr's circle application II

principal angle θ_p



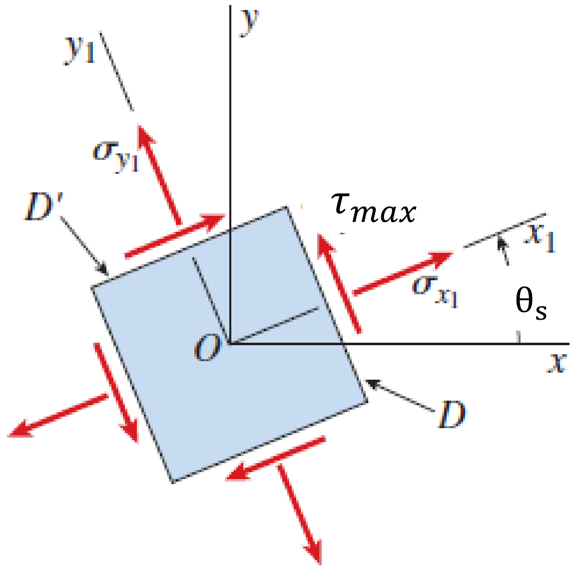
Point on Circle



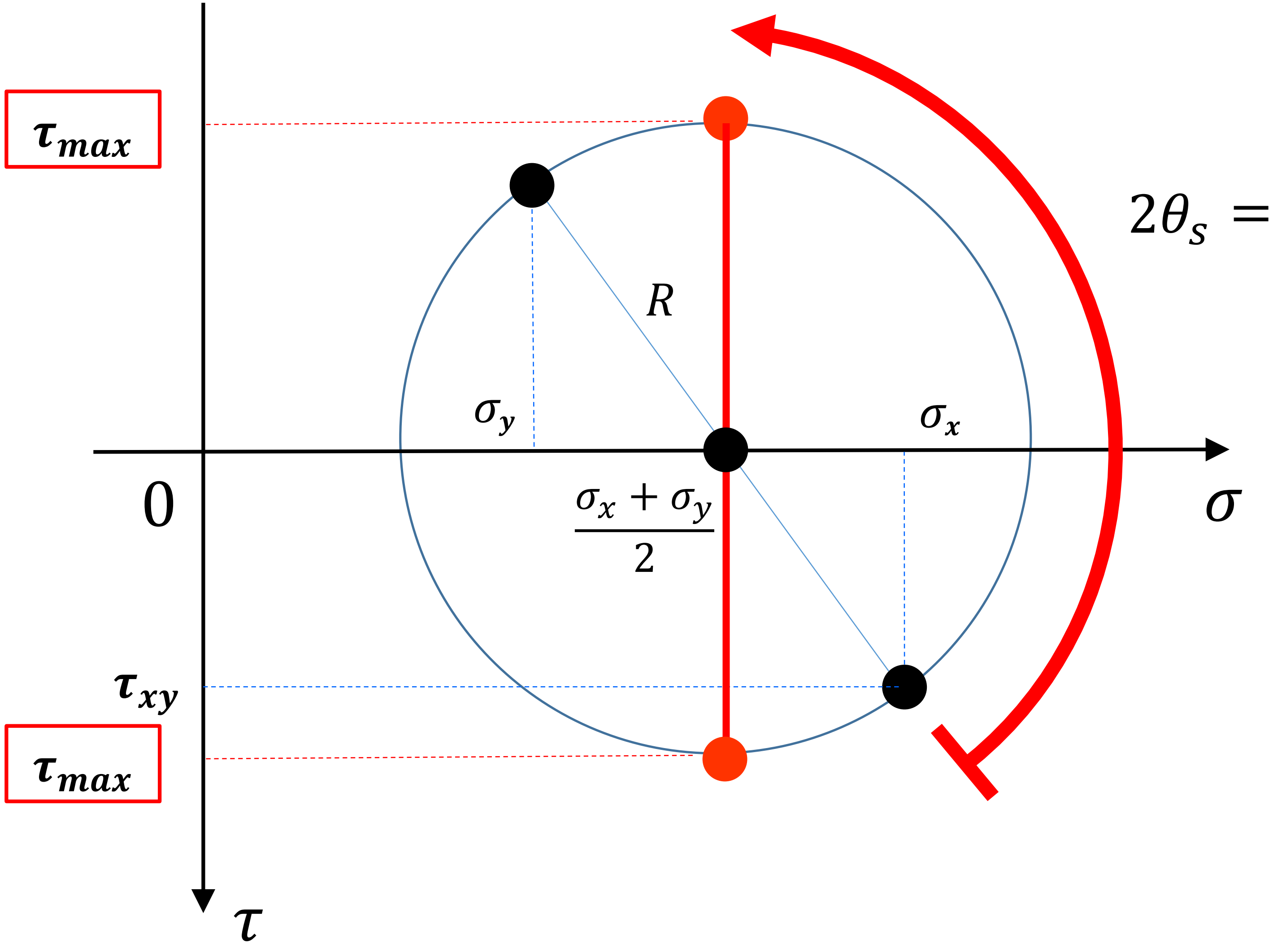
Mohr's circle application III

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

max. shear angle θ_s



Point on Circle



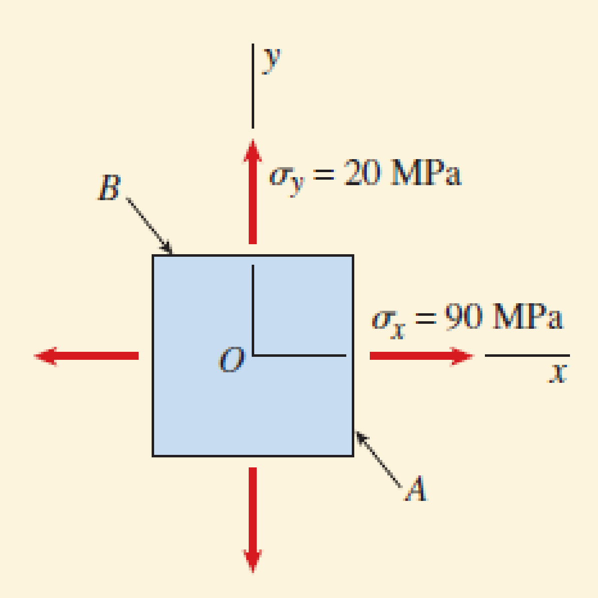
$$2\theta_s = 2(\theta_{p+45})$$

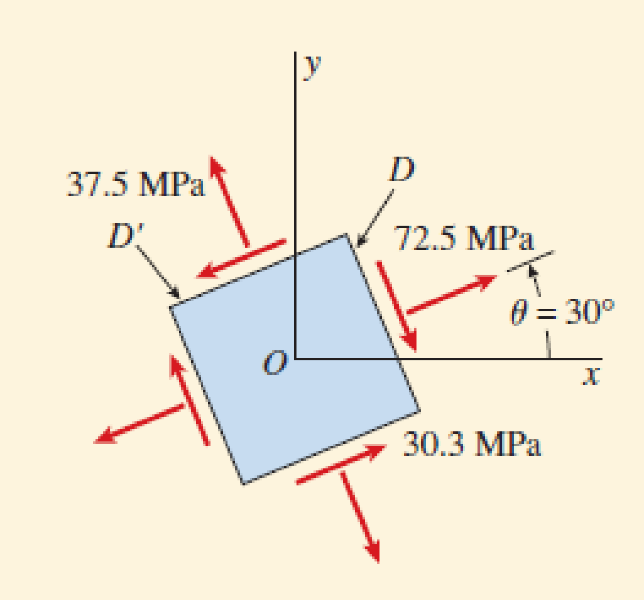
Mohr's circle Example

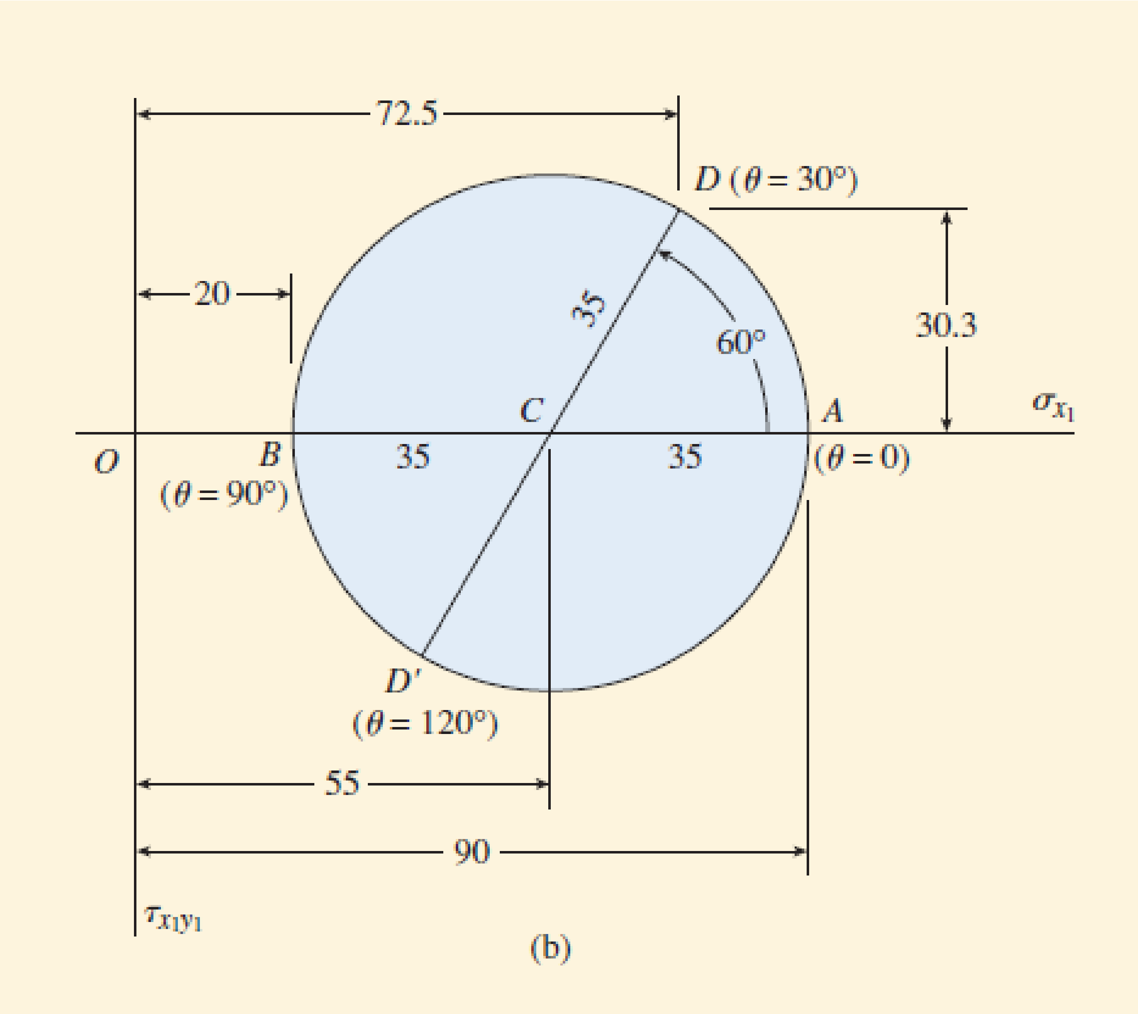
Example 7-4

At a point on the surface of a pressurized cylinder, the material is subjected to biaxial stresses $\sigma_x = 90$ MPa and $\sigma_y = 20$ MPa, as shown on the stress element of Fig. 7-18a.

Using Mohr's circle, determine the stresses acting on an element inclined at an angle $\theta = 30°$. (Consider only the in-plane stresses, and show the results on a sketch of a properly oriented element.)







Mohr's circle Example

Example 7-6

At a point on the surface of a generator shaft the stresses are $\sigma_x = -50$ MPa, $\sigma_y = 10$ MPa, and $\tau_{xy} = -40$ MPa, as shown in Fig. 7-22a.

Using Mohr's circle, determine the following quantities: (a) the stresses acting on an element inclined at an angle $\theta = 45°$, (b) the principal stresses, and (c) the maximum shear stresses. (Consider only the in-plane stresses, and show all results on sketches of properly oriented elements.)