

Chapter 13

The Nature of Thermodynamics

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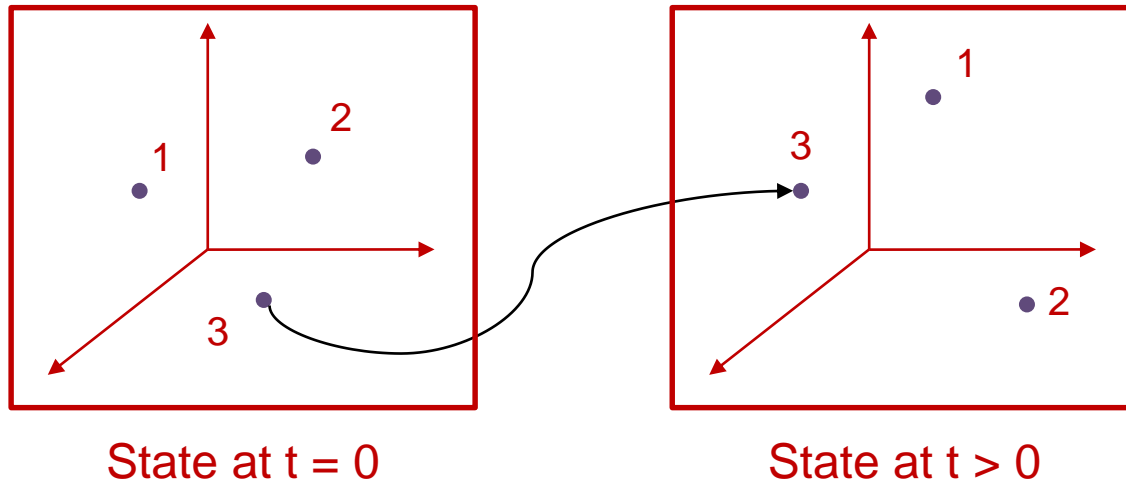
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13.1 Boltzmann Statistics

- Distinguishability : **Classical Statistics**

In classical mechanics, trajectories can be built up from the information of states of particles.

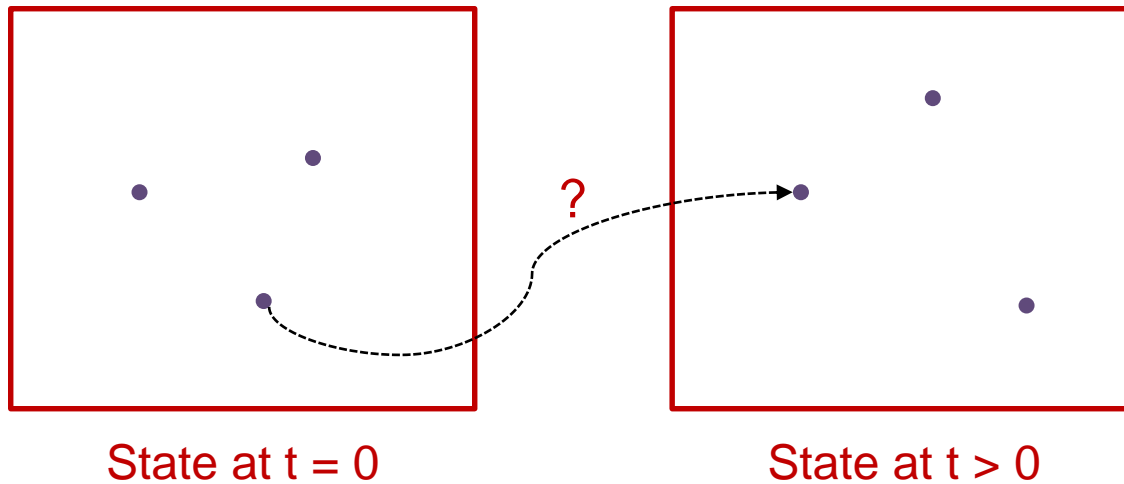
The trajectories allow us to distinguish particle whether they are identical or not.



13.1 Boltzmann Statistics

- Distinguishability : **Quantum Statistics**

In quantum mechanics, Our knowledge of states is imperfect because the states are hobbled according to Heisenberg's uncertainty principle. It means that it is impossible to distinguish identical particles.



13.1 Boltzmann Statistics

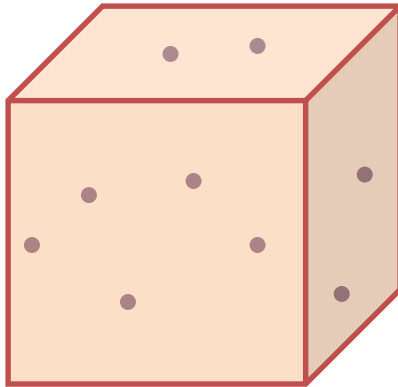
- Boltzmann statistics

Boltzmann statistics is for distinguishable particles.

Therefore, Boltzmann statistics is applied to particles of **classical gas** or on there positions in **solid lattice**.

Consider N molecules with internal energy E in cubic volume V

Each energy level, ϵ_i has N_i molecules with g_i degeneracies.



$$\left. \begin{aligned} \sum N_i &= N \\ \sum N_i \epsilon_i &= E \end{aligned} \right\} \text{two constraints of the system}$$

13.1 Boltzmann Statistics

- Number of rearrangement

First, select N_1 distinguishable particles from a total of N to be placed **in the first energy level** with arrangement among g_1 choices.

Ex) seven particles for 1st energy level of $g_i = 6$



$$w_1 = {}_N C_{N_1} \cdot g_1^{N_1} = \frac{N! \cdot g_1^{N_1}}{(N - N_1)! N_1!}$$

13.1 Boltzmann Statistics

Next step is to do same work for 2nd energy level among $(N - N_1)$ particles

These works are done in sequence until last N_n particles are distributed.

Thus, the number of rearrangement becomes

$$\begin{aligned}w_B &= \prod w_i = ({}_N C_{N_1} \cdot g_1^{N_1}) \times ({}_{N-N_1} C_{N_2} \cdot g_2^{N_2}) \times \dots \times ({}_{N_n} C_{N_n} \cdot g_n^{N_n}) \\ &= \left(\frac{N!}{(N - N_1)! N_1!} g_1^{N_1} \right) \times \left(\frac{(N - N_1)!}{(N - N_1 - N_2)! N_2!} g_2^{N_2} \right) \times \dots \times \left(\frac{N_n!}{0! N_n!} g_n^{N_n} \right)\end{aligned}$$

$$\longrightarrow w_B = N! \prod \frac{g_i^{N_i}}{N_i!}$$

13.3 Boltzmann Distributions

- Boltzmann distributions

From Stirling's approximation, $\ln(N!) = N \ln(N) - N$

$$\begin{aligned}\ln(w_B) &= \ln(N!) + \sum [N_i \ln(g_i) - \ln(N_i!)] \\ &= \ln(N!) + \sum [N_i \ln(g_i) - N_i \ln(N_i) + N_i]\end{aligned}$$

N_i for j^{th} energy level is undetermined yet


→ **Method of Lagrange multiplier** is used to obtain most probable macro state under two constraints, $\sum N_i = N$, $\sum N_i \epsilon_i = E$

$$\frac{\partial(\ln(w_B))}{\partial N_i} + \alpha \frac{\partial(\sum N_i - N)}{\partial N_i} + \beta \frac{\partial(\sum N_i \epsilon_i - E)}{\partial N_i} = 0$$

13.3 Boltzmann Distributions


Applying method of Lagrange multipliers to Boltzmann distributions,


$$\frac{\partial(\sum N_i \ln(g_i) - \sum N_i \ln(N_i) + \sum N_i)}{\partial N_i} + \alpha \frac{\partial(\sum N_i)}{\partial N_i} + \beta \frac{\partial(\sum N_i \epsilon_i)}{\partial N_i} = 0$$

 $\ln(g_i) - \ln(N_i) - \cancel{\frac{N_i}{N_i}} + 1 + \alpha + \beta \epsilon_i = 0$

Then, number distribution becomes

$$\ln\left(\frac{N_i}{g_i}\right) = \alpha + \beta \epsilon_i \quad \longrightarrow \quad \frac{N_i}{g_i} = e^{\alpha + \beta \epsilon_i} = f_i(\epsilon_i)$$

 # of particles per each quantum state for the equilibrium configuration

 Boltzmann distribution function

13.3 Boltzmann Distributions

- Physical relation of constant β

$$\sum N_i \ln g_i - \sum N_i \ln N_i + \alpha \sum N_i + \beta \sum N_i \epsilon_i = 0$$

$$\sum N_i \ln g_i - \sum N_i \ln N_i = -\alpha N - \beta U$$

$$\begin{aligned} \ln(w_B) &= \ln(N!) + \sum [N_i \ln(g_i) - N_i \ln(N_i) + N_i] \\ &= \ln(N!) + \sum [N_i \ln(N_i e^{-\alpha - \beta \epsilon_i}) - N_i \ln(N_i) + N_i] \\ &= \ln(N!) + \sum [N_i \ln(N_i) - \alpha N_i - \beta N_i \epsilon_i - N_i \ln(N_i) + N_i] \\ &= \ln(N!) + N - \alpha N - \beta U \end{aligned}$$

13.3 Boltzmann Distributions

Using the statistical definition of entropy,

$$S = k \ln(w_B) = k \ln(N!) + k(1 - \alpha)N - k\beta U = S_0 - k\beta U$$

In classical thermodynamics,

$$dS(U, V) = \frac{1}{T} dU + \frac{P}{T} dV = \left(\frac{\partial S}{\partial U} \right)_V dU + \left(\frac{\partial S}{\partial V} \right)_U dV \rightarrow \left(\frac{\partial S}{\partial U} \right)_V = \frac{1}{T}$$

From the previous result, $S = k \ln(N!) + k(1 - \alpha)N - k\beta U = S_0 - k\beta U$

$$\left(\frac{\partial S}{\partial U} \right)_V = -k\beta$$

Comparing these two results, the constant β becomes

$$\beta = -\frac{1}{kT}$$

13.3 Boltzmann Distributions

$$N_i = g_i e^{\alpha + \beta \varepsilon_j} = g_i e^{\alpha} e^{-\varepsilon_i/kT}$$

For the value of e^{α} ,

$$N = \sum_i N_i = e^{\alpha} \sum_i g_j e^{-\varepsilon_i/kT}$$

$$e^{\alpha} = \frac{N}{\sum g_i e^{-\varepsilon_i/kT}}$$

And hence,

$$f_i = \frac{N_i}{g_i} = \frac{N e^{-\varepsilon_i/kT}}{\sum g_i e^{-\varepsilon_i/kT}} \quad (\text{Boltzmann distribution})$$

Partition function Z

13.3 Boltzmann Distributions

- Partition function

Partition function is defined to

$$Z \equiv \sum_{i=1}^{\infty} g_i e^{\beta \epsilon_i}$$

Partition function has information of degeneracy and energy level.

There are two consequences of partition function.

$$1) N = \sum_{i=1}^{\infty} N_i = \sum_{i=1}^{\infty} g_i e^{\alpha + \beta \epsilon_i} = e^{\alpha} Z \quad e^{\alpha} = \frac{N}{Z}$$

$$2) E = \sum_{i=1}^{\infty} N_i \epsilon_i = \sum_{i=1}^{\infty} g_i \epsilon_i e^{\alpha + \beta \epsilon_i} = e^{\alpha} \left(\frac{\partial Z}{\partial \beta} \right)_V = \frac{N}{Z} \left(\frac{\partial Z}{\partial \beta} \right)_V = N \left(\frac{\partial \ln(Z)}{\partial \beta} \right)_V$$

13.3 Boltzmann Distributions

- Distribution function

From previous results, the number distributions N_i

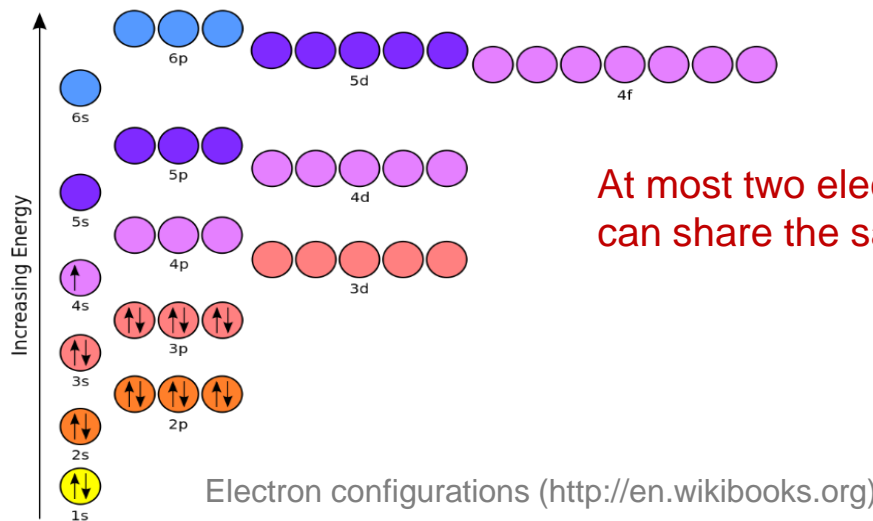
$$N_i = g_i e^{\alpha} e^{\beta \epsilon_i} = g_i \frac{N}{Z} e^{-\frac{\epsilon_i}{kT}}$$

Then, the **Boltzmann distribution function** is defined as below.

$$f(\epsilon_i) \equiv \frac{N_i}{g_i} = \frac{N e^{-\frac{\epsilon_i}{kT}}}{Z}$$

13.4 Fermi-Dirac Distribution

- Fermion
 - 1) Fermion is indistinguishable particle which obeys Pauli's exclusion principle.
 - 2) **Pauli's exclusion principle** means that no quantum state can accept more than one particle.
 - 3) Examples of fermions are electron, positron, proton, neutron, and neutrino.



At most two electrons (with different spins) can share the same orbitals

Periodic Table: Radioactive Elements

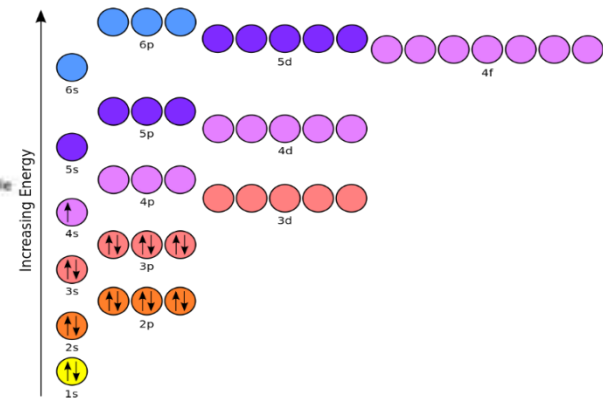
1 H 1.008 Hydrogen																	2 He 4.003 Helium						
3 Li 6.94 Lithium	4 Be 9.012 Beryllium																	5 B 10.81 Boron	6 C 12.011 Carbon	7 N 14.007 Nitrogen	8 O 15.999 Oxygen	9 F 18.998 Fluorine	10 Ne 20.180 Neon
11 Na 22.990 Sodium	12 Mg 24.305 Magnesium																	13 Al 26.982 Aluminum	14 Si 28.086 Silicon	15 P 30.974 Phosphorus	16 S 32.06 Sulfur	17 Cl 35.45 Chlorine	18 Ar 39.948 Argon
19 K 39.098 Potassium	20 Ca 40.078 Calcium	21 Sc 44.956 Scandium	22 Ti 47.887 Titanium	23 V 50.942 Vanadium	24 Cr 51.996 Chromium	25 Mn 54.938 Manganese	26 Fe 55.845 Iron	27 Co 58.933 Cobalt	28 Ni 58.693 Nickel	29 Cu 63.546 Copper	30 Zn 65.38 Zinc	31 Ga 69.723 Gallium	32 Ge 72.630 Germanium	33 As 74.922 Arsenic	34 Se 78.971 Selenium	35 Br 79.904 Bromine	36 Kr 83.796 Krypton						
37 Rb 85.468 Rubidium	38 Sr 87.62 Strontium	39 Y 88.906 Yttrium	40 Zr 91.224 Zirconium	41 Nb 92.906 Niobium	42 Mo 95.94 Molybdenum	43 Tc 98 Technetium	44 Ru 101.07 Ruthenium	45 Rh 102.906 Rhodium	46 Pd 106.42 Palladium	47 Ag 107.868 Silver	48 Cd 112.414 Cadmium	49 In 114.818 Indium	50 Sn 118.710 Tin	51 Sb 121.760 Antimony	52 Te 127.60 Tellurium	53 I 126.905 Iodine	54 Xe 131.29 Xenon						
55 Cs 132.905 Cesium	56 Ba 137.327 Barium	57 / 71	72 Hf 178.49 Hafnium	73 Ta 180.948 Tantalum	74 W 183.84 Tungsten	75 Re 186.207 Rhenium	76 Os 190.22 Osmium	77 Ir 192.222 Iridium	78 Pt 195.084 Platinum	79 Au 196.967 Gold	80 Hg 200.592 Mercury	81 Tl 204.38 Thallium	82 Pb 207.2 Lead	83 Bi 208.980 Bismuth	84 Po (209) Polonium	85 At (210) Astatine	86 Rn (222) Radon						
87 Fr (223) Francium	88 Ra (226) Radium	89 / 103	104 Rf (261) Rutherfordium	105 Db (262) Dubnium	106 Sg (263) Seaborgium	107 Bh (264) Bohrium	108 Hs (265) Hassium	109 Mt (266) Meitnerium	110 Ds (271) Darmstadtium	111 Rg (272) Roentgenium	112 Cn (285) Copernicium	113 Nh (286) Nihonium	114 Fl (289) Flerovium	115 Mc (290) Moscovium	116 Lv (293) Livermorium	117 Ts (294) Tennessine	118 Og (294) Oganesson						

Atomic Number
SYMBOL
Atomic Weight*
Name

Lanthanide Series	57 La 138.905 Lanthanum	58 Ce 140.12 Cerium	59 Pr 140.908 Praseodymium	60 Nd 144.242 Neodymium	61 Pm (145) Promethium	62 Sm 150.36 Samarium	63 Eu 151.964 Europium	64 Gd 157.25 Gadolinium	65 Tb 158.925 Terbium	66 Dy 162.50 Dysprosium	67 Ho 164.930 Holmium	68 Er 167.259 Erbium	69 Tm 168.934 Thulium	70 Yb 173.054 Ytterbium	71 Lu 174.967 Lutetium
Actinide Series	89 Ac (227) Actinium	90 Th 232.038 Thorium	91 Pa 231.036 Protactinium	92 U 238.029 Uranium	93 Np (237) Neptunium	94 Pu (244) Plutonium	95 Am (243) Americium	96 Cm (247) Curium	97 Bk (247) Berkelium	98 Cf (251) Californium	99 Es (252) Einsteinium	100 Fm (257) Fermium	101 Md (288) Mendelevium	102 No (289) Nobelium	103 Lr (260) Lawrencium

*() indicates the mass number of the longest-lived isotope.

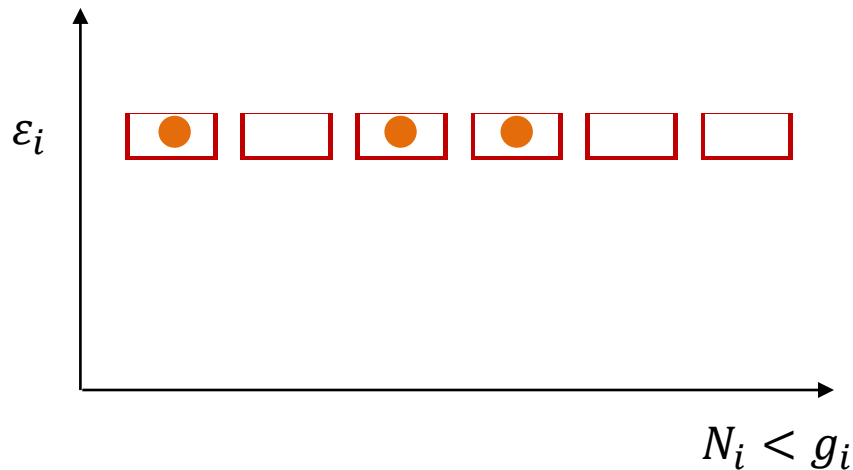
Based on NIST 2017 Periodic Table



13.4 Fermi-Dirac Distribution

- Number of rearrangement

Distribution of n_i particles among g_i state boxes.



Ex) three particles for
 i^{th} energy level of $g_i = 6$

$$W_{FD} = \prod g_i C_{N_i} = \prod \frac{g_i!}{(g_i - N_i)! N_i!}$$

13.4 Fermi-Dirac Distribution

- Fermi-Dirac distributions

From Stirling's approximation, $\ln(N!) = N \ln(N) - N$

$$\begin{aligned}\ln(w_{FD}) &= \sum [\ln(g_i!) - \ln(N_i!) - \ln((g_i - N_i)!)] \\ &= \sum [g_i \ln(g_i) - g_i - N_i \ln(N_i) + N_i - (g_i - N_i) \ln(g_i - N_i) + (g_i - N_i)]\end{aligned}$$

N_i for j^{th} energy level is undetermined yet.

→ **Method of Lagrange multiplier** is used to obtain most probable macro state under two constraints, $\sum N_i = N$, $\sum N_i \epsilon_i = E$

$$\frac{\partial(\ln(w_{FD}))}{\partial N_i} + \alpha \frac{\partial(\sum N_i - N)}{\partial N_i} + \beta \frac{\partial(\sum N_i \epsilon_i - E)}{\partial N_i} = 0$$

13.4 Fermi-Dirac Distribution

Applying method of Lagrange multipliers to Fermi-Dirac distributions,

$$\frac{\partial(\sum[g_i \ln(g_i) - N_i \ln(N_i) - (g_i - N_i) \ln(g_i - N_i)])}{\partial N_i} + \alpha \frac{\partial(\sum N_i)}{\partial N_i} + \beta \frac{\partial(\sum N_i \epsilon_i)}{\partial N_i} = 0$$

$$\longrightarrow -\ln(N_i) - \frac{N_i}{N_i} + \ln(g_i - N_i) + \frac{g_i - N_i}{g_i - N_i} + \alpha + \beta \epsilon_i = 0$$

Then, number distribution becomes

$$\ln\left(\frac{g_i}{N_i} - 1\right) = -\alpha - \beta \epsilon_i \longrightarrow N_i = g_i \frac{1}{e^{-\alpha - \beta \epsilon_i} + 1}$$

13.4 Fermi-Dirac Distribution

- Distribution function

Provisionally, we associated α with the chemical potential μ divided by kT , and reserve for later the physical interpretation of this connection.

$$\alpha = \frac{\mu}{kT}$$

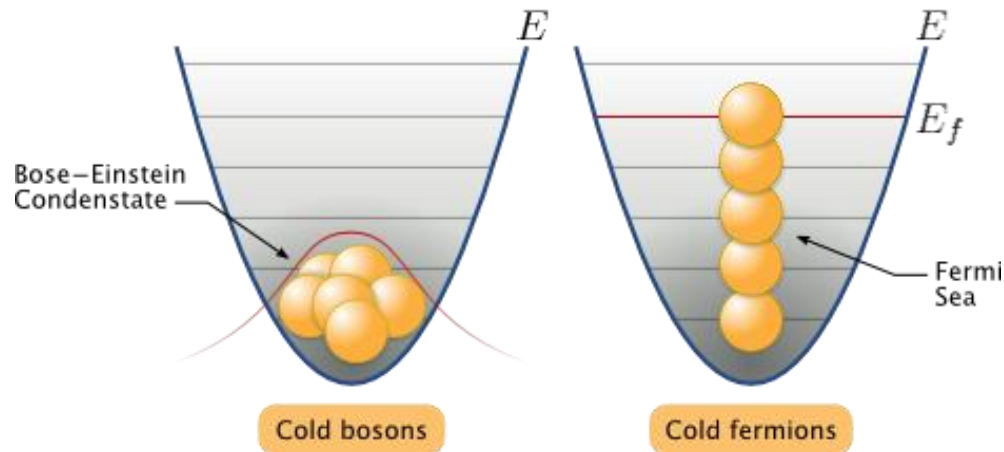
Then, the **Fermi-Dirac distribution function** is defined as below.

$$f(\epsilon_i) \equiv \frac{N_i}{g_i} = \frac{1}{e^{-\alpha - \beta\epsilon_i} + 1} = \frac{1}{e^{(\epsilon_i - \mu)/kT} + 1}$$

13.5 Bose-Einstein Distribution

- Boson

- 1) Boson is indistinguishable particle not obeying Pauli's exclusion principle.
- 2) Thus, one micro-state can be occupied by several Bosons.
- 3) Photon is the most notable example of Boson.

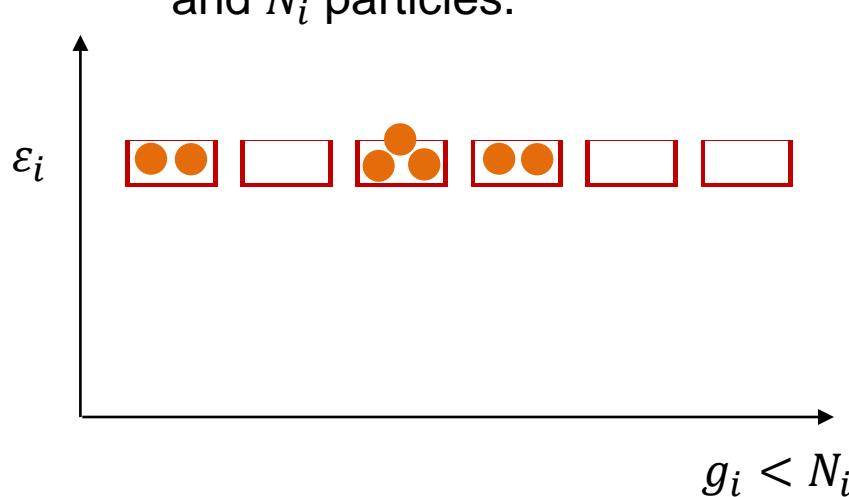


Difference between fermions and bosons
(<http://quantum-bits.org/>)

13.5 Bose-Einstein Distribution

- Number of rearrangement

Rearrangement of $N_i + g_i - 1$ symbols into $g_i - 1$ partitions (degeneracy) and N_i particles.



Ex) seven particles for i^{th} energy level of $g_i = 6$

$$W_{BE} = \prod N_i + g_i - 1 C_{g_i - 1} = \prod \frac{(N_i + g_i - 1)!}{N_i! (g_i - 1)!}$$

13.5 Bose-Einstein Distribution

- Bose-Einstein distributions

From Stirling's approximation, $\ln(N!) = N\ln(N) - N$

$$\begin{aligned}\ln(w_{BE}) &= \sum [\ln((N_i + g_i - 1)!) - \ln(N_i!) - \ln((g_i - 1)!)] \\ &= \sum [(N_i + g_i - 1) \ln(N_i + g_i - 1) - N_i \ln(N_i) - (g_i - 1) \ln(g_i - 1)]\end{aligned}$$

N_i for i^{th} energy level is undetermined yet

→ **Method of Lagrange multiplier** is used to obtain the most probable macro state under two constraints,

$$\sum N_i = N, \sum N_i \epsilon_i = E$$

$$\frac{\partial(\ln(w_{BE}))}{\partial N_i} + \alpha \frac{\partial(\sum N_i - N)}{\partial N_i} + \beta \frac{\partial(\sum N_i \epsilon_i - E)}{\partial N_i} = 0$$

13.5 Bose-Einstein Distribution

Applying method of Lagrange multipliers to Bose-Einstein distributions,

$$\frac{\partial(\sum[(N_i+g_i-1)\ln(N_i+g_i-1) - \sum N_i \ln(N_i)])}{\partial N_i} + \alpha \frac{\partial(\sum N_i)}{\partial N_i} + \beta \frac{\partial(\sum N_i \epsilon_i)}{\partial N_i} = 0$$

$$\longrightarrow \ln(N_i+g_i-1) + \frac{g_i+N_i-1}{g_i+N_i-1} - \ln(N_i) - \frac{N_i}{N_i} + \alpha + \beta \epsilon_i = 0$$

Then, number distribution becomes

$$\ln\left(\frac{N_i+g_i-1}{N_i}\right) = -\alpha - \beta \epsilon_i \longrightarrow N_i = g_i \frac{1}{e^{-\alpha-\beta \epsilon} - 1}$$

13.5 Bose-Einstein Distribution

- Distribution function

$$N_i = g_i \frac{1}{e^{-\alpha - \beta \epsilon_i} - 1} \quad \left(\alpha = \frac{\mu}{kT}, \beta = -\frac{1}{kT} \right)$$

Then, the **Bose-Einstein distribution function** is defined as below.

$$f(\epsilon_i) \equiv \frac{N_i}{g_i} = \frac{1}{e^{-\alpha - \beta \epsilon_i} - 1} = \frac{1}{e^{(\epsilon_i - \mu)/kT} - 1}$$

13.6 Dilute Gases and the Maxwell-Boltzmann Distribution

- Maxwell-Boltzmann Statistics

For dilute system, $N_i \ll g_i$ for all i , which is called dilute gas.

$$w_{BE} = \prod \frac{(g_i + N_i - 1)!}{N_i! (g_i - 1)!} = \prod \frac{(g_i + N_i - 1) \cdot (g_i + N_i - 2) \cdots (g_i + 1) \cdot (g_i)}{N_i!} \approx \prod \frac{g_i^{N_i}}{N_i!}$$

$$w_{FD} = \prod \frac{(g_i)!}{N_i! (g_i - N_i)!} = \prod \frac{(g_i) \cdot (g_i - 1) \cdots (g_i - N_i + 2) \cdot (g_i - N_i + 1)}{N_i!} \approx \prod \frac{g_i^{N_i}}{N_i!}$$

Therefore, both Fermion and Boson follow Maxwell-Boltzmann statistics for dilute gas.

$$w_{MB} = \prod \frac{g_i^{N_i}}{N_i!}$$

13.6 Dilute Gases and the Maxwell-Boltzmann Distribution

- Maxwell-Boltzmann distributions

From Stirling's approximation, $\ln(N!) = N \ln(N) - N$

$$\ln(w_{MB}) = \sum [N_i \ln(g_i) - \ln(N_i!)] = \sum [N_i \ln(g_i) - N_i \ln(N_i) + N_i]$$

N_i for i^{th} energy level is undetermined yet.

→ **Method of Lagrange multiplier** is used to obtain the most probable macro state under two constraints,

$$\sum N_i = N, \sum N_i \epsilon_i = E$$

$$\frac{\partial(\ln(w_{MB}))}{\partial N_i} + \alpha \frac{\partial(\sum N_i - N)}{\partial N_i} + \beta \frac{\partial(\sum N_i \epsilon_i - E)}{\partial N_i} = 0$$

13.6 Dilute Gases and the Maxwell-Boltzmann Distribution

Applying method of Lagrange multipliers to Maxwell-Boltzmann distributions,

$$\frac{\partial(\ln(\sum[N_i \ln(g_i) - N_i \ln(N_i) + N_i]))}{\partial N_i} + \alpha \frac{\partial(\sum N_i)}{\partial N_i} + \beta \frac{\partial(\sum N_i \epsilon_i)}{\partial N_i} = 0$$

$$\longrightarrow \ln(g_i) - \ln(N_i) - \frac{N_i}{N_i} + 1 + \alpha + \beta \epsilon_i = 0$$

Then, number distribution becomes

$$\ln\left(\frac{g_i}{N_i}\right) = -\alpha - \beta \epsilon_i \longrightarrow N_i = g_i e^{\alpha + \beta \epsilon_i}$$

$$f(\epsilon_i) \equiv \frac{N_i}{g_i} = \frac{1}{e^{-\alpha - \beta \epsilon_i} + 0}$$

13.6 Dilute Gases and the Maxwell-Boltzmann Distribution

- Distribution function

$$N_i = g_i e^{-\alpha - \beta \epsilon} \quad \left(\alpha = \frac{\mu}{kT}, \quad \beta = -\frac{1}{kT} \right)$$

Then, the **Maxwell-Boltzmann distribution function** is defined as below.

$$f(\epsilon_i) \equiv \frac{N_i}{g_i} = e^{\alpha + \beta \epsilon_i} = e^{-\frac{(\epsilon_i - \mu)}{kT}} = \frac{N}{Z} e^{-\epsilon_i/kT} \quad \left(e^{\frac{\mu}{kT}} = \frac{N}{Z} \right)$$

13.7 The Connection of Classical and Statistical Thermodynamics

- Energy transition

$$U = \sum N_i \epsilon_i$$

$$dU = \sum N_i d\epsilon_i + \sum \epsilon_i dN_i = \sum N_i \frac{d\epsilon_i(V)}{dV} dV + \sum \epsilon_i dN_i$$

This statistical expression can be matched with classical expression.

$$dU = \delta Q - \delta W = TdS - PdV$$

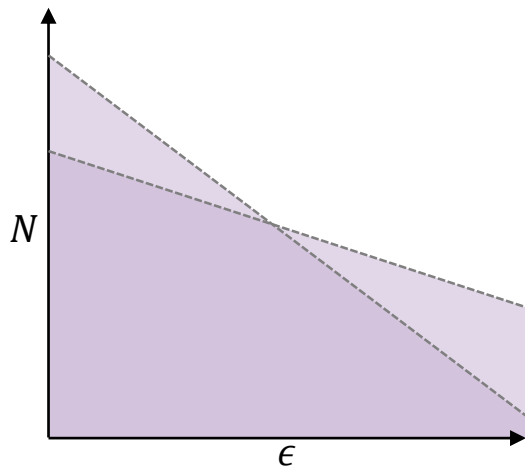
$$\sum N_i \frac{d\epsilon_i(V)}{dV} dV + \sum \epsilon_i dN_i = -PdV + TdS$$

$$\sum N_i d\epsilon_i = -PdV \quad \sum \epsilon_i dN_i = TdS$$

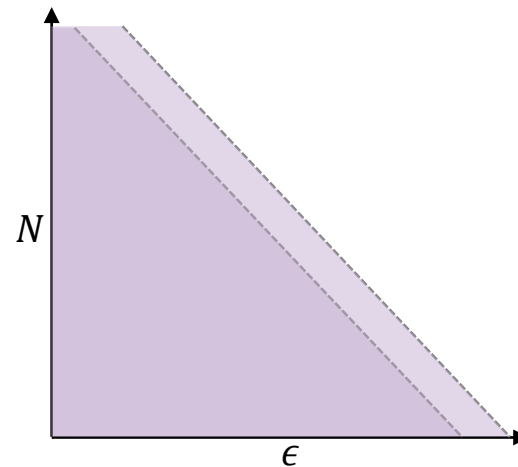
13.7 The Connection of Classical and Statistical Thermodynamics

Heat transfer to the system : particles are re-distributed so that particles are shifted from lower to higher energy level.

Isentropic process with **work done** : the energy levels are shifted to higher values with no re-distribution.



Heat transfer



Work done

13.7 The Connection of Classical and Statistical Thermodynamics

- Physical relations of constant α

For a dilute gas,

$$\begin{aligned} S &= k \ln(w_{MB}) = k \sum \left[N_i \ln \left(\frac{g_i}{N_i} \right) + N_i \right] = k \sum \left[N_i \ln(e^{-\alpha - \beta \epsilon_i}) + N_i \right] \\ &= k \sum \left[N_i \left(\ln \left(\frac{Z}{N} \right) + 1 \right) - \frac{1}{kT} N_i \epsilon_i \right] \\ &\quad \left(\because e^\alpha = \frac{N}{Z}, \beta = -\frac{1}{kT} \right) \end{aligned}$$

$$\longrightarrow S = Nk \left(\ln \left(\frac{Z}{N} \right) + 1 \right) + \frac{U}{T}$$

13.7 The Connection of Classical and Statistical Thermodynamics

In classical thermodynamics,

$$dF(U, V, N) = -SdT - PdV + \mu dN \rightarrow \left(\frac{\partial F}{\partial N} \right)_{V, T} = \mu$$

From the previous result, $S = Nk \left(\ln \left(\frac{Z}{N} \right) + 1 \right) + \frac{U}{T}$

$$F = U - TS = -NkT \left(\ln \left(\frac{Z}{N} \right) + 1 \right)$$

$$\left(\frac{\partial F}{\partial N} \right)_{V, T} = -kT \left(\ln \left(\frac{Z}{N} \right) + 1 \right) + \frac{NkT}{N}$$

$$\longrightarrow \mu = -kT \left(\ln \left(\frac{Z}{N} \right) \right)$$

13.7 The Connection of Classical and Statistical Thermodynamics

Recalling that $\frac{N}{Z} = e^\alpha$, constant α is associated with chemical potential and temperature as it is previously introduced.

$$\alpha = \ln \left(\frac{N}{Z} \right) = \frac{\mu}{kT}$$

13.8 Comparison of the Distributions

- Number distributions for identical indistinguishable particles

$$\frac{N_i}{g_i} = \frac{1}{e^{(\epsilon_i - \mu)/kT} + a} \quad a = \begin{cases} +1 & \text{for FD statistics} \\ -1 & \text{for BE statistics} \\ 0 & \text{for MB statistics} \end{cases}$$

