211130
2021년 11월 30일 화요일 오전 10:48

Last lecture
laminar boundary layer analysis
δ^* displacement thickness
$H \equiv \delta^*/\theta$ shape factor
C_D drag coefficient

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turbulent boundary layer.
selected topics
course review

Final Exam on Dec 9
Suggested problems 7.7, 7.23, 7.33, 7.43

Integral analysis of turbulent boundary layer (TBL)
↳ no rigorous solution

integral relations valid for laminar & turbulent boundary layers

$$\tau_w(x) = \rho U_0^2 \frac{d\theta}{dx}$$

$$C_f(x) = \frac{\tau_w(x)}{\left(\frac{\rho U_0^2}{2}\right)} = 2\frac{d\theta}{dx}$$

Assume
1. Overlap layer constitutes the entire turbulent boundary layer

$$\frac{u}{u^*} \approx \frac{1}{\kappa} \ln \frac{y u^*}{\nu} + B$$

$$u^+ \approx \frac{1}{\kappa} \ln y^+ + B \qquad \kappa = 0.41 \;\&\; B = 5.0$$
from experiment

Boundary condition: at $y = \delta$, $u = U_0$

$$\boxed{\frac{U_0}{u^*} = \frac{1}{\kappa} \ln \frac{\delta u^*}{\nu} + B} \qquad (1)$$

but $\quad \dfrac{\tau_w}{\rho U_0^2} = \dfrac{2\rho u^{*2}}{U_0^2} \qquad u^{*2} = \dfrac{\tau_w}{\rho}$

but $\frac{\tau_w}{\rho u_o^2} = \frac{2\rho u^{*2}}{u_o^2}$ $\qquad u^* \equiv \frac{\tau_w}{\rho}$

$$\downarrow$$

$$C_f = 2\left(\frac{u^*}{u_o}\right)^2$$

$$\downarrow$$

$$\boxed{\frac{u_o}{u^*} = \left(\frac{2}{C_f}\right)^{1/2}}$$

sub into (1)

and

$$\boxed{\frac{\delta u^*}{\nu} = \frac{\delta u_o}{\nu} \cdot \frac{u^*}{u_o} = Re_\delta \left(\frac{C_f}{2}\right)^{1/2}}$$

Sub above into Eq. (1),

$$\left(\frac{2}{C_f}\right)^{1/2} \approx \frac{1}{0.41} \ln\left[Re_\delta \left(\frac{C_f}{2}\right)^{1/2}\right] + 5.0 \qquad (2)$$

$C_f(Re_\delta)$ for turbulent boundary layer over flat plate

(2) inconvenient because
1) implicit
2) based on Re_δ and not Re_x

Professor Prandtl's curve-fit of Eqn (2)

$$\boxed{C_f \approx 0.02\, Re_\delta^{-1/6}} \qquad (3)$$

$\to$ $C_f(Re_\delta)$ explicit

$C_f(Re_x)$? $\to$ $\frac{\delta}{x}$ for turbulent boundary layer

Professor Prandtl suggested (from experimental observation)

$$\left(\frac{u}{u_o}\right) \approx \left(\frac{y}{\delta}\right)^{1/7} \quad \text{for turbulent boundary layers}$$

$\to$ then $\boxed{\theta \approx \frac{7}{72}\delta}$

→ then $\boxed{\theta \simeq \frac{7}{72}\delta}$

then from eqn (3) and integral relation,

$$C_f \simeq 0.02\, Re_\delta^{-1/6} = 2\frac{d\theta}{dx} = 2\frac{d}{dx}\left(\frac{7}{72}\delta\right)$$

$$Re_\delta^{-1/6} \simeq 9.72\frac{d\delta}{dx} = 9.72\frac{d(Re_\delta)}{d(Re_x)}$$

integrate

$$\boxed{\frac{\delta}{x} \simeq \frac{0.16}{Re_x^{1/7}}}$$ for turbulent boundary layers

$\left(\frac{\delta}{x} \simeq \frac{5.0}{\sqrt{Re_x}}\right.$ for laminar b.l.$\left.\right)$

$\therefore$ $\delta \propto x^{1/2}$ for laminar b.l.
$\delta \propto x^{6/7}$ " turbulent b.l. → boundary layers thickens more rapidly with x for turbulent boundary layers.

$$\boxed{C_f \simeq 0.02\, Re_\delta^{-1/6} \simeq \frac{0.027}{Re_x^{1/7}}}$$

$$\boxed{C_D(x) = \frac{2}{x}\theta(x) = \frac{0.031}{Re_x^{1/7}} \simeq \frac{7}{6}C_f(x)}$$

→ turbulent boundary layer

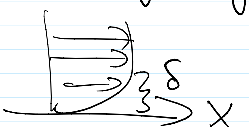
$C_D(x) = 2C_f(x)$ for laminar b.l.

$$\boxed{\delta^* = \frac{\delta}{8}}$$ → turbulent b.l.

$$\boxed{H \equiv \frac{\delta^*}{\theta} = \frac{1/8}{7/72} \simeq 1.3}$$

$H = 2.59$ for laminar b.l.

Flow separation occurs when boundary layers encounter a sufficiently strong adverse pressure gradient



$+ \quad \frac{dp}{dx} > 0 \quad \rightarrow$ risk of flow separation.

Ch. 6. → pressure drop in internal flows < laminar / turbulent flows.

Ch. 7 → friction (τ_w) drag $D = \int_A \tau_w dA$ in external flows (flat plate) < laminar / turbulent flows.

In Ch. 7, we [measure / assume] velocity profiles $\frac{u(y)}{U_0}$ for integral analysis

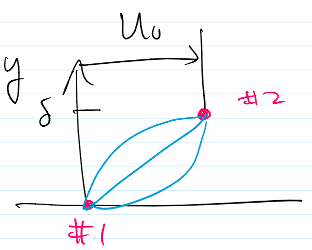
Flow separation

→ high drag → high loss, loss of lift → aerodynamically not good.

→ enhances mixing → improve heat transfer; improve mixing of fuel & air in combustor → reduce pollution.

velocity profile boundary conditions (stationary flat plate)

must satisfy [1. at $y=0$, $u=0$
2. at $y=\delta$, $u=U_0$ freestream velocity]



in addition, good to satisfy

3. $\frac{\partial u}{\partial y} = \frac{\tau_w}{\mu}$ at $y=0$ (finite τ_w at wall)

4 $\frac{\partial u}{\partial y} = 0$ at $y=\delta$ (zero shear at $y=\delta$)

5. $\frac{\partial^2 u}{\partial y^2}$

6. $\frac{\partial^2 u}{\partial y^2}$

211202

2021년 12월 2일 목요일 오전 10:23

Last lecture

Integral analysis of turbulent boundary layer (TBL)

$$\underbrace{\frac{u}{U_0} \approx \left(\frac{y}{\delta}\right)^{1/7} \quad + \quad C_f(Re_\delta)}$$

$$\downarrow$$

$$\frac{\delta}{x},\ \frac{\delta^*}{\delta},\ H,\ C_f,\ C_D,\ \text{etc.}$$

Today: diffusers & flowmeters (Ch. 6) & some energy issues

Dec 7: Course review & Q&A

DEC 9: FINAL EXAM 11:00 – 13:30 (will cover everything)

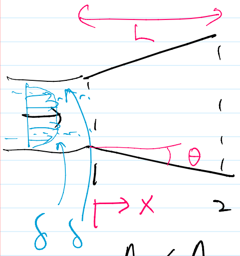
LOCATION: TBA

Revisit Ch. 6.

adverse pressure gradient

Keep in mind (Ch. 7) 1. boundary layer & 2. $\frac{dp}{dx} > 0$ → flow separation risk

diffuser. → 1. slow down velocity → combustors
2. increase pressure



1-D

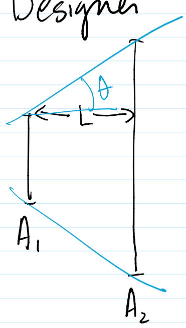
steady

incompressible $\rho_1 = \rho_2$

diffuser
- $A_1 < A_2$
- $V_1 > V_2$ (mass)
- $P_1 < P_2$ (Bernoulli) → $\frac{dp}{dx} > 0$ adverse pressure gradient

θ determines magnitude of $\frac{dp}{dx}$ → large θ → large $\frac{dp}{dx}$ → higher risk of separation
→ small θ → small $\frac{dp}{dx}$ → lower " " "

Designer



Given A_1 & A_2

Vary L → θ key variable

small L → large θ → flow separation

large L → small θ

"shorter" → smaller → lighter

Pressure coefficient

$C_p = P_{exit} - P_{inlet}$ ← static pressure change in diffuser < 1

Pressure coefficient

$$C_p = \frac{P_{exit} - P_{inlet}}{(P_t - P_s)_{inlet}} < 1$$

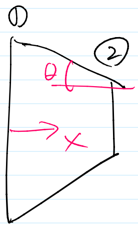
static pressure change in nozzle

dynamic pressure at inlet

$t \rightarrow$ total
$s \rightarrow$ static

aircraft engines
rockets
vacuum cleaners

Nozzle



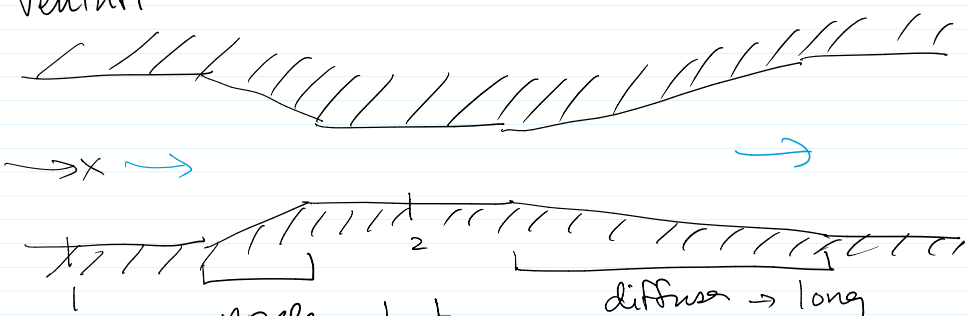
Is 2θ important in nozzles?

$A_1 > A_2$

$V_1 < V_2$

$P_1 > P_2 \rightarrow \frac{dp}{dx} < 0$ favorable gradient $\rightarrow$ no risk of flow separation

∴ Venturi



nozzle → short   diffuser → long

ΔP between 1 & 2 $\rightarrow \dot{m}$

Flowmeters → measure V, A $\rightarrow$ $\dot{m}$, Q

1. Venturi
2. Pitot tube → aircraft
3. turbine

Fig 6.29

2. Pitot tube → aircraft
3. turbine
4. hot wire
5. hot film
⋮
↑ intrusive

6. laser (optical) method ↓ non-intrusive

Velocity meters

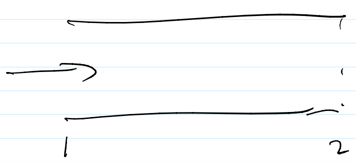
Venturi vs. nozzle vs. orifice

longest — shortest

smallest loss — highest loss.

Discharge coefficient $C_D \equiv \dfrac{\dot{m}_{actual}}{\dot{m}_{ideal}}$

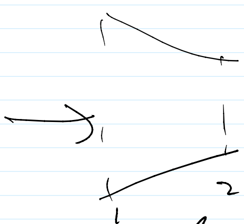
$C_D(Re, Ma)$ ← calibration needed (expensive)



$A_1 = A_2$

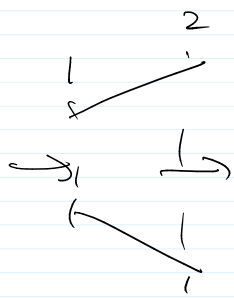
entrance

no nozzle



nozzle

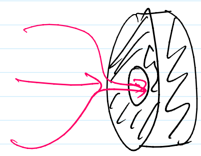
$A_1 > A_2$



diffuser

$A_1 < A_2$

orifice → measure velocity → pressure drop



diffuser → increase p

↳ slow down velocity