Announcement

To be updated

Chap. 9 Deflections of Beams

9.1 Introduction

9.2 Differential equations of the deflection curve

9.3 Deflection by integration of the bending-moment equation

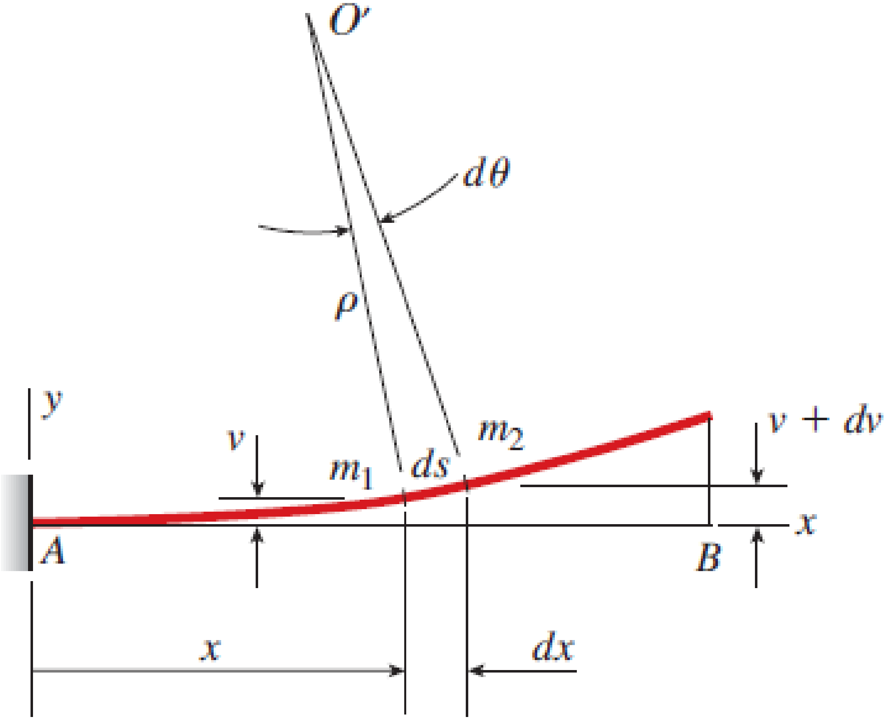
9.4 Deflections by integration of the shear-force and load equations

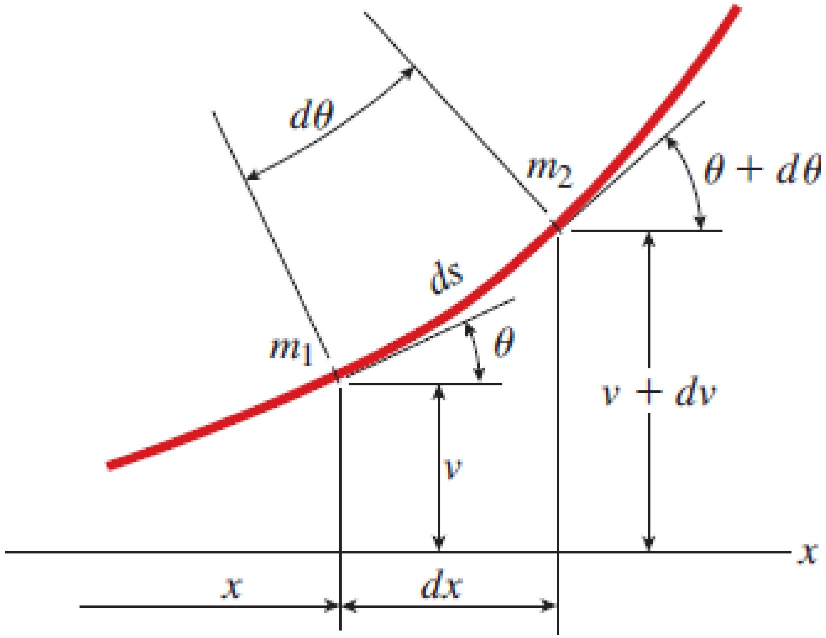
9.5 Method of superposition

9.7 Nonprismatic beams

Differential equations of the deflection curve

Geometry of deflected beam





$$\rho \, d\theta = ds$$

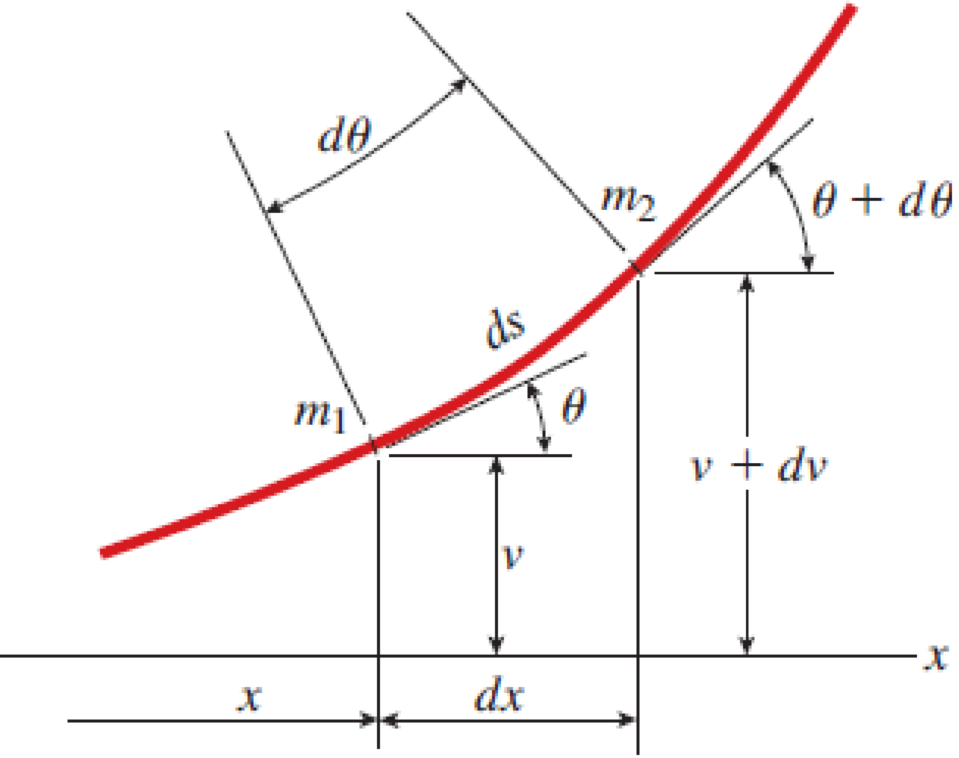
$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds}$$

$$\cos\theta = \frac{dx}{ds} \qquad \sin\theta = \frac{dv}{ds}$$

$$\frac{dv}{dx} = \tan\theta \qquad \theta = \arctan\frac{dv}{dx}$$

Differential equations of the deflection curve

Beams with small angles of rotation



$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds}$$

$$ds \approx dx$$

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{dx}$$

$$\theta \approx \tan\theta = \frac{dv}{dx}$$

$$\frac{d\theta}{dx} = \frac{d^2v}{dx^2}$$

$$\kappa = \frac{1}{\rho} = \frac{d^2v}{dx^2}$$

$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

Differential equations of the deflection curve

Beams with small angles of rotation

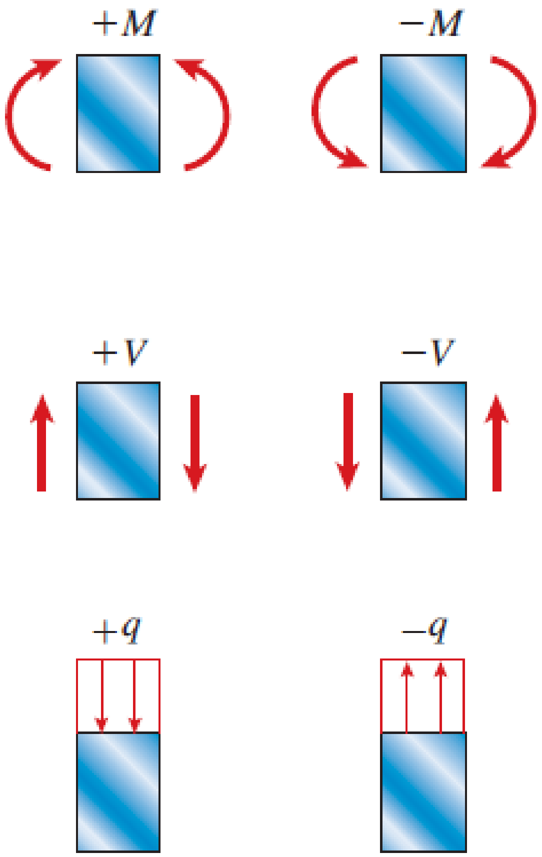


FIG. 9-4 Sign conventions for bending moment M, shear force V, and intensity q of distributed load

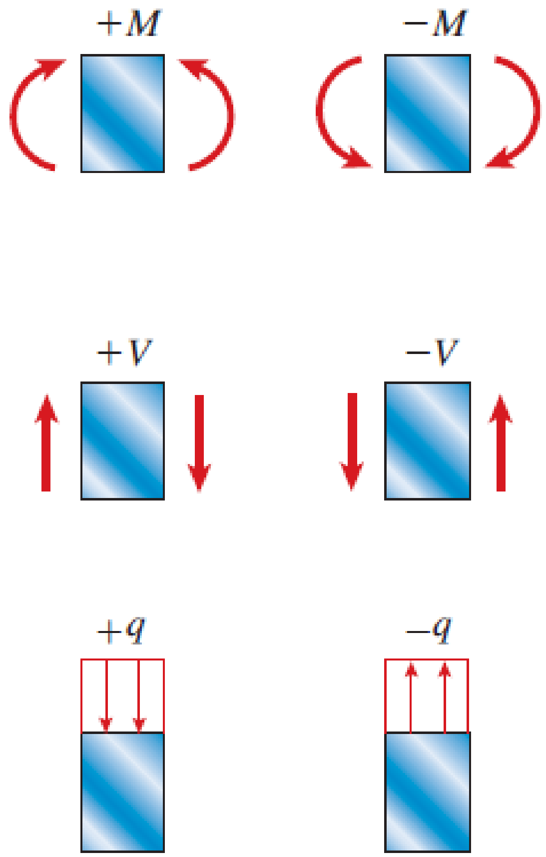
$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

$$\frac{dM}{dx} = V$$

$$\frac{dV}{dx} = -q$$

Differential equations of the deflection curve

Beams with small angles of rotation



$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

$$\frac{dM}{dx} = V$$

$$\frac{dV}{dx} = -q$$

FIG. 9-4 Sign conventions for bending moment M, shear force V, and intensity q of distributed load

$$v' \equiv \frac{dv}{dx} \qquad v'' \equiv \frac{d^2v}{dx^2} \qquad v''' \equiv \frac{d^3v}{dx^3} \qquad v'''' \equiv \frac{d^4v}{dx^4}$$

Nonprismatic Beams

$$EI_x \frac{d^2v}{dx^2} = M$$

$$\frac{d}{dx}\left(EI_x \frac{d^2v}{dx^2}\right) = \frac{dM}{dx} = V$$

$$\frac{d^2}{dx^2}\left(EI_x \frac{d^2v}{dx^2}\right) = \frac{dV}{dx} = -q$$

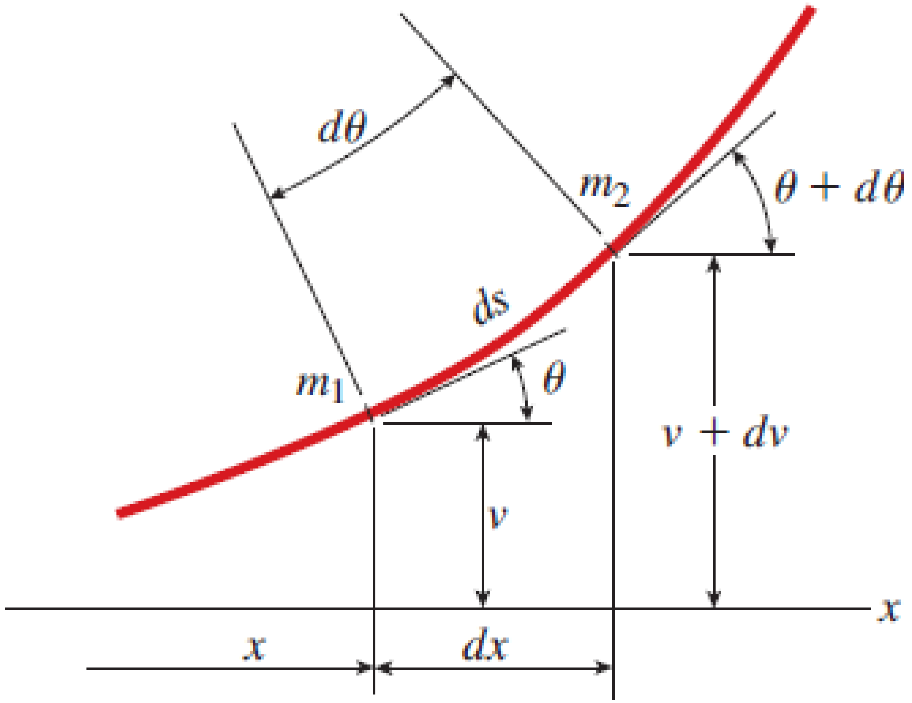
Prismatic Beams

$$EI\frac{d^2v}{dx^2} = M \qquad EI\frac{d^3v}{dx^3} = V \qquad EI\frac{d^4v}{dx^4} = -q$$

$$EIv'' = M \qquad EIv''' = V \qquad EIv'''' = -q$$

Differential equations of the deflection curve

Beams with large angles of rotation (Exact expression for curvature)



$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d(\arctan v')}{dx}\frac{dx}{ds}$$

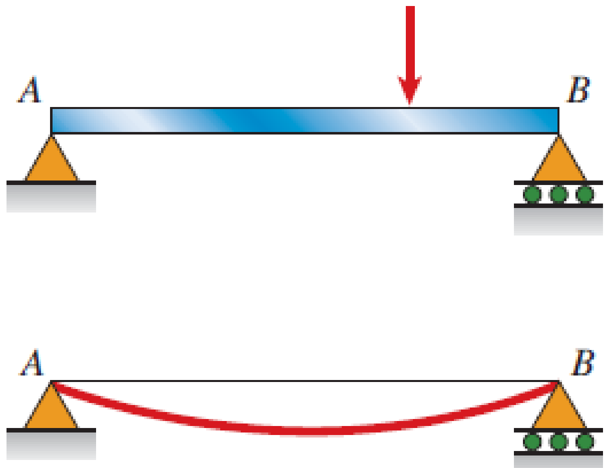
$$ds^2 = dx^2 + dv^2 \quad \text{or} \quad ds = [dx^2 + dv^2]^{1/2}$$

$$\frac{ds}{dx} = \left[1 + \left(\frac{dv}{dx}\right)^2\right]^{1/2} = [1 + (v')^2]^{1/2} \quad \text{or} \quad \frac{dx}{ds} = \frac{1}{[1 + (v')^2]^{1/2}}$$

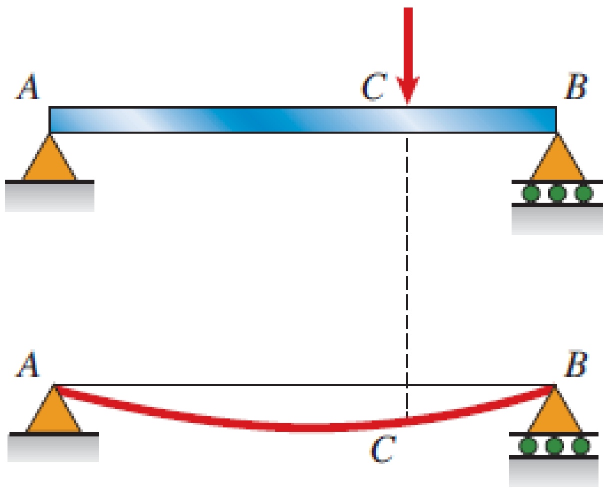
$$\frac{d}{dx}(\arctan v') = \frac{v''}{1 + (v')^2}$$

$$\kappa = \frac{1}{\rho} = \frac{v''}{[1 + (v')^2]^{3/2}}$$

Deflections by integration of the bending-moment equation

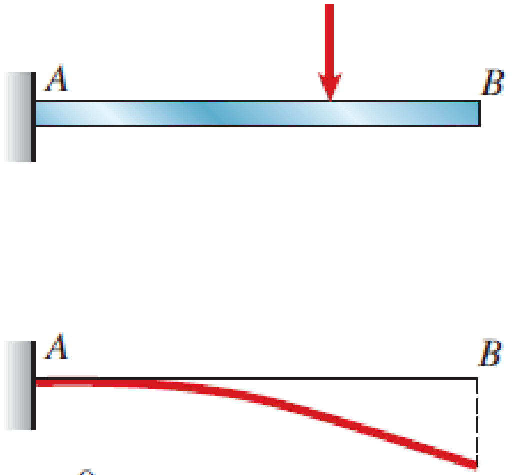


$v_A = 0$ $\qquad$ $v_B = 0$



At point C: $(v)_{AC} = (v)_{CB}$

$(v')_{AC} = (v')_{CB}$



$v_A = 0$

$v'_A = 0$

$$\frac{d\theta}{dx} = \frac{d^2v}{dx^2}$$

$$\theta \approx \tan\theta = \frac{dv}{dx}$$

$$\kappa = \frac{1}{\rho} = \frac{d^2v}{dx^2}$$

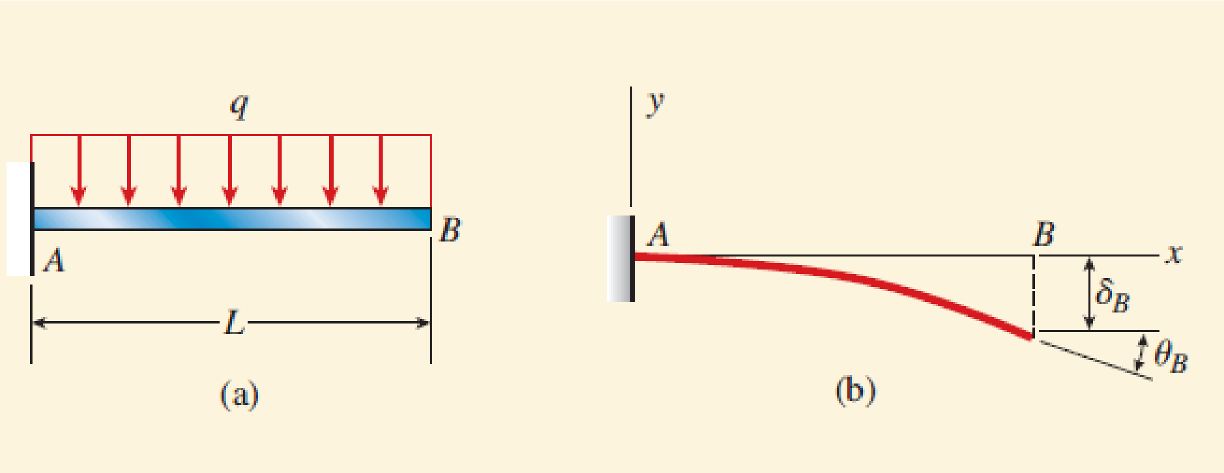
$$\frac{dM}{dx} = V$$

$$\frac{dV}{dx} = -q$$

$$v \rightleftarrows \frac{dv}{dx} = \theta \rightleftarrows \frac{d^2v}{dx^2} = \frac{M}{EI} \rightleftarrows \frac{d^3v}{dx^3} = \frac{V}{EI} \rightleftarrows \frac{d^4v}{dx^4} = \frac{-q}{EI}$$

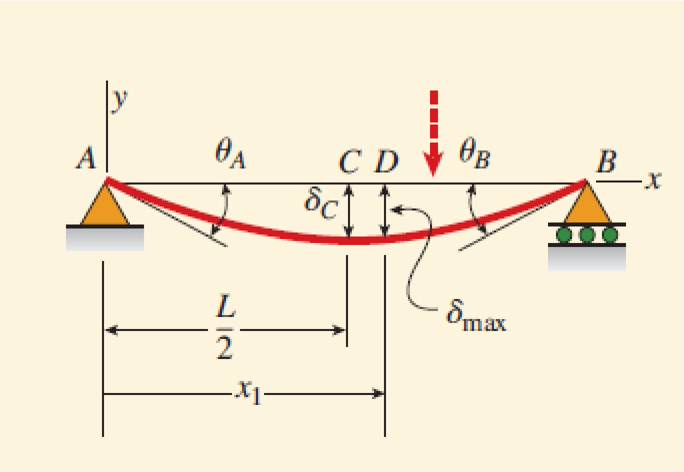
Deflections by integration of the bending-moment equation

$$v \rightleftarrows \frac{dv}{dx} = \theta \rightleftarrows \frac{d^2v}{dx^2} = \frac{M}{EI} \rightleftarrows \frac{d^3v}{dx^3} = \frac{V}{EI} \rightleftarrows \frac{d^4v}{dx^4} = \frac{-q}{EI}$$



$$\theta_B = -v'(L) = \frac{qL^3}{6EI}$$

$$\delta_B = -v(L) = \frac{qL^4}{8EI}$$



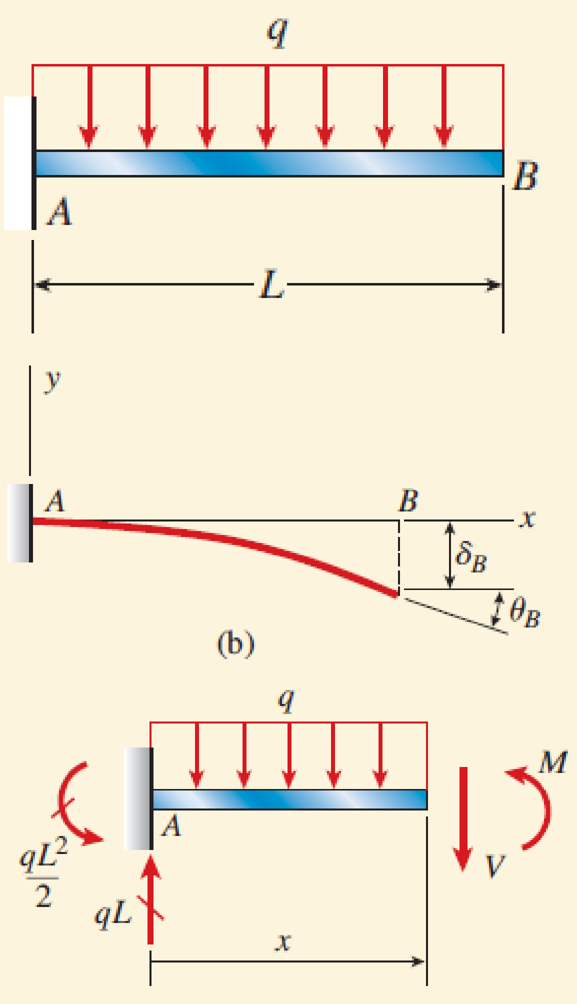
$$\theta_A = -v'(0)$$

$$\delta_C = -v\left(\frac{L}{2}\right)$$

$$\theta_B = v'(L)$$

Deflections by integration of the bending-moment equation

Example 9-2



$$M = -\frac{qL^2}{2} + qLx - \frac{qx^2}{2}$$

$$EIv'' = -\frac{qL^2}{2} + qLx - \frac{qx^2}{2}$$

$$EIv' = -\frac{qL^2}{2}x + \frac{qLx^2}{2} - \frac{qx^3}{6} + C_1$$

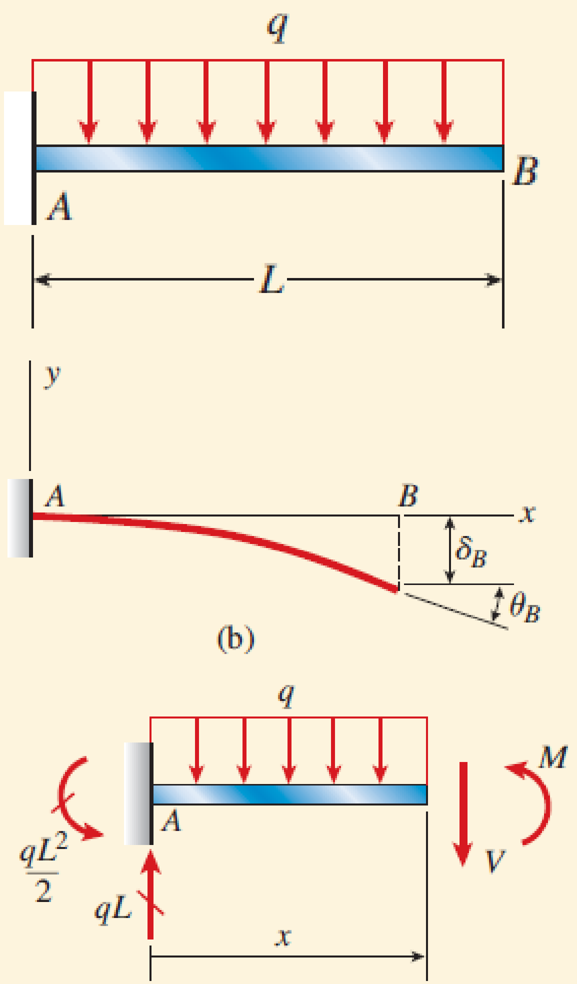
$$v'(0) = 0$$

$$EIv' = -\frac{qL^2}{2}x + \frac{qLx^2}{2} - \frac{qx^3}{6}$$

$$v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2)$$

Deflections by integration of the bending-moment equation

Example 9-2



$$v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2) \Rightarrow \theta_B = -v'(L) = \frac{qL^3}{6EI}$$

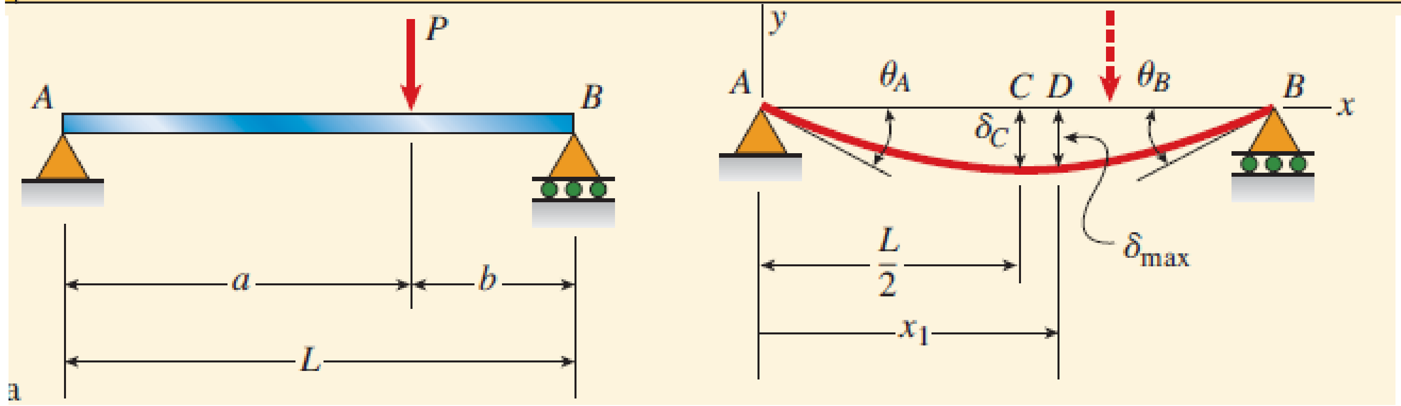
$$EIv = -\frac{qL^2x^2}{4} + \frac{qLx^3}{6} - \frac{qx^4}{24} + C_2$$

$$v(0) = 0$$

$$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \Rightarrow \delta_B = -v(L) = \frac{qL^4}{8EI}$$

Deflections by integration of the bending-moment equation

Example 9-3



$$M = \frac{Pbx}{L} \qquad (0 \le x \le a)$$

$$M = \frac{Pbx}{L} - P(x-a) \qquad (a \le x \le L)$$

$$EIv'' = \frac{Pbx}{L} \qquad (0 \le x \le a)$$

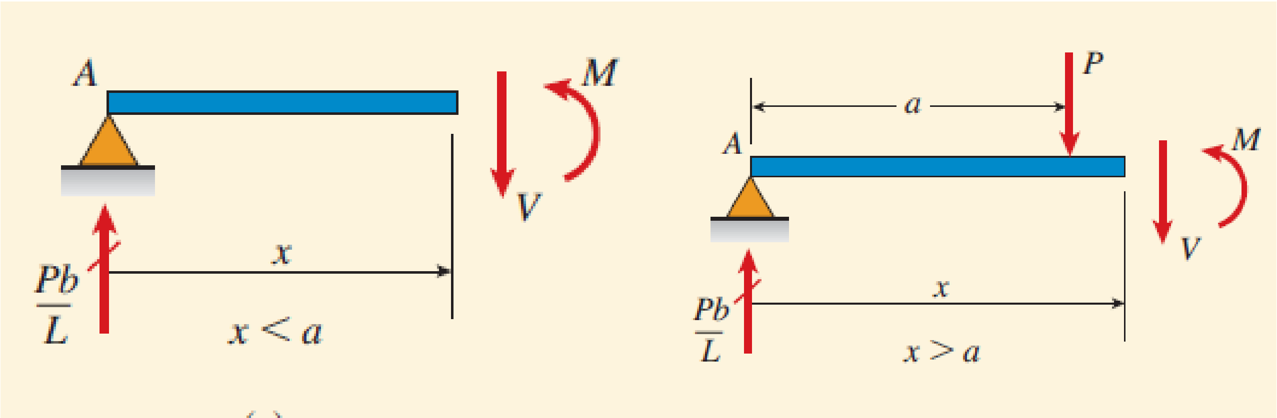
$$EIv'' = \frac{Pbx}{L} - P(x-a) \qquad (a \le x \le L)$$

$$EIv' = \frac{Pbx^2}{2L} + C_1 \qquad (0 \le x \le a)$$

$$EIv' = \frac{Pbx^2}{2L} - \frac{P(x-a)^2}{2} + C_2 \qquad (a \le x \le L)$$

$$EIv = \frac{Pbx^3}{6L} + C_1 x + C_3 \qquad (0 \le x \le a)$$

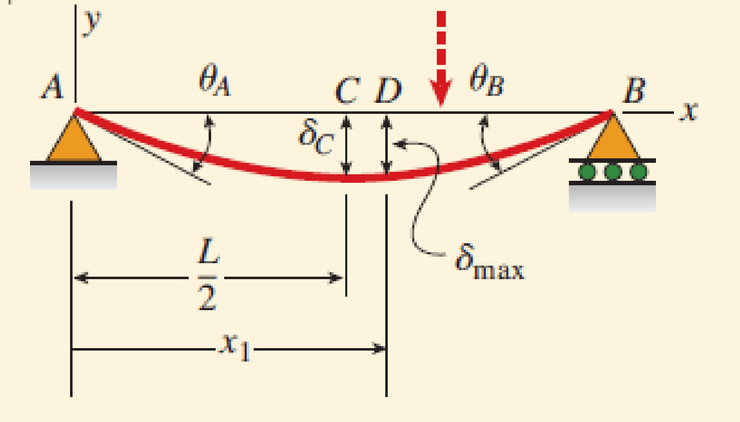
$$EIv = \frac{Pbx^3}{6L} - \frac{P(x-a)^3}{6} + C_2 x + C_4 \quad (a \le x \le L)$$



(a) (b)

Deflections by integration of the bending-moment equation

Example 9-3



Deflection equations

$$EIv = \frac{Pbx^3}{6L} + C_1x + C_3 \qquad (0 \le x \le a)$$

$$EIv = \frac{Pbx^3}{6L} - \frac{P(x-a)^3}{6} + C_2x + C_4 \qquad (a \le x \le L)$$

Angle equations

$$EIv' = \frac{Pbx^2}{2L} + C_1 \qquad (0 \le x \le a)$$

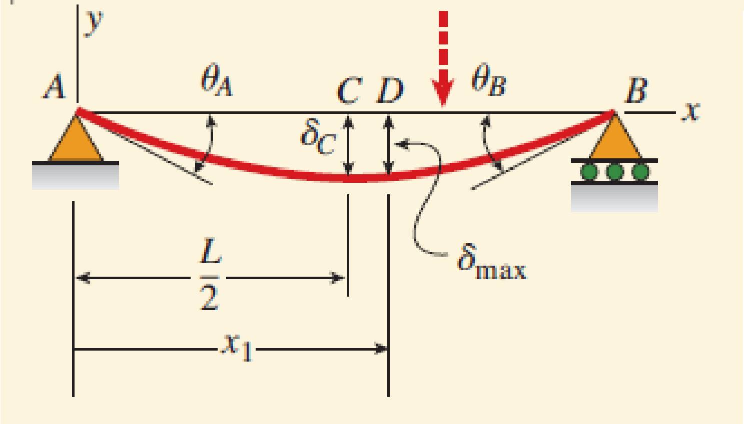
$$EIv' = \frac{Pbx^2}{2L} - \frac{P(x-a)^2}{2} + C_2 \qquad (a \le x \le L)$$

Boundary condition (BC)

1. At x = a, the slopes v′ for the two parts of the beam are the same.
2. At x = a, the deflections v for the two parts of the beam are the same.
3. At x = 0, the deflection v is zero.
4. At x = L, the deflection v is zero.

Deflections by integration of the bending-moment equation

Example 9-3



Deflection equations

$$EIv = \frac{Pbx^3}{6L} + C_1x + C_3 \qquad (0 \leq x \leq a)$$

$$EIv = \frac{Pbx^3}{6L} - \frac{P(x-a)^3}{6} + C_2x + C_4 \qquad (a \leq x \leq L)$$

Angle equations

$$EIv' = \frac{Pbx^2}{2L} + C_1 \qquad (0 \leq x \leq a)$$

$$EIv' = \frac{Pbx^2}{2L} - \frac{P(x-a)^2}{2} + C_2 \qquad (a \leq x \leq L)$$

Boundary condition (BC)

1. At x = a, the slopes v′ for the two parts of the beam are the same.

$$\frac{Pba^2}{2L} + C_1 = \frac{Pba^2}{2L} + C_2 \qquad \text{or} \qquad C_1 = C_2$$

2. At x = a, the deflections v for the two parts of the beam are the same.

$$\frac{Pba^3}{6L} + C_1a + C_3 = \frac{Pba^3}{6L} + C_2a + C_4 \qquad \text{or} \qquad C_3 = C_4$$

3. At x = 0, the deflection v is zero.

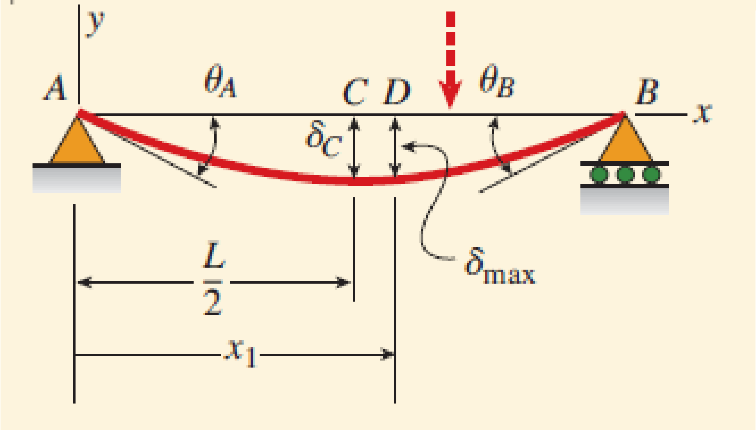
$$C_3 = C_4 = 0$$

4. At x = L, the deflection v is zero.

$$\frac{PbL^3}{6L} - \frac{Pb^3}{6} + C_2L = 0 \qquad \text{or} \qquad C_1 = C_2 = -\frac{Pb(L^2 - b^2)}{6L}$$

Deflections by integration of the bending-moment equation

Example 9-3



Finally..

$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad (0 \leq x \leq a)$$

$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) - \frac{P(x-a)^3}{6LEI} \qquad (a \leq x \leq L)$$

$$\Rightarrow \quad \delta_C = v(L/2) = -\frac{Pbx}{6LEI}(L^2 - b^2 - L^4/4)$$

$$v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \qquad (0 \leq x \leq a)$$

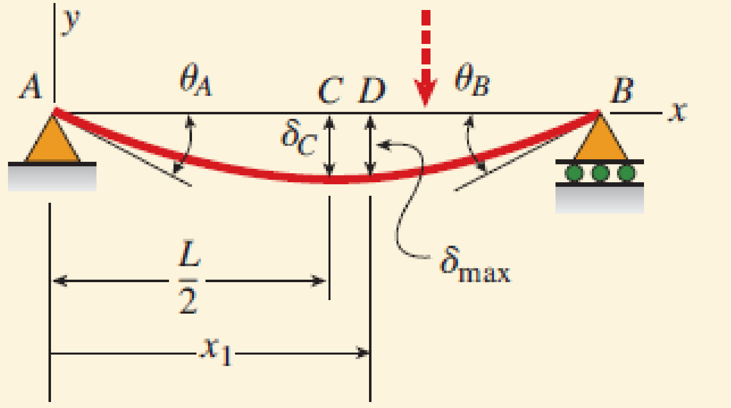
$$v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) - \frac{P(x-a)^2}{2EI} \qquad (a \leq x \leq L)$$

$$\Rightarrow \quad \theta_A = -v'(0) = \frac{Pb(L^2 - b^2)}{6LEI} = \frac{Pab(L+b)}{6LEI}$$

$$\theta_B = v'(L) = \frac{Pb(2L^2 - 3bL + b^2)}{6LEI} = \frac{Pab(L+a)}{6LEI}$$

Deflections by integration of the bending-moment equation

Example 9-3



Maximum angle of rotation

$$\theta_A = -v'(0) = \frac{Pb(L^2 - b^2)}{6LEI}$$

Condition: $\frac{d\theta}{db} = 0$; $b = L/\sqrt{3}$

$$\Rightarrow \quad (\theta_A)_{\max} = \frac{PL^2\sqrt{3}}{27EI}$$

Maximum deflection of the beam

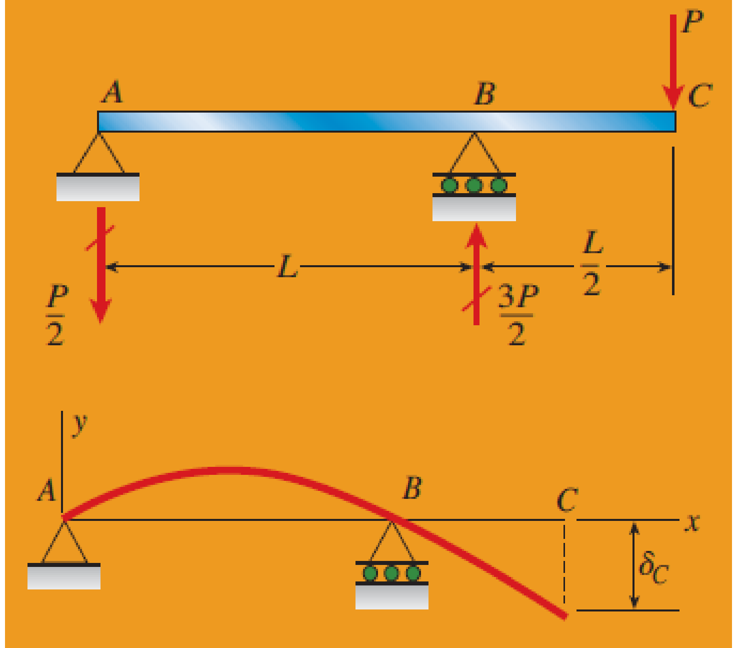
$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad (0 \le x \le a)$$

Condition: $v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) = 0$; $x_1 = \sqrt{\frac{L^2 - b^2}{3}}$

$$\Rightarrow \quad \delta_{\max} = -(v)_{x=x_1} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI} \qquad (0 \le x \le a)$$

Deflections by integration of the shear-force and load equations

Example 9-5



(b)

$$V = -\frac{P}{2} \qquad (0 < x < L)$$

$$V = P \qquad \left(L < x < \frac{3L}{2}\right)$$

$$EIv''' = -\frac{P}{2} \qquad (0 < x < L)$$

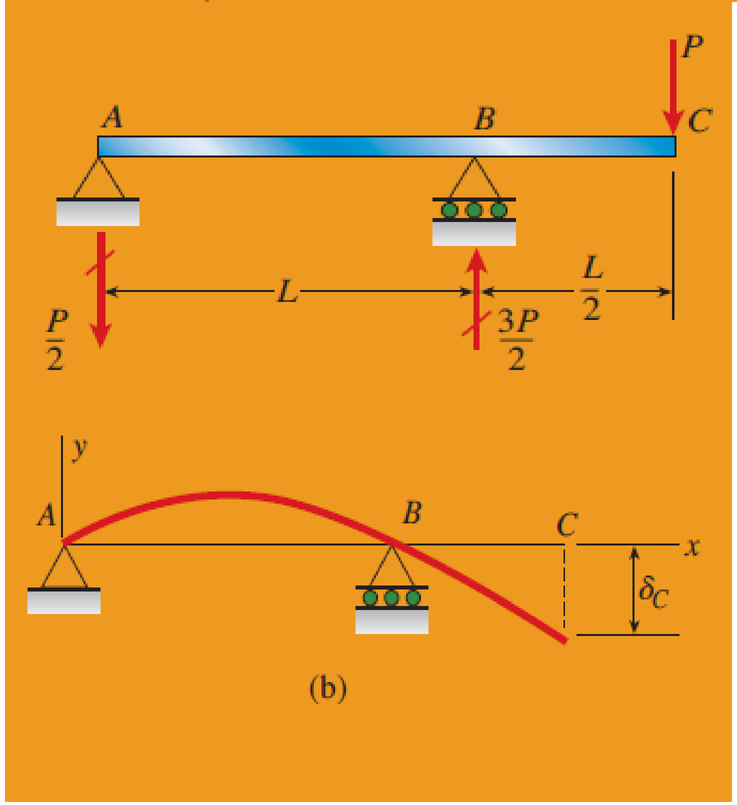
$$EIv''' = P \qquad \left(L < x < \frac{3L}{2}\right)$$

$$M = EIv'' = -\frac{Px}{2} + C_1 \qquad (0 < x < L)$$

$$M = EIv'' = Px + C_2 \qquad \left(L < x < \frac{3L}{2}\right)$$

Deflections by integration of the shear-force and load equations

Example 9-5



$$M = EIv'' = -\frac{Px}{2} + C_1 \qquad (0 < x < L)$$

$$M = EIv'' = Px + C_2 \qquad \left(L < x < \frac{3L}{2}\right)$$

$$v''(0) = 0 \quad \Rightarrow \quad C_1 = 0$$

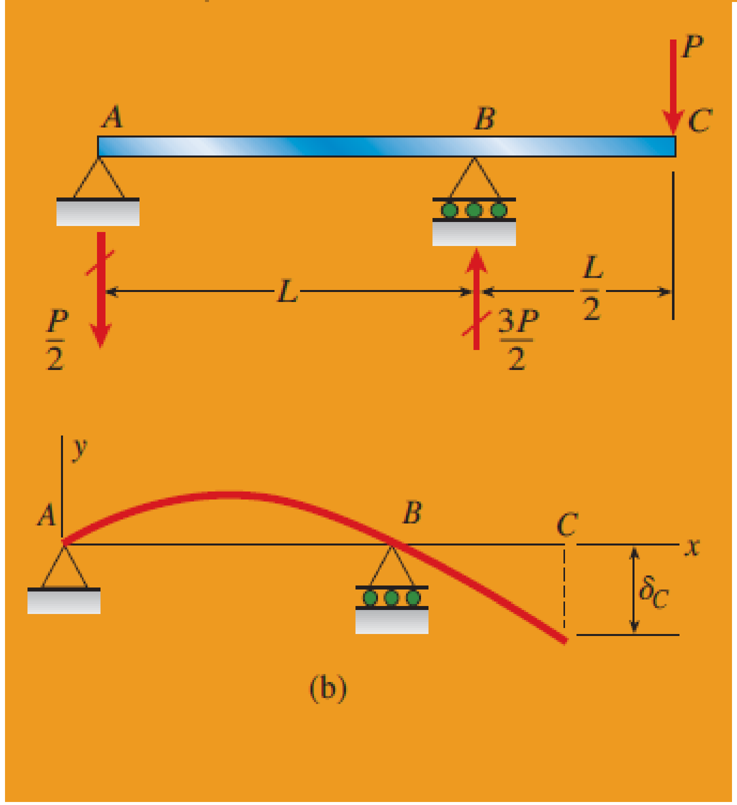
$$v''\left(\frac{3L}{2}\right) = 0 \quad \Rightarrow \quad C_2 = -\frac{3PL}{2}$$

$$M = EIv'' = -\frac{Px}{2} \qquad (0 < x < L)$$

$$M = EIv'' = Px - \frac{3PL}{2} \qquad \left(L < x < \frac{3L}{2}\right)$$

Deflections by integration of the shear-force and load equations

Example 9-5



$$M = EIv'' = -\frac{Px}{2} \qquad (0 < x < L)$$

$$M = EIv'' = Px - \frac{3PL}{2} \qquad \left(L < x < \frac{3L}{2}\right)$$

$$EIv' = -\frac{Px^2}{4} + C_3 \qquad (0 < x < L)$$

$$EIv' = -\frac{Px(3L - x)}{2} + C_4 \qquad \left(L < x < \frac{3L}{2}\right)$$

$$v'(L) = v'(L)$$

$$EIv = -\frac{Px^3}{12} + C_3x + C_5 \qquad (0 < x < L)$$

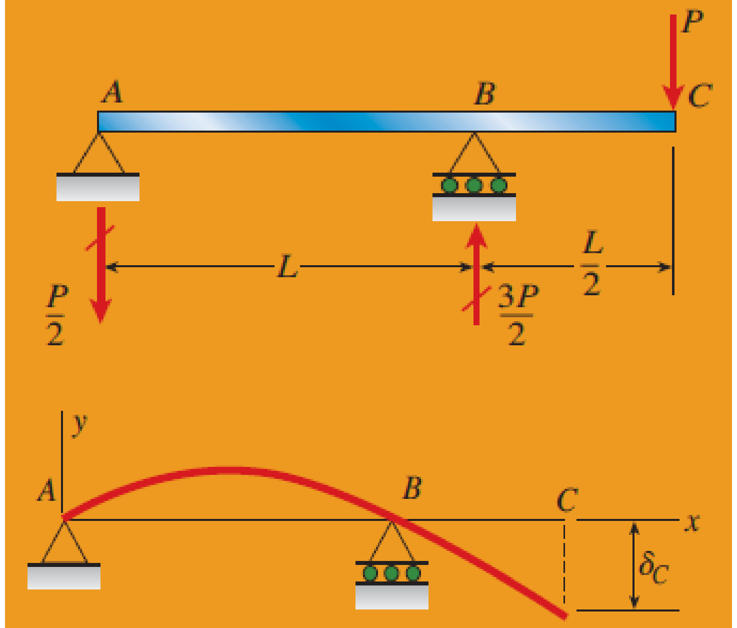
$$v(0) = 0$$

$$EIv = -\frac{Px^2(9L - 2x)}{12} + C_4x + C_6 \qquad \left(L < x < \frac{3L}{2}\right)$$

$$v(L) = 0$$

Deflections by integration of the shear-force and load equations

Example 9-5



(b)

$$EIv = -\frac{Px^3}{12} + C_3x + C_5 \qquad (0 < x < L)$$

$$EIv = -\frac{Px^2(9L-2x)}{12} + C_4x + C_6 \qquad \left(L < x < \frac{3L}{2}\right)$$

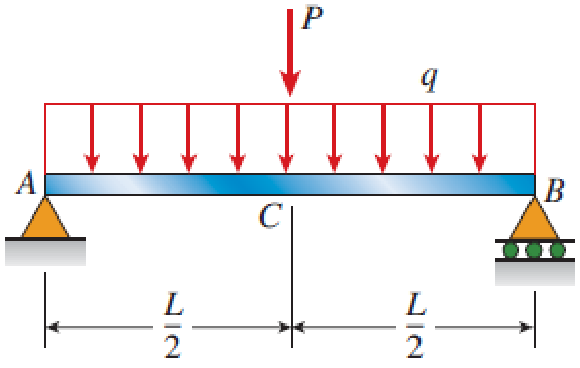
$$\Rightarrow \quad C_5 = 0;\ C_3 = \frac{PL^2}{12};\ C_4 = \frac{5PL^2}{6};\ C_6 = -\frac{PL^3}{4}$$

$$EIv = -\frac{Px^3}{12} + \frac{PL^2}{12}x \qquad (0 < x < L)$$

$$EIv = -\frac{Px^2(9L-2x)}{12} + \frac{5PL^2}{6}x - \frac{PL^3}{4} \qquad \left(L < x < \frac{3L}{2}\right)$$

$$\Rightarrow \quad \delta_C = -v\left(\frac{3L}{2}\right) = \frac{PL^3}{8EI}$$

Method of superposition



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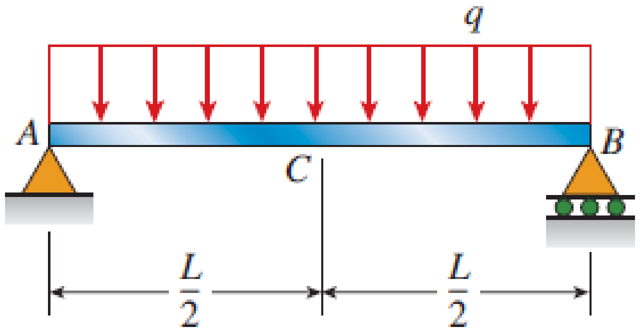

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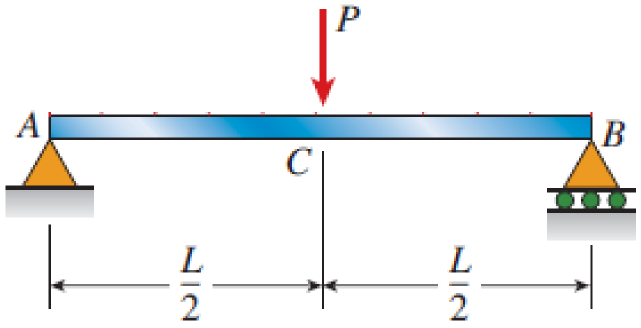

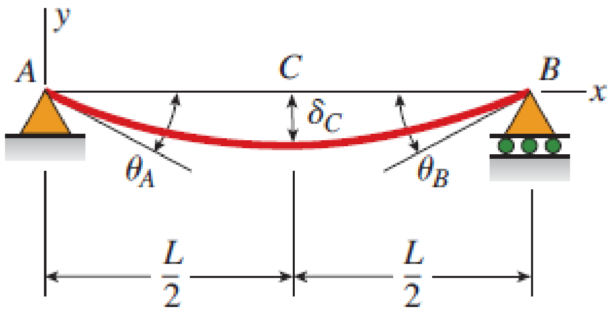

(b)

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+

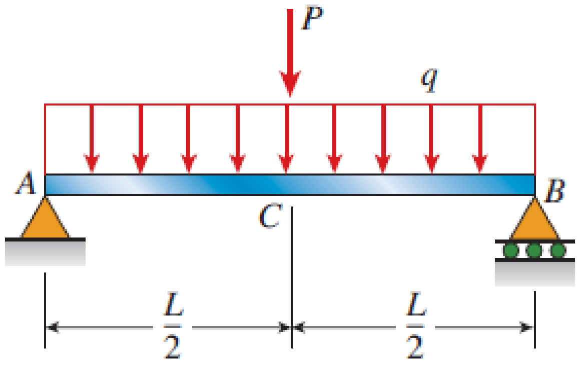
$$v' = -\frac{q}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$v = -\frac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = -\frac{P}{16EI}(L^2 - 4x^2)$$

$$v = -\frac{Px}{48EI}(3L^2 - 4x^2)$$

Method of superposition



(a)

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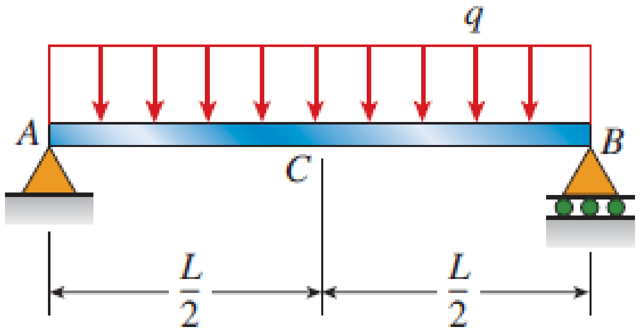

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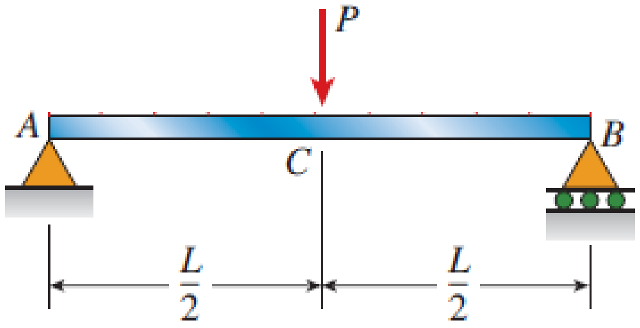
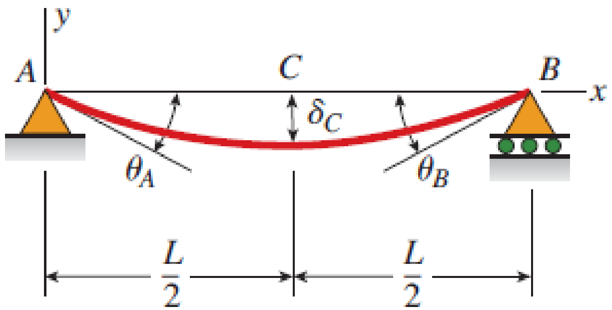




(b)

$$v' = -\frac{q}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$v = -\frac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$(\theta_A)_1 = (\theta_B)_1 = \frac{qL^3}{24EI}$$

$$(\delta_C)_1 = \frac{5qL^4}{384EI}$$

$$v' = -\frac{P}{16EI}(L^2 - 4x^2)$$

$$v = -\frac{Px}{48EI}(3L^2 - 4x^2)$$

$$(\theta_A)_2 = (\theta_B)_2 = \frac{PL^2}{16EI}$$

$$(\delta_C)_2 = \frac{PL^3}{48EI}$$

⇨

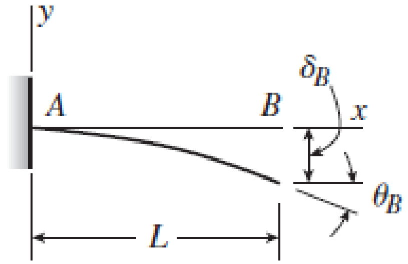
$$\theta_A = \theta_B = (\theta_A)_1 + (\theta_A)_2 = \frac{qL^3}{24EI} + \frac{PL^2}{16EI}$$

$$\delta_C = (\delta_C)_1 + (\delta_C)_2 = \frac{5qL^4}{384EI} + \frac{PL^3}{48EI}$$

Method of superposition

Appendix G

TABLE G-1 DEFLECTIONS AND SLOPES OF CANTILEVER BEAMS



v = deflection in the y direction (positive upward)

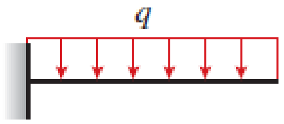
$v' = dv/dx$ = slope of the deflection curve

$\delta_B = -v(L)$ = deflection at end B of the beam (positive downward)

$\theta_B = -v'(L)$ = angle of rotation at end B of the beam (positive clockwise)

EI = constant

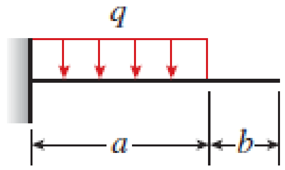
1



$$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \qquad v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{qL^4}{8EI} \qquad \theta_B = \frac{qL^3}{6EI}$$

2



$$v = -\frac{qx^2}{24EI}(6a^2 - 4ax + x^2) \qquad (0 \le x \le a)$$

$$v' = -\frac{qx}{6EI}(3a^2 - 3ax + x^2) \qquad (0 \le x \le a)$$

$$v = -\frac{qa^3}{24EI}(4x - a) \qquad v' = -\frac{qa^3}{6EI} \qquad (a \le x \le L)$$

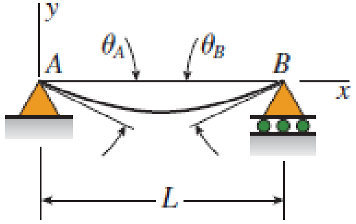
$$\text{At } x = a\text{:} \quad v = -\frac{qa^4}{8EI} \qquad v' = -\frac{qa^3}{6EI}$$

$$\delta_B = \frac{qa^3}{24EI}(4L - a) \qquad \theta_B = \frac{qa^3}{6EI}$$

Method of superposition

Appendix G

TABLE G-2 DEFLECTIONS AND SLOPES OF SIMPLE BEAMS



EI = constant

v = deflection in the y direction (positive upward)

$v' = dv/dx$ = slope of the deflection curve

$\delta_C = -v(L/2)$ = deflection at midpoint C of the beam (positive downward)

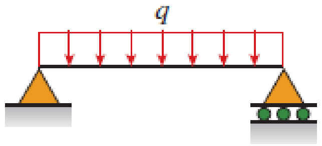
x_1 = distance from support A to point of maximum deflection

$\delta_{max} = -v_{max}$ = maximum deflection (positive downward)

$\theta_A = -v'(0)$ = angle of rotation at left-hand end of the beam (positive clockwise)

$\theta_B = v'(L)$ = angle of rotation at right-hand end of the beam (positive counterclockwise)

1

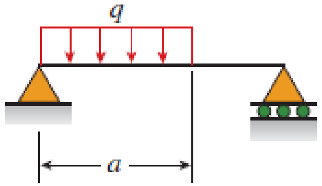


$$v = -\frac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = -\frac{q}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\delta_C = \delta_{max} = \frac{5qL^4}{384EI} \qquad \theta_A = \theta_B = \frac{qL^3}{24EI}$$

3



$$v = -\frac{qx}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3) \qquad (0 \le x \le a)$$

$$v' = -\frac{q}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 6a^2x^2 - 12aLx^2 + 4Lx^3) \qquad (0 \le x \le a)$$

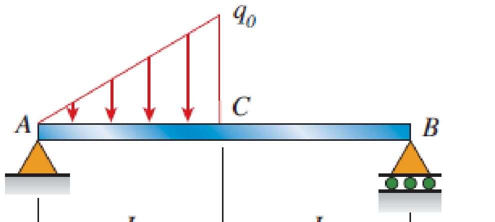
$$v = -\frac{qa^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3) \qquad (a \le x \le L)$$

$$v' = -\frac{qa^2}{24LEI}(4L^2 + a^2 - 12Lx + 6x^2) \qquad (a \le x \le L)$$

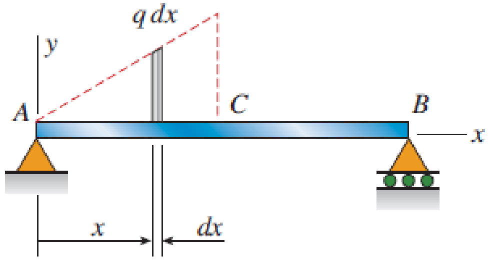
$$\theta_A = \frac{qa^2}{24LEI}(2L - a)^2 \qquad \theta_B = \frac{qa^2}{24LEI}(2L^2 - a^2)$$

Method of superposition

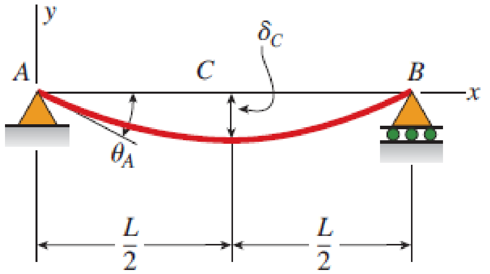
Distributed loads



(a)



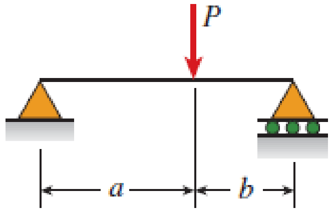
(b)



(c)

Use Table G-2, Appendix G

5



$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \qquad (0 \le x \le a)$$

$$\theta_A = \frac{Pab(L+b)}{6LEI} \qquad \theta_B = \frac{Pab(L+a)}{6LEI}$$

$$\text{If } a \ge b, \quad \delta_C = \frac{Pb(3L^2 - 4b^2)}{48EI} \qquad \text{If } a \le b, \quad \delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$$

$$\text{If } a \ge b, \quad x_1 = \sqrt{\frac{L^2 - b^2}{3}} \quad \text{and} \quad \delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}\,LEI}$$

Midpoint deflection

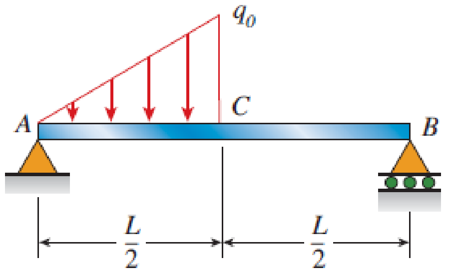
$$\text{If } a \le b, \quad \delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$$

Angle of rotation

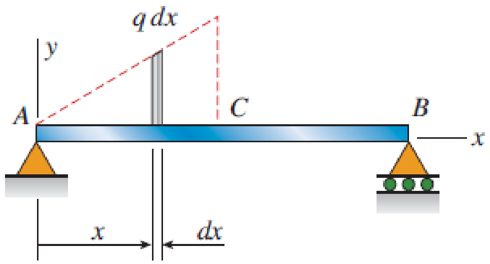
$$\theta_A = \frac{Pab(L+b)}{6LEI}$$

Method of superposition

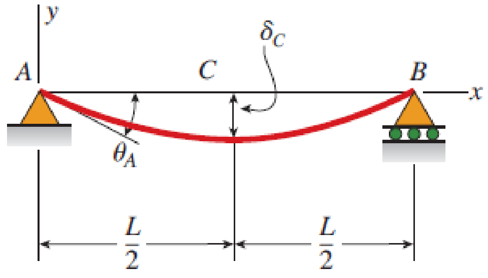
Distributed loads



(a)



(b)



(c)

Midpoint deflection

$$\frac{Pa}{48EI}(3L^2 - 4a^2)$$

$$\frac{(q\,dx)(x)}{48EI}(3L^2 - 4x^2)$$

$$q = \frac{2q_0 x}{L}$$

$$\frac{q_0 x^2}{24LEI}(3L^2 - 4x^2)dx$$

$$\Rightarrow \delta_C = \int_0^{L/2} \frac{q_0 x^2}{24LEI}(3L^2 - 4x^2)dx$$

$$= \frac{q_0}{24LEI}\int_0^{L/2}(3L^2 - 4x^2)x^2\,dx = \frac{q_0 L^4}{240EI}$$

Angle of rotation

$$\frac{Pab(L+b)}{6LEI}$$

Replacing P with $2q_0 x\,dx/L$, a with x, and b with $L - x$,

$$\frac{2q_0 x^2(L-x)(L+L-x)}{6L^2EI}dx$$

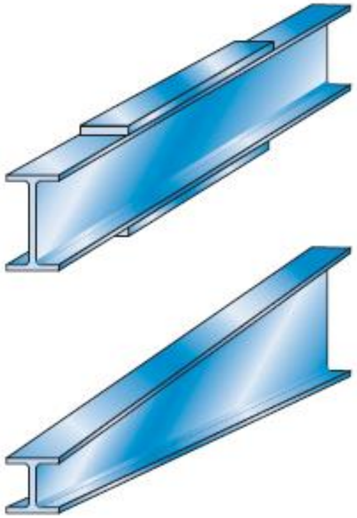
$$\frac{q_0}{3L^2EI}(L-x)(2L-x)x^2dx$$

$$\Rightarrow \theta_A = \int_0^{L/2} \frac{q_0}{3L^2EI}(L-x)(2L-x)x^2dx$$

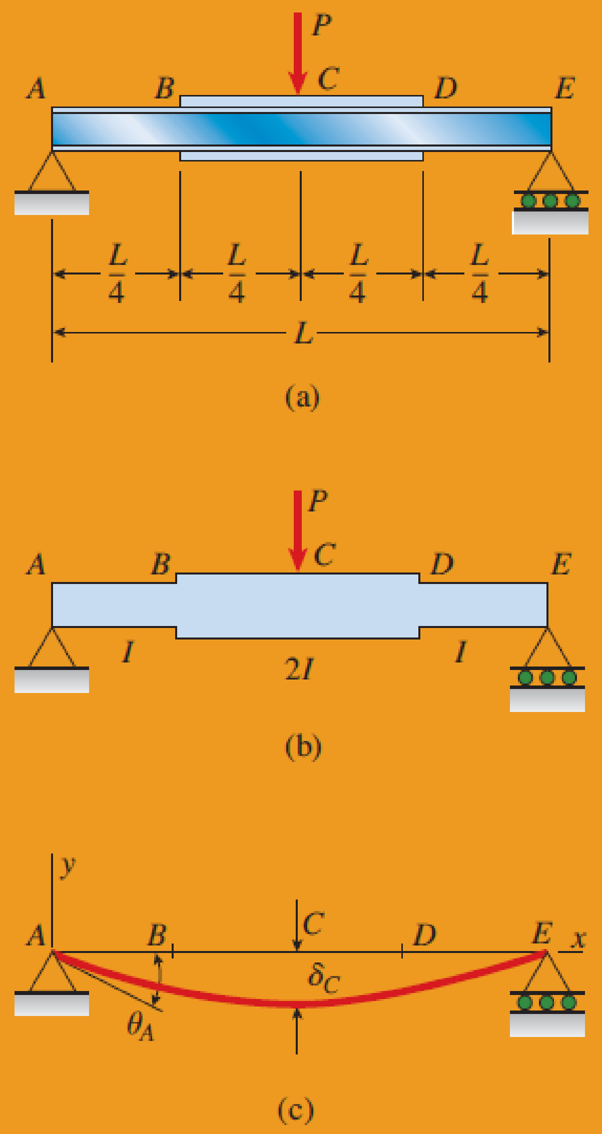
$$= \frac{41q_0L^3}{2880EI}$$

Nonprismatic beams

Nonprismatic beams with cutouts in their webs



Example 9-13



Equation for Moment

$$M = \frac{Px}{2} \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$EIv'' = \frac{Px}{2} \qquad \left(0 \le x \le \frac{L}{4}\right)$$

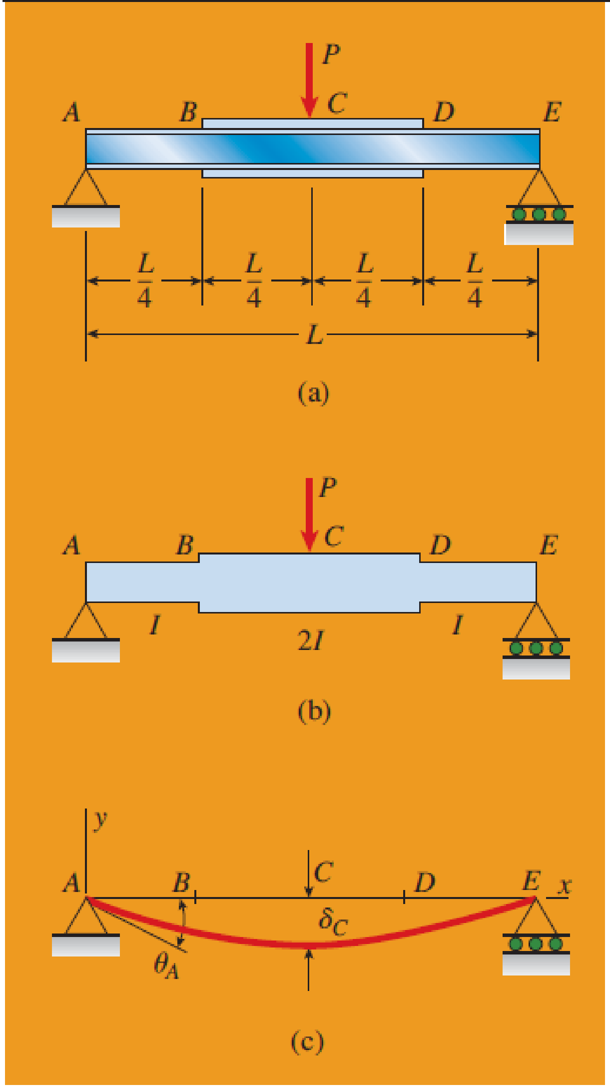
$$E(2I)v'' = \frac{Px}{2} \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right)$$

Conditions

1. At support A (x=0), deflection is zero (v=0)
2. At point C (x=L/2), slope is zero (v'=0)
3. At point B (x=L/4), slope should be continuous
4. At point B (x=L/4), deflection should be continuous

Nonprismatic beams

Example 9-13



$$v' = \frac{Px^2}{4EI} + C_1 \qquad \left(0 \le x \le \frac{L}{4}\right)$$

$$v' = \frac{Px^2}{8EI} + C_2 \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right)$$

2. At point C (x=L/2), slope is zero (v'=0)

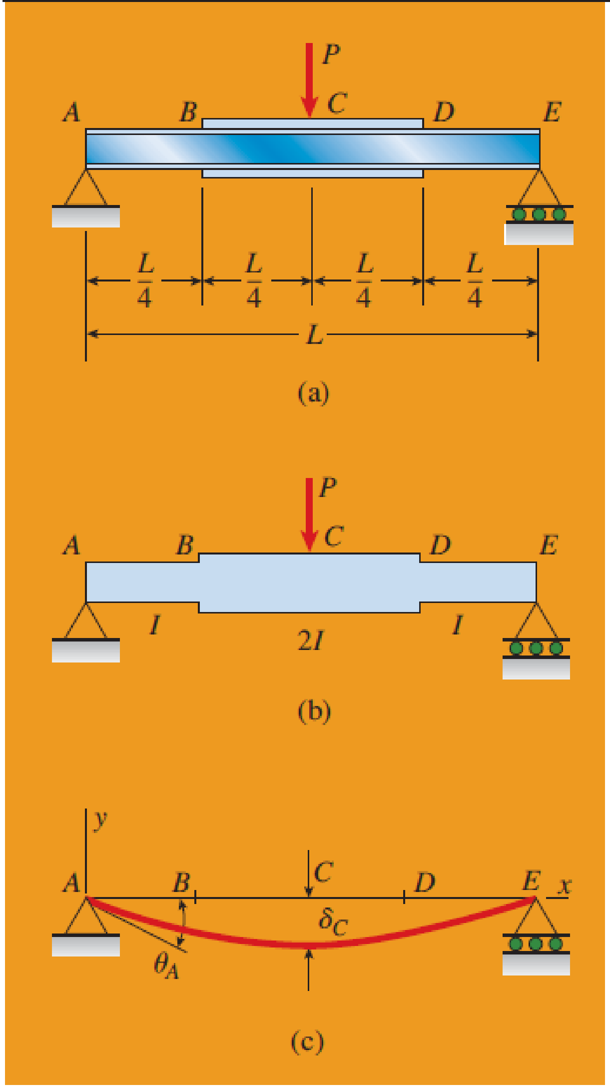
$$\Rightarrow \quad C_2 = -\frac{PL^2}{32EI}$$

3. At point B (x=L/4), slope should be continuous

$$\frac{P}{4EI}\left(\frac{L}{4}\right)^2 + C_1 = -\frac{3PL^2}{128EI} \quad \Rightarrow \quad C_1 = -\frac{5PL^2}{128EI}$$

Nonprismatic beams

Example 9-13



$$v' = \frac{Px^2}{4EI} - \frac{5PL^2}{128EI} \qquad \left(0 \le x \le \frac{L}{4}\right)$$

$$v' = \frac{Px^2}{8EI} - \frac{PL^2}{32EI} \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right)$$

$$v = -\frac{P}{128EI}\left(5L^2x - \frac{32x^3}{3}\right) + C_3 \qquad \left(0 \le x \le \frac{L}{4}\right)$$

$$v = -\frac{P}{32EI}\left(L^2x - \frac{4x^3}{3}\right) + C_4 \qquad \left(\frac{L}{4} \le x \le \frac{L}{2}\right)$$

1. At support A (x=0), deflection is zero (v=0) ⇨ $C_3 = 0$

4. At point B (x=L/4), deflection should be continuous

$$-\frac{P}{32EI}\left[L^2\left(\frac{L}{4}\right) - \frac{4}{3}\left(\frac{L}{4}\right)^3\right] + C_4 = v\left(\frac{L}{4}\right) = -\frac{13PL^3}{1536EI} \quad \Rightarrow \quad C_4 = -\frac{PL^3}{768EI}$$