7.4. Biaxial folding

- Booklet folding : based on traditional book binding technique
  - difficult to unfold / fold
  - difficult to implement through a global actuation scheme

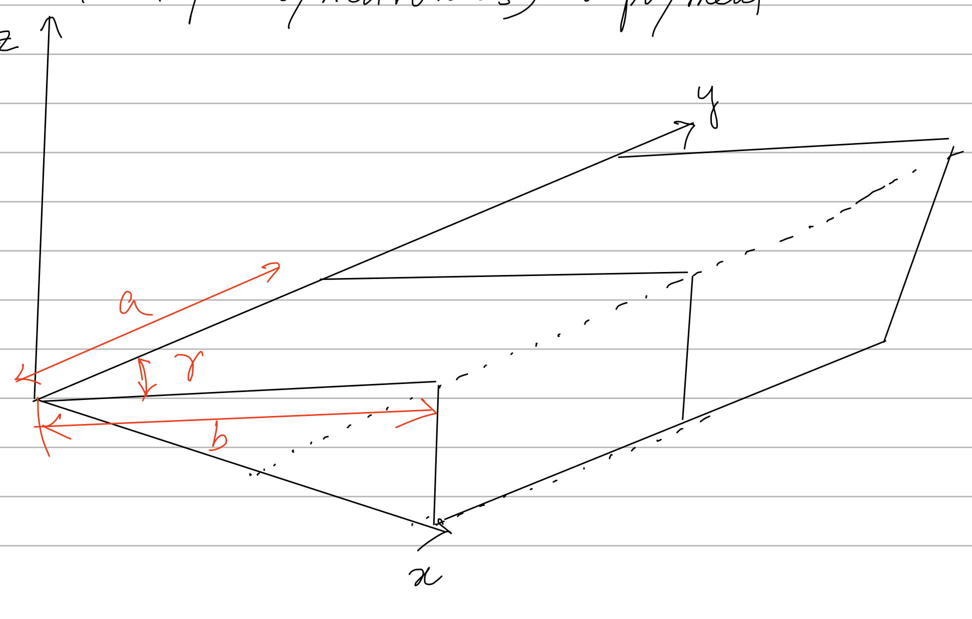
- Map folding : distribution of folds repeats periodically in both horizontal and vertical directions.
  - relatively easy deployment
  - two-step actuation process

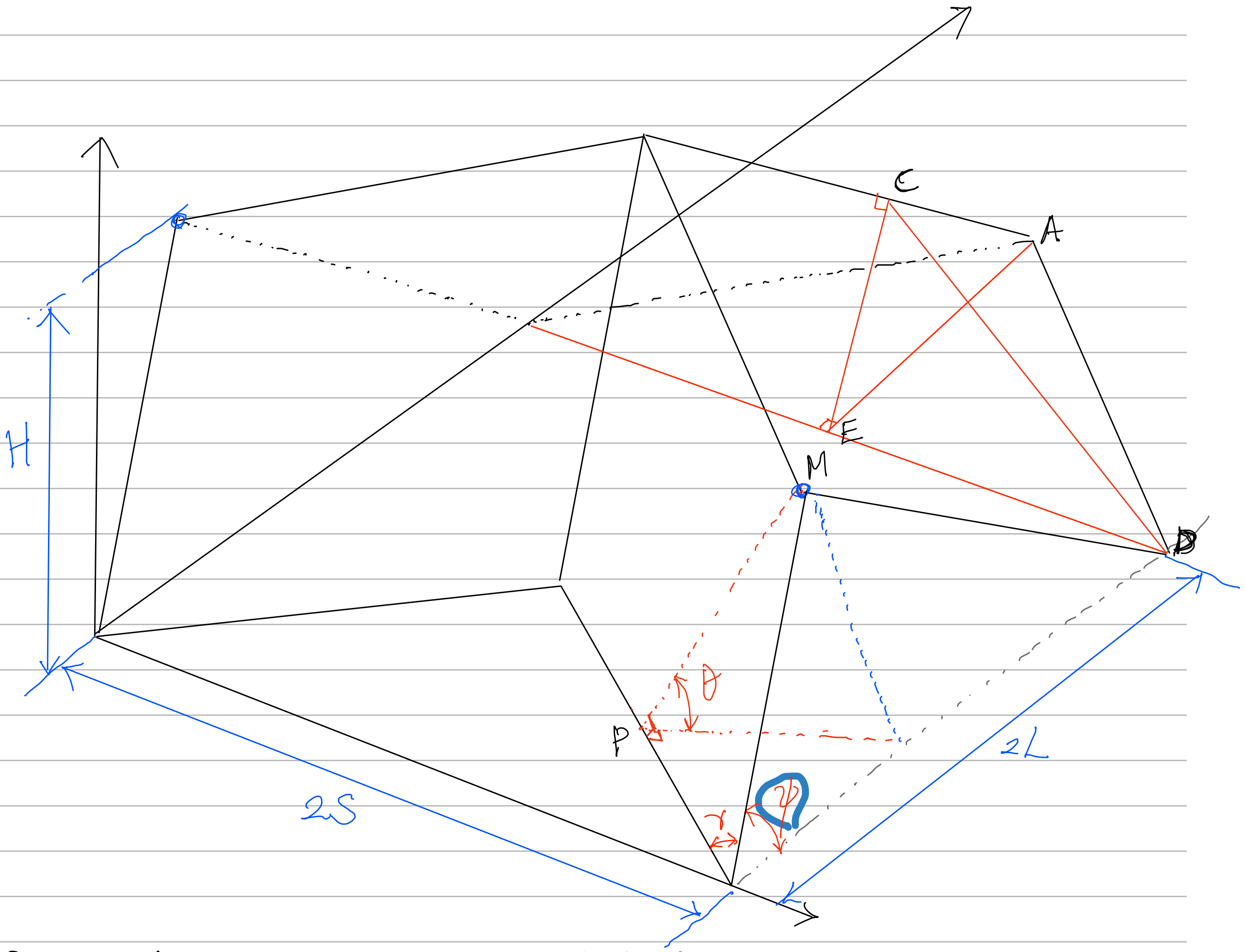
Thickness effects

- Folding patterns that are translationally periodic
  → thickness effects are not compounded

- " " not " "
  → " " are compounded

- Developable double corrugation surface : Miura-ori
  : 1-step (synchronous) deployment





θ : deployment angle (dihedral fold angle)

$$H = a \sin\psi = \overline{MP} \cdot \sin\theta = a \sin\gamma \cdot \sin\theta$$

$$a^2 = H^2 + L^2$$

$$L = \sqrt{a^2 - H^2} = a\sqrt{1 - \sin^2\gamma \sin^2\theta}$$

$$\tan\psi = \frac{H}{L} = \frac{\sin\gamma \sin\theta}{\sqrt{1 - \sin^2\gamma \sin^2\theta}}$$

$$\overline{CD} = \overline{AD} \sin\gamma = b \sin\gamma$$

$$\overline{AC} = \overline{AD} \cos\gamma = b \cos\gamma$$

$$\overline{CE} = \overline{AC} \tan\psi = b\cos\gamma \cdot \frac{\sin\gamma \sin\theta}{\sqrt{1 - \sin^2\gamma \sin^2\theta}}$$

$$S^2 = \overline{CD}^2 - \overline{CE}^2 = b^2 \sin^2\gamma - b^2 \cos^2\gamma \cdot \frac{\sin^2\gamma \sin^2\theta}{1 - \sin^2\gamma \sin^2\theta}$$

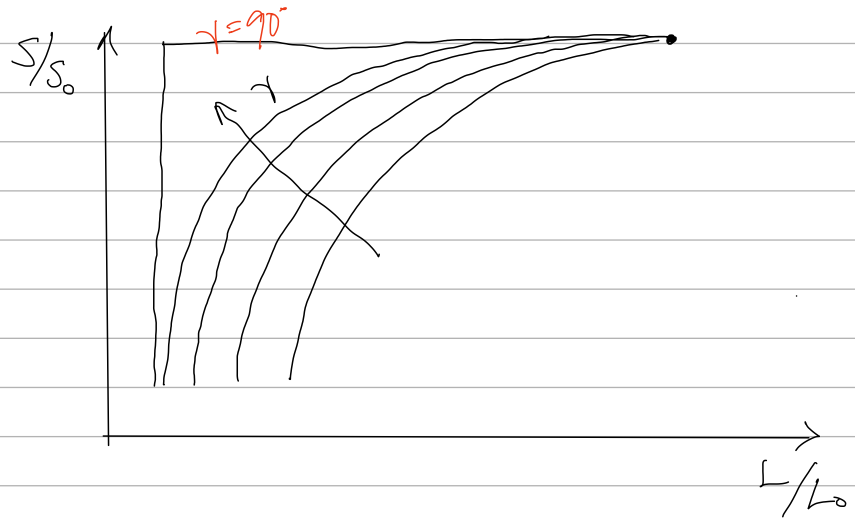
$$= b^2 \sin^2\gamma \cdot \frac{1 - \sin^2\gamma \sin^2\theta - \cos^2\gamma \sin^2\theta}{1 - \sin^2\gamma \sin^2\theta}$$

$$= b^2 \frac{\sin^2\gamma \cos^2\theta}{1 - \sin^2\gamma \sin^2\theta}$$

$$= b^2 \frac{\sin^2\gamma \cos^2\theta / \cos^2\gamma}{(\cos^2\gamma + \sin^2\gamma - \sin^2\gamma \sin^2\theta)/\cos^2\gamma}$$

$$= b^2 \frac{\tan^2\gamma \cos^2\theta}{1 + \tan^2\gamma \cos^2\theta}$$

$$S = b \cdot \frac{\tan\gamma \cos\theta}{\sqrt{1 + \tan^2\gamma \cos^2\theta}}$$



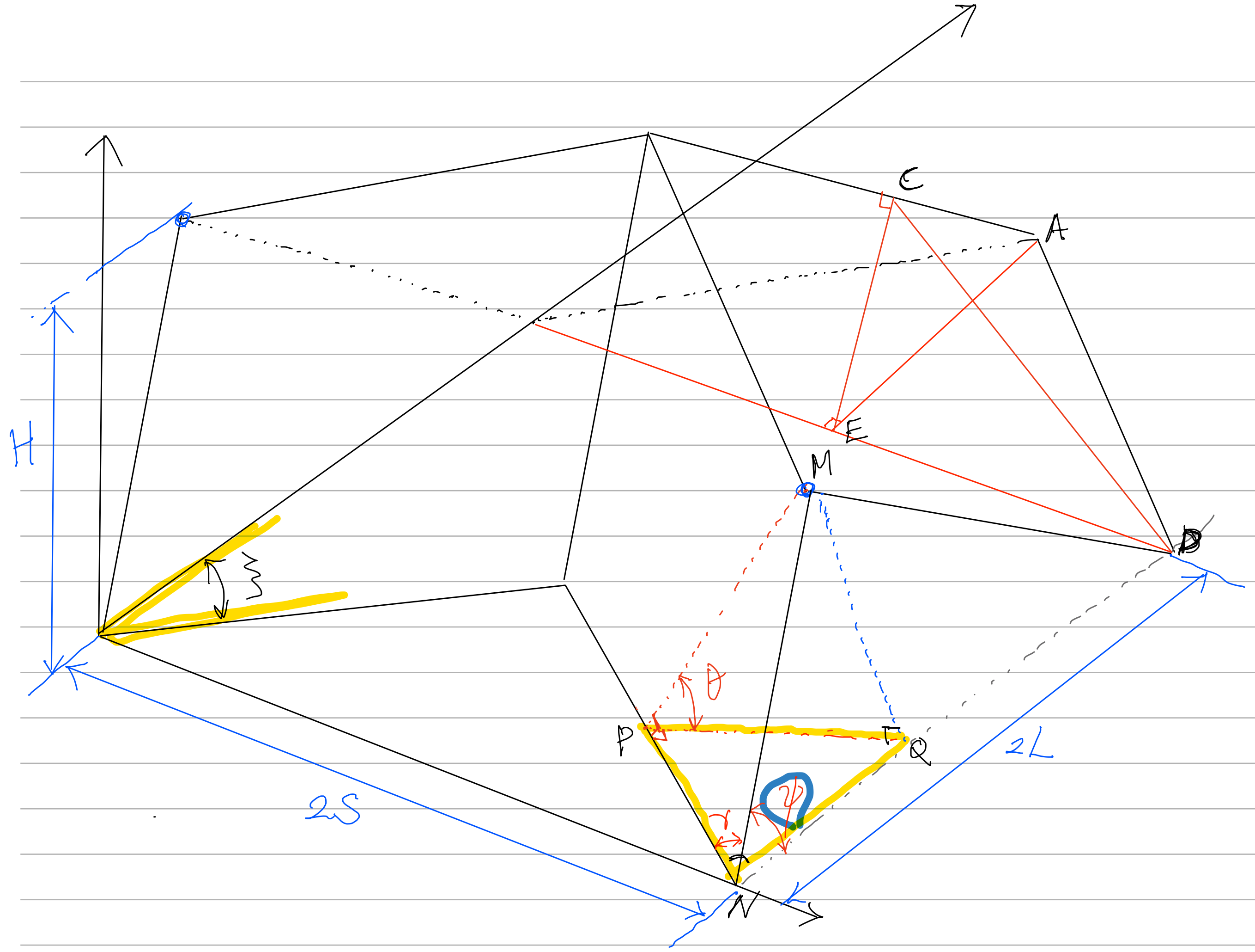
$\gamma \uparrow$: coupling becomes weaker

$\gamma \downarrow$: packaging efficiency goes down.

In-plane kinematics

$$\nu_{SL} = -\frac{\varepsilon_L}{\varepsilon_S} = -\frac{dL/L}{dS/S} = -\frac{dL/d\theta}{dS/d\theta} \cdot \frac{S}{L}$$

$$= -\cos^2\theta \tan^2\gamma$$



$$\cos\theta \tan\gamma = \frac{\overline{PQ}}{\overline{PM}} \cdot \frac{\overline{PM}}{\overline{PN}} = \frac{\overline{PQ}}{\overline{PN}} = \tan\xi$$

$$\nu_{SL} = -\tan^2\xi$$

$$\nu_{LH} = -\frac{\varepsilon_H}{\varepsilon_L} = -\frac{dH/d\theta}{d\psi/d\theta} \cdot \frac{L}{H} = \frac{1-\sin^2\psi}{\sin^2\psi}$$

$$\nu_{SH} = -\frac{\varepsilon_H}{\varepsilon_S} = -\left(-\varepsilon_H/\varepsilon_L\right)\cdot\left(-\varepsilon_L/\varepsilon_S\right)$$

$$= -\nu_{LH} \cdot \nu_{SL}$$

Relative density

$$\bar{\rho} = \frac{t \cdot ab \sin\gamma}{HSL}$$

Minimal density?

$$d\bar{\rho}/d\theta = 0 \quad \rightarrow \quad \theta = \pi/4$$