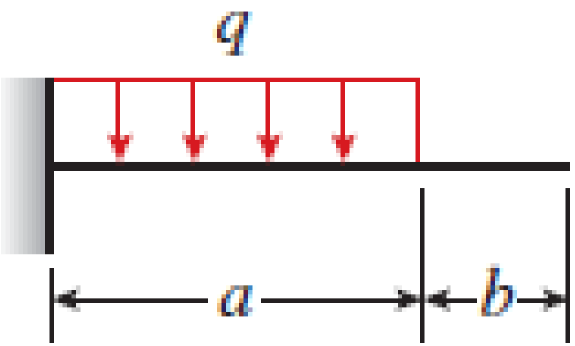
Announcement

To be updated

More problem 9. Deflection

Deflection at the tip



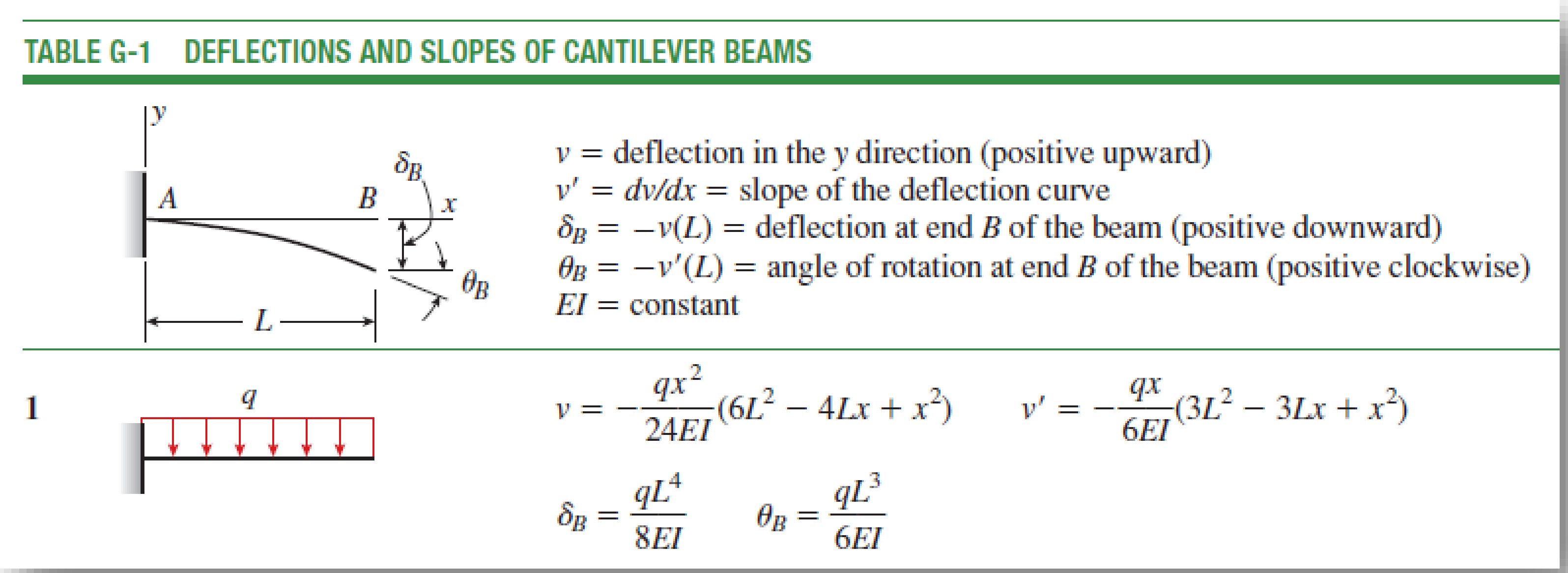


TABLE G-1 DEFLECTIONS AND SLOPES OF CANTILEVER BEAMS

v = deflection in the y direction (positive upward)
$v' = dv/dx$ = slope of the deflection curve
$\delta_B = -v(L)$ = deflection at end B of the beam (positive downward)
$\theta_B = -v'(L)$ = angle of rotation at end B of the beam (positive clockwise)
EI = constant

1

$$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \qquad v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{qL^4}{8EI} \qquad \theta_B = \frac{qL^3}{6EI}$$

$$\delta_{total} = \delta_{deflection} + \delta_{angle}$$

$$\delta_{total} = \frac{qa^4}{8EI} + \frac{qa^4}{6EI} \times (L - a) = \frac{qa^3}{24EI} \times (4L - a)$$

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Chap. 11 Columns

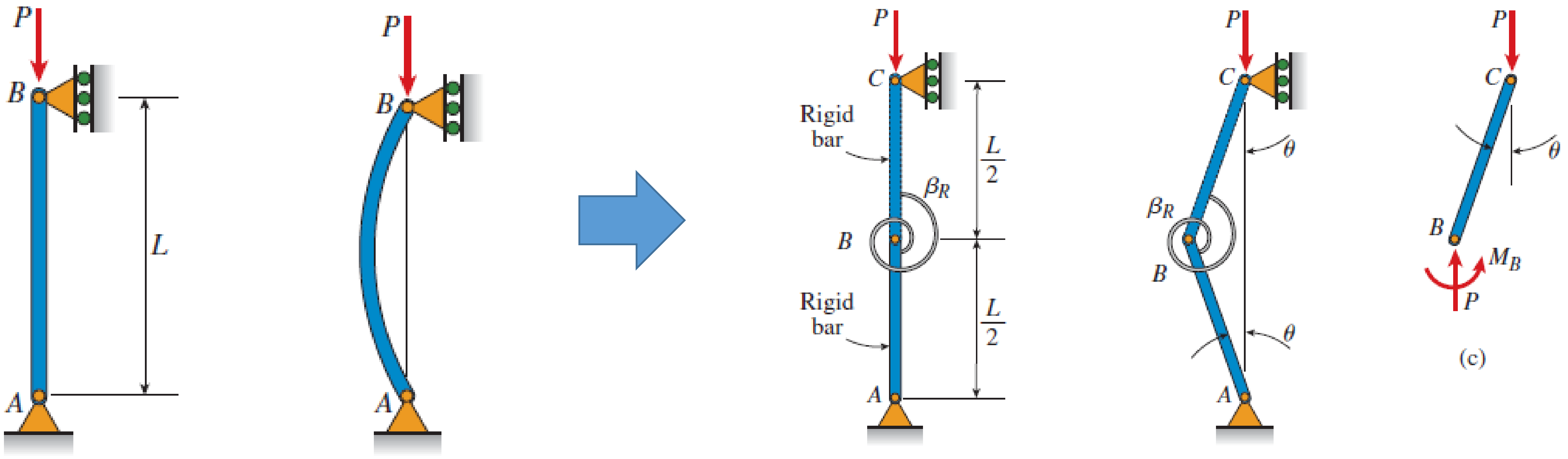
11.1 Introduction

11.2 Buckling and stability

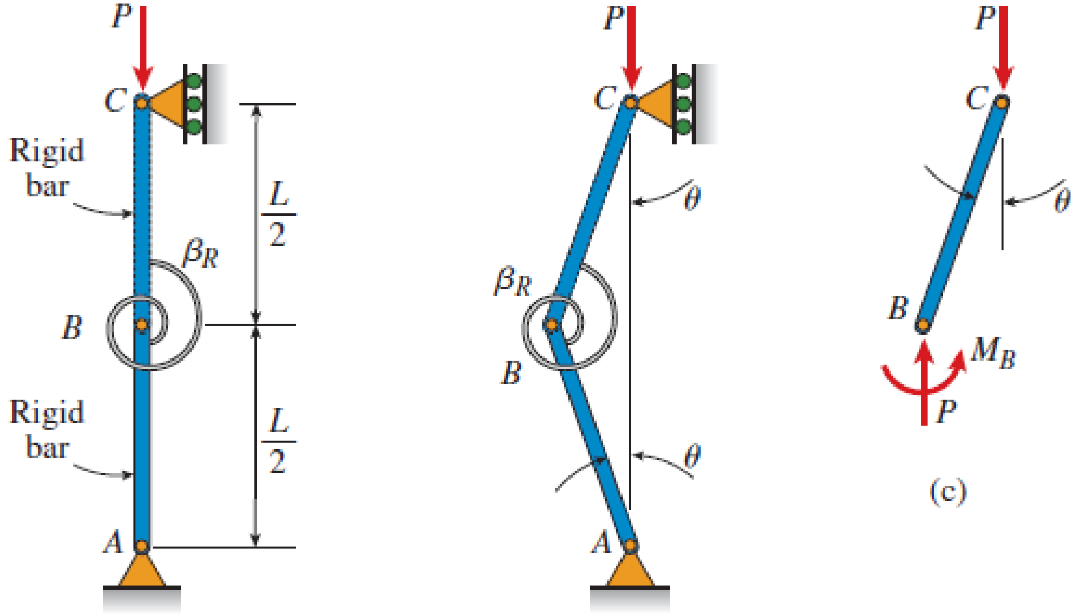
11.3 Columns with pinned ends

11.4 Columns with other support conditions

11.2 Buckling and stability



11.2 Buckling and stability



Hooke's spring model

$$M_B = 2\beta_R\theta$$

Equilibrium eq.

$$M_B - P\left(\frac{\theta L}{2}\right) = 0$$

Buckling eq.

$$\left(2\beta_R - \frac{PL}{2}\right)\theta = 0$$

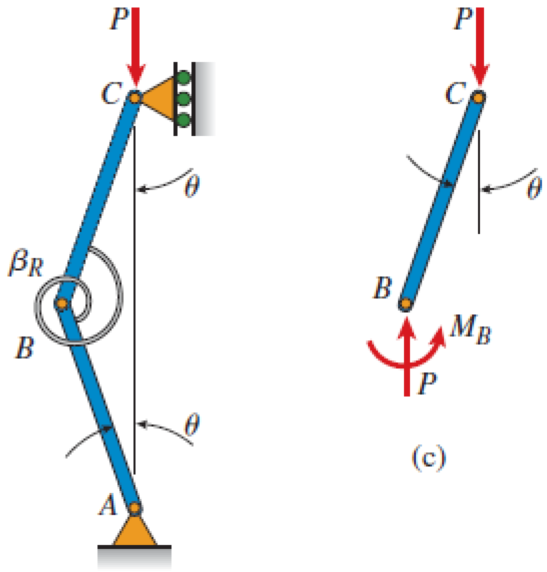
1st solution (Trivial solution)

$$\theta = 0$$

2nd solution

$$P_{cr} = \frac{4\beta_R}{L}$$ ***= Critical load***

11.2 Buckling and stability



$$P_{cr} = \frac{4\beta_R}{L}$$

Critical load

If $P < P_{cr}$, the structure is *stable*
If $P > P_{cr}$, the structure is *unstable*

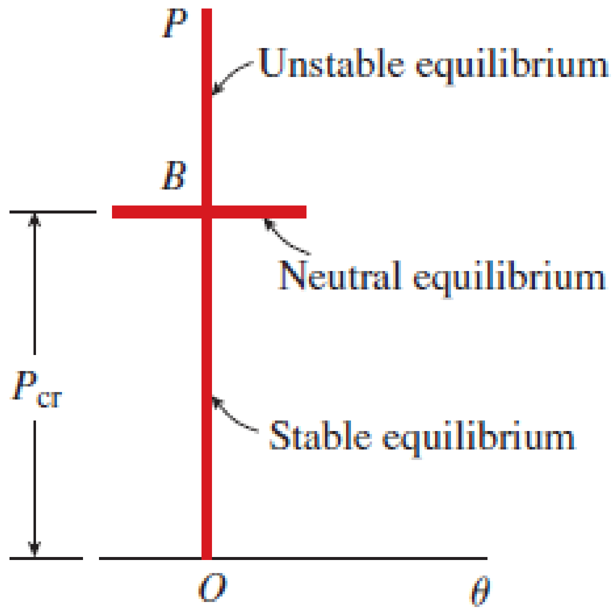


FIG. 11-3 Equilibrium diagram for buckling of an idealized structure

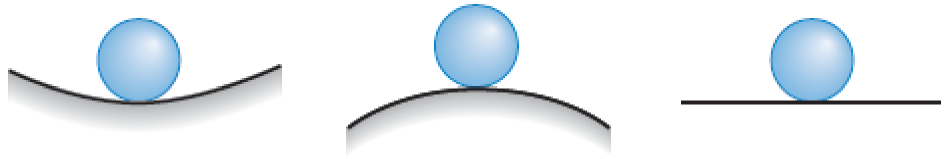
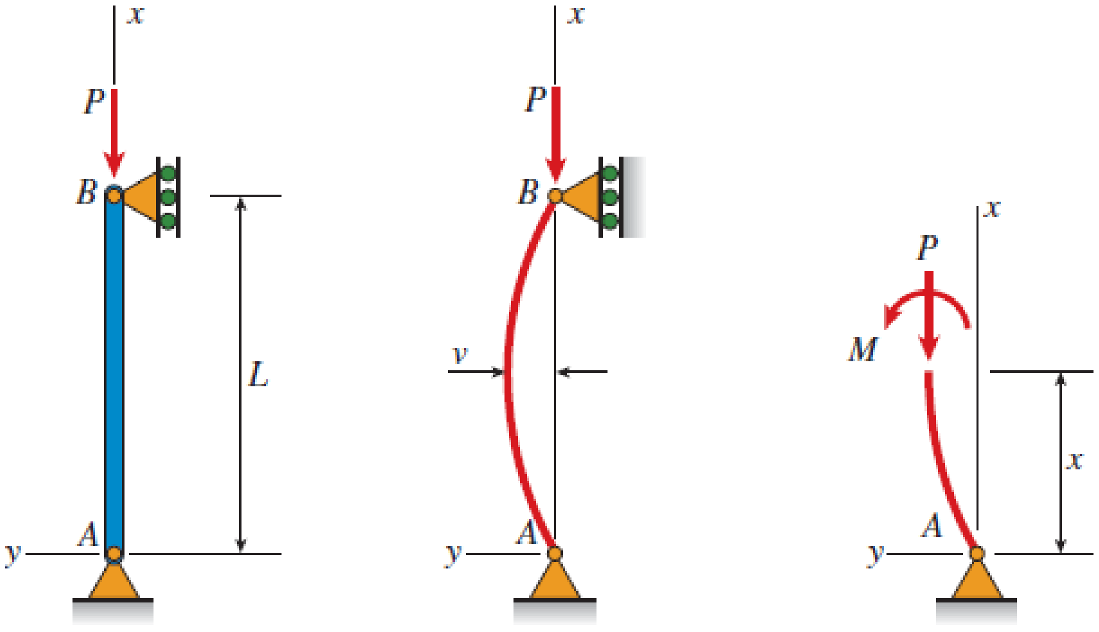


FIG. 11-4 Ball in stable, unstable, and neutral equilibrium

11.3 Columns with pinned ends



(a) (b) (c)

If $P < P_{cr}$, the column is in stable equilibrium in the straight position.
If $P = P_{cr}$, the column is in neutral equilibrium in either the straight or a slightly bent position.
If $P > P_{cr}$, the column is in unstable equilibrium in the straight position and will buckle under the slightest disturbance.

Equilibrium eq.

$$M + Pv = 0$$

Buckling eq.

$$EIv'' + Pv = 0 \qquad (using\ M = EIv'')$$

$$v'' + k^2 v = 0 \qquad (k^2 = \frac{P}{EI})$$

Solution

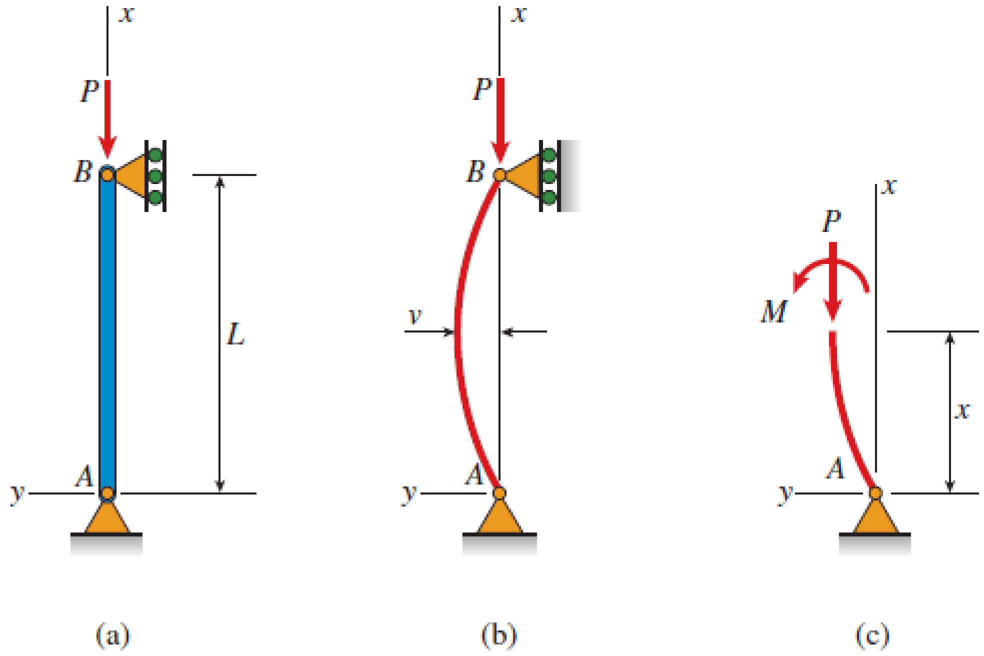
$$v = C_1 \sin kx + C_2 \cos kx$$

Boundary condition

$$v(0) = 0 \rightarrow C_2 = 0$$

$$C_1 \sin kL = 0$$

11.3 Columns with pinned ends



$v = C_1 \sin kx$ (where $C_1 \sin kL = 0$)

1st solution (Trivial solution)

$$C_1 = 0$$

2nd solution

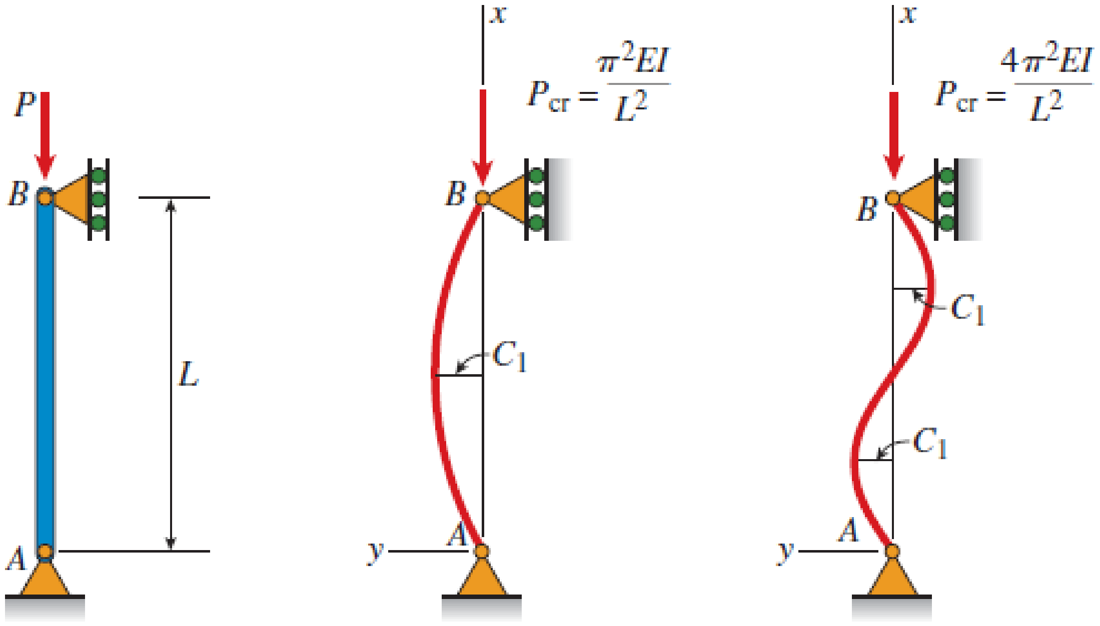
$$\sin kL = 0$$

$$kL = n\pi, \quad n = 1,2,3,\ldots$$

$$P = \frac{n^2\pi^2 EI}{L^2}, \quad n = 1,2,3,\ldots \qquad (k^2 = \frac{P}{EI})$$

$$v = C_1 \sin kx = C_1 \sin \frac{n\pi x}{L}, \quad n = 1,2,3,\ldots$$

11.3 Columns with pinned ends



Buckling load

$$P = \frac{n^2\pi^2 EI}{L^2}, \quad n = 1,2,3,\ldots \qquad (k^2 = \frac{P}{EI})$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$ ***= Critical load (smallest buckling load)***

Buckling shape

$$v = C_1 \sin\frac{n\pi x}{L}, \quad n = 1,2,3,\ldots$$

11.3 Columns with other support conditions

A. Both pinned ends

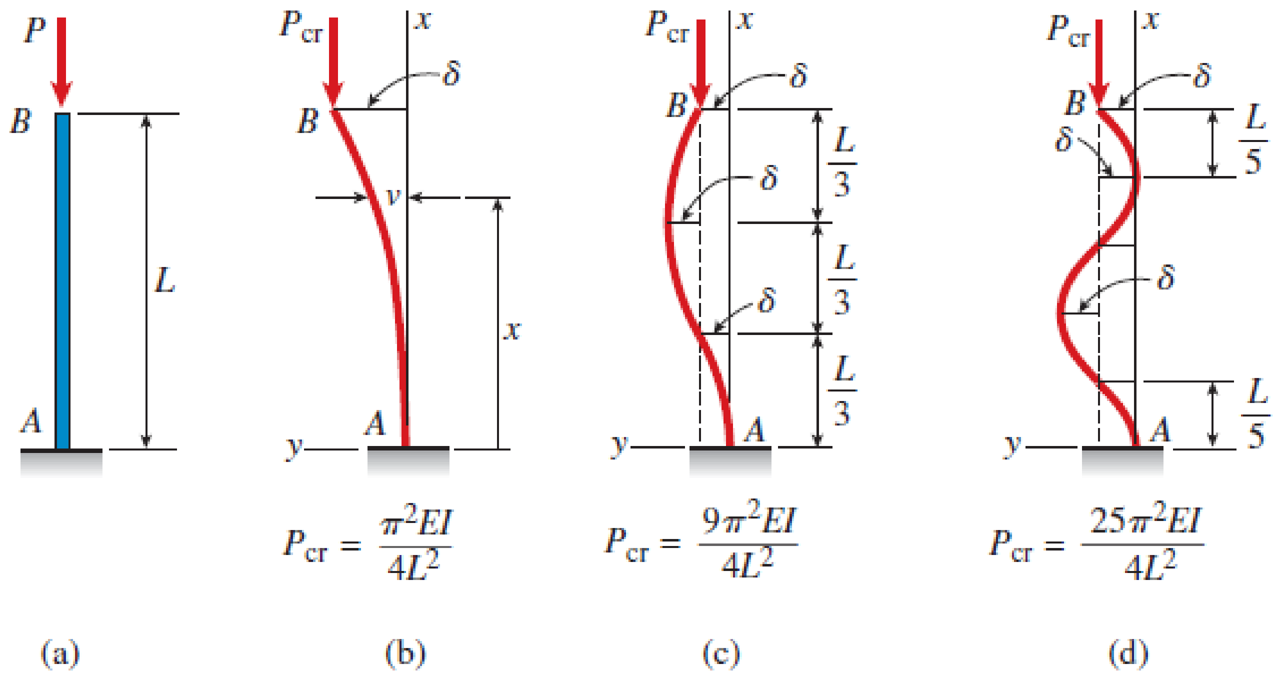
B. Fixed at the base and free at the top

C. Both ends fixed against rotation

D. Fixed at the base and pinned at the top

11.3 Columns with other support conditions

B. Fixed at the base and free at the top



$$M = P(\delta - v)$$

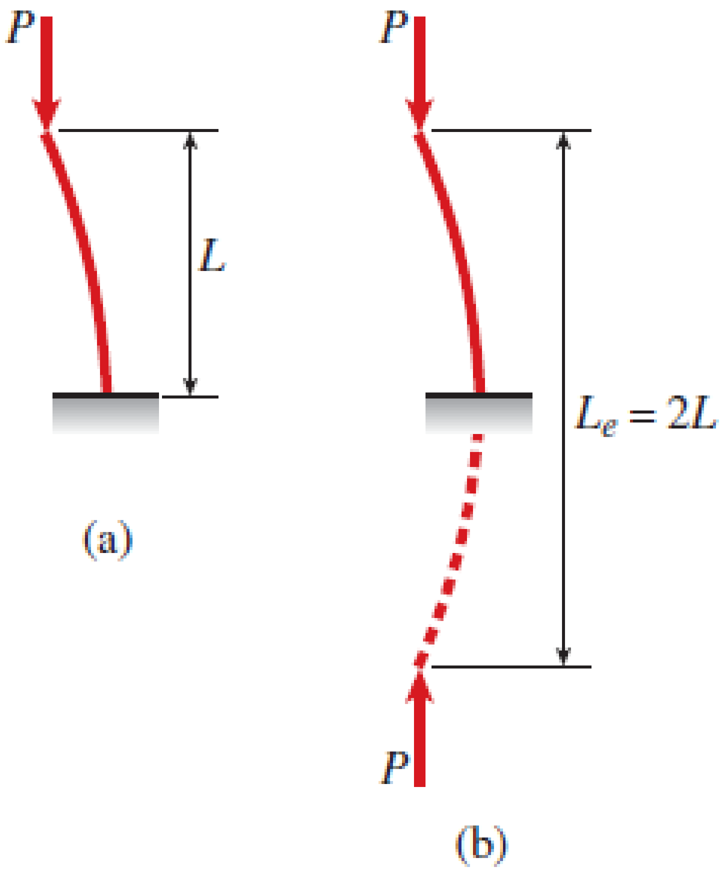
$$EIv'' = M = P(\delta - v)$$

$$v'' + k^2 v = k^2 \delta$$

$$P_{cr} = \frac{n^2\pi^2 EI}{4L^2} \quad n = 1,3,5,\ldots$$

11.3 Columns with other support conditions

B. Fixed at the base and free at the top

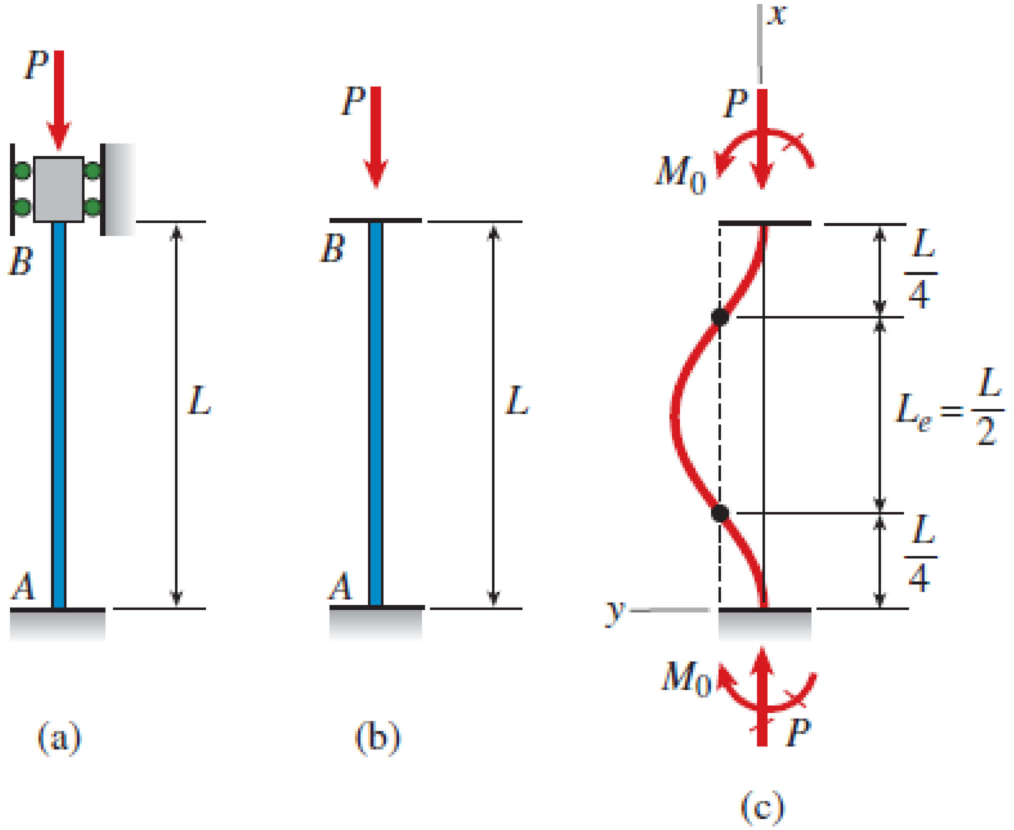


Effective length $L_e = KL$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{(KL)^2}$$

11.3 Columns with other support conditions

C. Column with both ends fixed against rotation



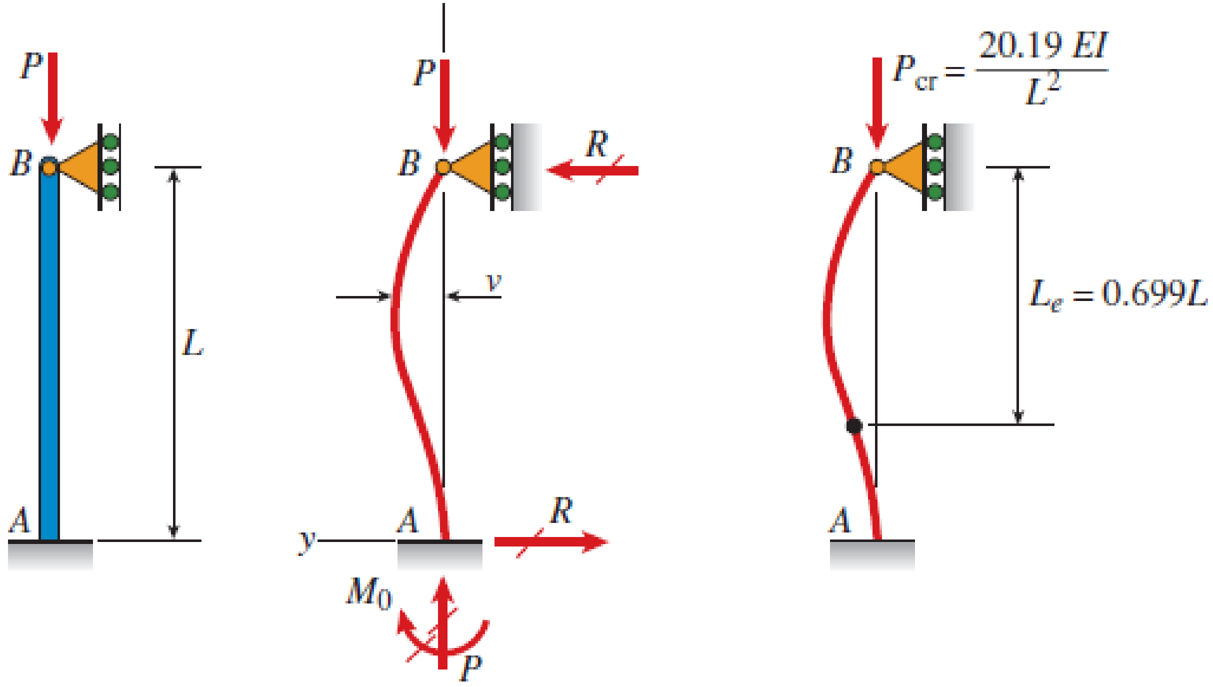
Effective length $L_e = L/2$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{(KL)^2}$$

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

11.3 Columns with other support conditions

D. Column fixed at the base and pinned at the top



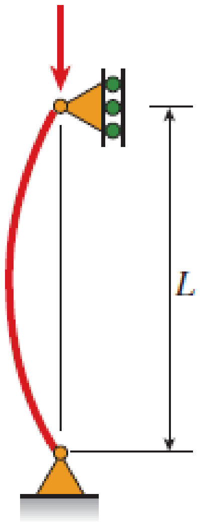
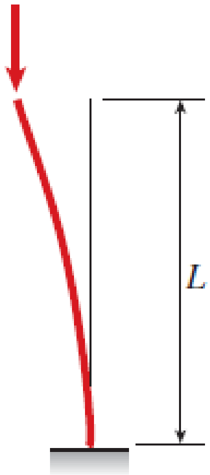
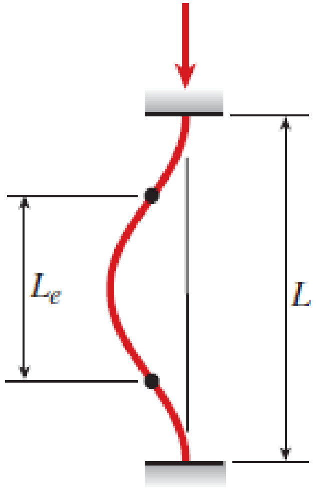
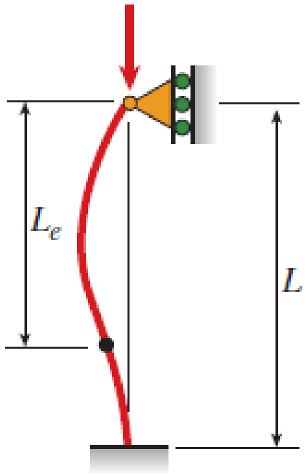
Effective length $L_e = 0.7L$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{(KL)^2}$$

$$P_{cr} = \frac{2.046\pi^2 EI}{L^2}$$

11.3 Columns with other support conditions

Summary

(a) Pinned-pinned column	(b) Fixed-free column	(c) Fixed-fixed column	(d) Fixed-pinned column
$P_{cr} = \frac{\pi^2 EI}{L^2}$	$P_{cr} = \frac{\pi^2 EI}{4L^2}$	$P_{cr} = \frac{4\pi^2 EI}{L^2}$	$P_{cr} = \frac{2.046\, \pi^2 EI}{L^2}$
			
$L_e = L$	$L_e = 2L$	$L_e = 0.5L$	$L_e = 0.699L$
$K = 1$	$K = 2$	$K = 0.5$	$K = 0.699$