Rock Mechanics & Experiment
암석역학 및 실험

Lecture 9. Fluid Flow and Heat Transfer in rock
Lecture 9. 암반에서의 유체유동 및 열전달

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Outline

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- Introduction
- Fluid flow in rock
  - Introduction
  - Darcy's law
  - Permeability
  - Fluid flow in fractured rock
- Heat transfer in rock
  - Introduction
  - Fourier's Law
  - Thermal conductivity

Introduction
T-H-M-C process - convenient truth

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Physical problem	Conservation Principle	State Variable	Flux σ	Material properties	Source	Constitutive equation
Elasticity	힘의 평형 (Equilibrium)	변위 (Displacement), u	응력 (Stress) σ	탄성계수 및 포아송비 (Young's modulus & Poisson's ratio)	체적력 (Body forces)	후크의 법칙 (Hooke's law)
Heat conduction	Conservation of energy	Temperature, T	Heat flux q	Thermal conductivity k	Heat sources	Fourier's law
Porous media flow	Conservation of mass	Hydraulic head, H	Fluid flux q	Permeability k	Fluid source	Darcy's law
Mass transport	Conservation of mass	Concentration, C	Diffusive flux q	Diffusion coefficient D	Chemical source	Fick's law

탄성체역학, 열전도, 다공성매질의 유체유동, 용질이동 등의 상태변수와 flux 의 구조는 비슷함 – 편리한 진실 *(a convenient truth)!*

Introduction
Constitutive Equation – T-H-M process

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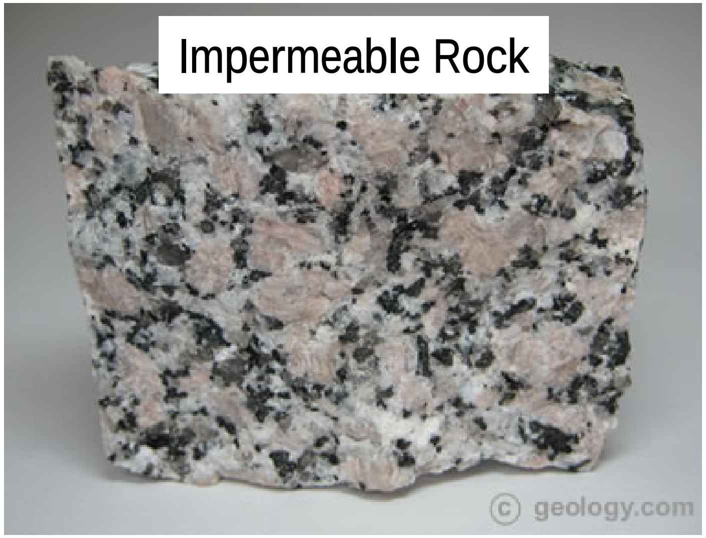
$$q = -\frac{k}{\mu}\frac{dP}{dl}$$

Heat conduction	Porous media fluid flow	Geomechanics
$q = -k\frac{dT}{dl}$	$q = -K\frac{dH}{dl}$	$\sigma = E\varepsilon = E\frac{du}{dx}$
Fourier's Law	Darcy's Law	Hooke's Law
Heat Flux	Fluid Flux	Stress (응력)
Temperature gradient	Head (or Pressure) gradient	strain (변형율)
Thermal conductivity	Hydraulic Conductivity (or Permeability)	Elastic modulus & Poisson's ratio
Conservation of Energy	Conservation of mass	Equilibrium Equation
T	H	M

Introduction

- Problem of great importance to geological/energy resources engineering;
  - Groundwater hydrology
    - groundwater migration, tunnel inflow, Contaminant transport,
  - Oil/gas extraction
    - Reservoir engineering
  - Rock/soil mechanics
    - Stability (pore pressure) of underground structure
    - Fault mechanics
  - Geothermal Energy
    - Water is a medium of heat transport

Fluid flow in rock
Introduction



Fractured Rock
(Forsmark, Sweden, 2003)

Fluid flow in rock

Constitutive Relation - Darcy's law

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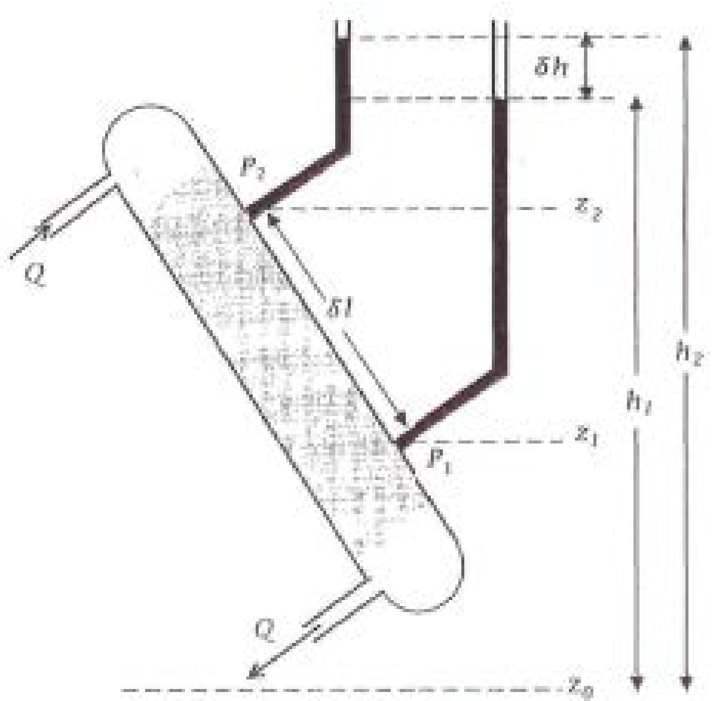

Fig. 6.1. Darcy's experiment (Hubbert's version).

- Q: volumetric flow rate (m^3/sec)
- q: volumetric flow rate per unit area (fluid flux or specific discharge) (m/sec)
- K: hydraulic conductivity (m/sec), the ease with which fluid can move through a porous rock
- h: hydraulic head (total head)

$$Q = -KA\frac{\delta h}{\delta l} = -KiA \qquad q = -K\frac{\delta h}{\delta l} = -Ki$$

$$q_x = -K_x\frac{\partial h}{\partial x} \qquad q_y = -K_y\frac{\partial h}{\partial y} \qquad q_z = -K_z\frac{\partial h}{\partial z} \qquad \mathbf{q} = -K\nabla h$$

Middleton and Wilcock, 1994

Fluid flow in rock
Constitutive Relation - Darcy's law

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- Expressed in terms of pressure and elevation head,

$$q = -K\nabla\left(z + \frac{P}{\rho_w g}\right)$$

- When all piezometers are bottomed at the same elevation

$$q = -\frac{k}{\mu}\nabla p \longleftarrow K = \frac{\rho g k}{\mu}$$

- In 1D,

$$q = -\frac{k}{\mu}\frac{dP}{dl}$$

Fluid flow in rock

Constitutive Relation - Hydraulic conductivity vs. Permeability

- K (hydraulic conductivity, 수리전도도) is related to medium & fluid, unit: m/sec

$$K = \frac{\rho g k}{\mu}$$

  – μ: viscosity (점성도) of fluid, unit: Pa·s, water: ~10^{-3} Pa·s = 1 cp

  – ρ: density of fluid, unit: kg/m^3, water: 10^3 kg/m^3

  – g: acceleration due to gravity

- k (permeability, 투수율) is related to only medium, unit: m^2

  – Perrmeability: the ease with which fluid can move through a porous rock

Fluid flow in rock
Constitutive Relation - Permeability

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Rock Type	k (m^2)	k (Darcy)	K (m/s)
Coarse gravels	10^{-9}-10^{-8}	10^{3}-10^{4}	10^{-2}-10^{-1}
Sands, gravels	10^{-12}-10^{-9}	10^{0}-10^{3}	10^{-5}-10^{-2}
Fine sands, silts	10^{-16}-10^{-12}	10^{-4}-10^{0}	10^{-9}-10^{-5}
Clays, shales	10^{-23}-10^{-16}	10^{-11}-10^{-4}	10^{-16}-10^{-9}
Dolomites	10^{-12}-10^{-10}	10^{0}-10^{2}	10^{-5}-10^{-3}
Limestones	10^{-22}-10^{-12}	10^{-10}-10^{0}	10^{-15}-10^{-5}
Sandstones	10^{-17}-10^{-11}	10^{-5}-10^{1}	10^{-10}-10^{-4}
Granites, Gneiss	10^{-20}-10^{-16}	10^{-8}-10^{-4}	10^{-13}-10^{-9}
Basalts	10^{-19}-10^{-13}	10^{-7}-10^{-1}	10^{-12}-10^{-6}

– k (permeability, 투수율) is a measure of only ‘medium’

– Also called, coefficient of permeability, intrinsic permeability

– 1 darcy = 0.987×10^{-12} m^2 ~ 10^{-12} m^2 = 10^{-5} m/sec

– 1 m/sec = 10^{-7} m^2

$$K = \frac{\rho g k}{\mu} = \frac{10^3 \times 10 \times k}{10^{-3}} = 10^7 \times k$$

– Permeability has very large variation → very important to characterize/determine its value

Jaeger et al., 2007

Fluid flow in fractured rock

- In many rock types (especially hard rocks), fractures are the main pathways of fluid flow – note that hard rocks are attractive for many applications.
- Understandings on fluid flow in fractures are essential for;
  - Underground structure (mines, tunnels and oil storages)
  - Geological repository of high level nuclear waste
  - Enhanced Geothermal System
  - Fractured Oil Reservoir

Fluid flow in rock
Fluid flow in fractured rock

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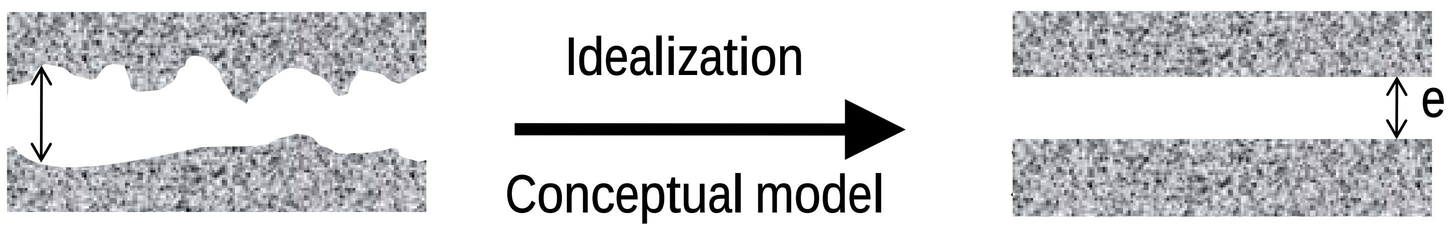
Aperture (e): size of opening measured normal to the fracture wall



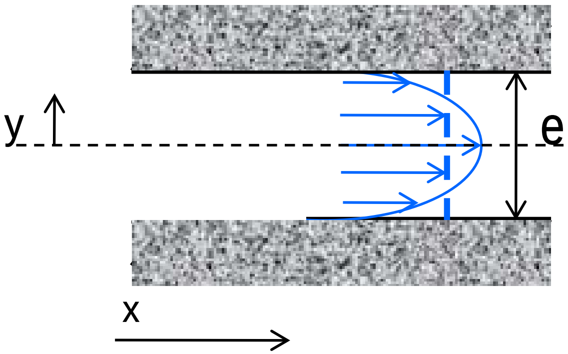
Real rock fracture

Idealized rock fracture

- Hard to estimate Q due mainly to complex geometry

-Analytical solution exist to calculate Q and velocity profile

Fluid flow in rock
Fluid flow in fractured rock - Cubic Law



Velocity (v)distribution between parallel plates

$$v = -\frac{1}{8\mu}\left(e^2 - 4y^2\right)\frac{d(\rho_w g h)}{dx}$$

$$= -\frac{1}{8\mu}\left(e^2 - 4y^2\right)\frac{dp}{dx}$$

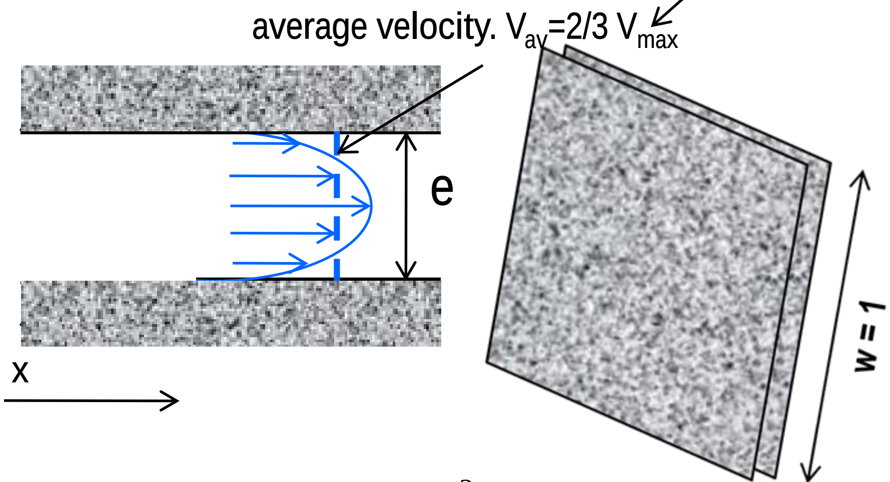
- Navier-Stokes' equation for laminar flow.
- Most of geological application involves laminar flow (low Reynolds number, <2000, de Marsily, 1986)

$$\mathrm{Re} = \frac{\rho V d}{\mu}$$

ρ: density of fluid
V: mean velocity of fluid
D: diameter of the pipe
μ: viscosity

Fluid flow in rock
Fluid flow in fractured rock - Cubic Law

$$v = -\frac{1}{8\mu}\left(e^2 - 4y^2\right)\frac{d(\rho_w gh)}{dx}$$



$$Q = \int_{-\frac{e}{2}}^{\frac{e}{2}} v(wdy) = -\frac{we^3}{12\mu}\frac{d}{dx}(\rho_w gh)$$

$$Q = -\frac{e^3}{12\mu}\frac{d}{dx}(\rho_w gh) = -\frac{\rho_w g e^3}{12\mu}\frac{dh}{dx}$$

Hydraulic conductivity (K) of parallel plate model

$$q = Q/A = Q/ew = -\frac{\rho_w g e^2}{12\mu}\frac{\partial h}{\partial x}$$

$$Q = -\frac{\rho_w g e^3}{12\mu}\frac{\partial h}{\partial x}$$

$$Q = -\frac{e^3}{12\mu}\frac{\partial p}{\partial x}$$ ← with zero elevation

ρ_w: density of fluid
g: acceleration of gravity
μ: viscosity

– Cubic law: for a given gradient in head and unit width (w), flow rate through a fracture is proportional to the <u>cube</u> of the fracture aperture.

Fluid flow in rock

Fluid flow in fractured rock - Equivalent permeability

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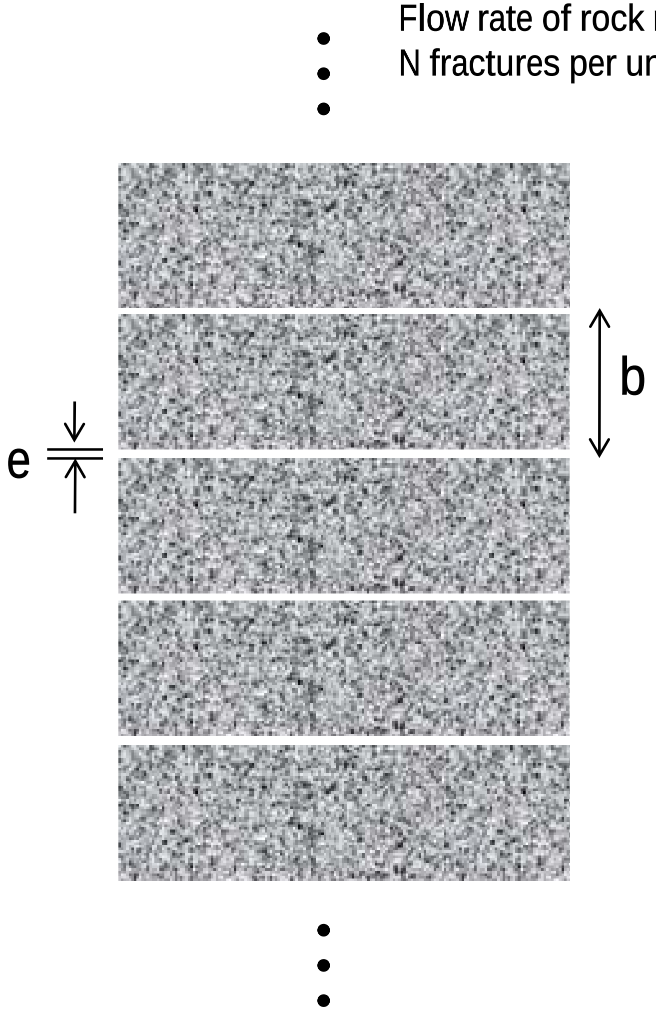


Flow rate of rock mass with N fractures per unit length

Equivalent hydraulic conductivity (K) of multiple parallel plate models

$$Q = -\frac{\rho_w g e^3}{12\mu}\frac{\partial h}{\partial x} \times N = -\frac{\rho_w g e^3}{12\mu}\frac{1}{b} bN \frac{\partial h}{\partial x}$$

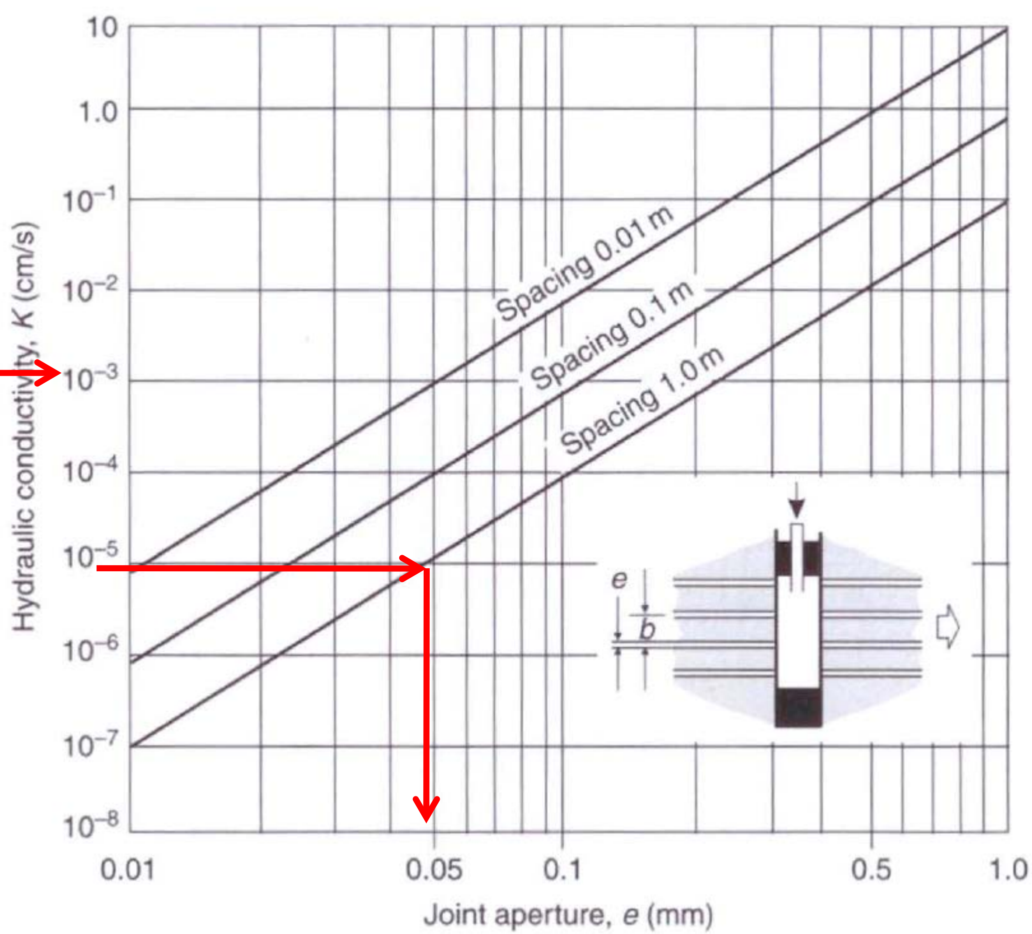
$$K = \frac{\rho_w g N e^3}{12\mu} = \frac{\rho_w g e^3}{12\mu b} \qquad k = \frac{Ne^3}{12} = \frac{e^3}{12b}$$

N = 1/b

- K: hydraulic conductivity
- k: permeability
- e: aperture
- N: number of fracture per unit distance = frequency, (L^{-1})
- b: spacing (L)

Fluid flow in rock
Fluid flow in fractured rock - Equivalent permeability

10^{-5} m/s
$= 10^{-12}$ m^2
= 1 D



$$K = \frac{\rho_w g e^3}{12 \mu b}$$

A sandstone with K of 10^{-5} cm/s (which is 10^{-7} m/s ~10^{-14} m^2~10^{-2}Darcy~10mD) correspond to aperture 50μm in 1 m interval.

Influence of fracture aperture e and spacing b on hydraulic conductivity K in the direction of a set of smooth parallel fractures in a rock mass (Hoek et al., 2004)

Fluid flow in rock
Governing equations

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– Diffusion equation for fluid flow in porous media

$$\frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right)+\dot{q}=S_s\frac{\partial h}{\partial t}$$

$$\frac{k}{\mu}\left[\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}\right]=S\frac{\partial p}{\partial t}$$

– Heat diffusion equation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

Fluid flow in rock
Implication for mechanics

- Effective stress
- Subsidence (or heaving)
- Stress dependent permeability

Fluid flow in rock
Implication for mechanics: Effective stress

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- Mechanical behavior of saturated reservoir will be controlled by the effective stress (Terzaghi, 1923).

$$\sigma' = \sigma - p$$

$$\sigma'_x = \sigma_x - p \qquad \sigma'_y = \sigma_y - p \qquad \sigma'_z = \sigma_z - p$$

$$\tau'_{xy} = \tau_{xy} \qquad \tau'_{yz} = \tau_{yz} \qquad \tau'_{zx} = \tau_{zx}$$

  – Principal assumptions:
    - Interconnected pore system uniformly saturated with fluid
    - Total volume of pore system is small compared to the volume of the rock as a whole
    - We consider;
      - ☑ Pressure in the pores
      - ☑ The total stress acting on the rock externally
      - ☑ The stresses acting on individual grains (in terms of statistically averaged uniform values)

Fluid flow in rock
Implication for mechanics: Effective stress

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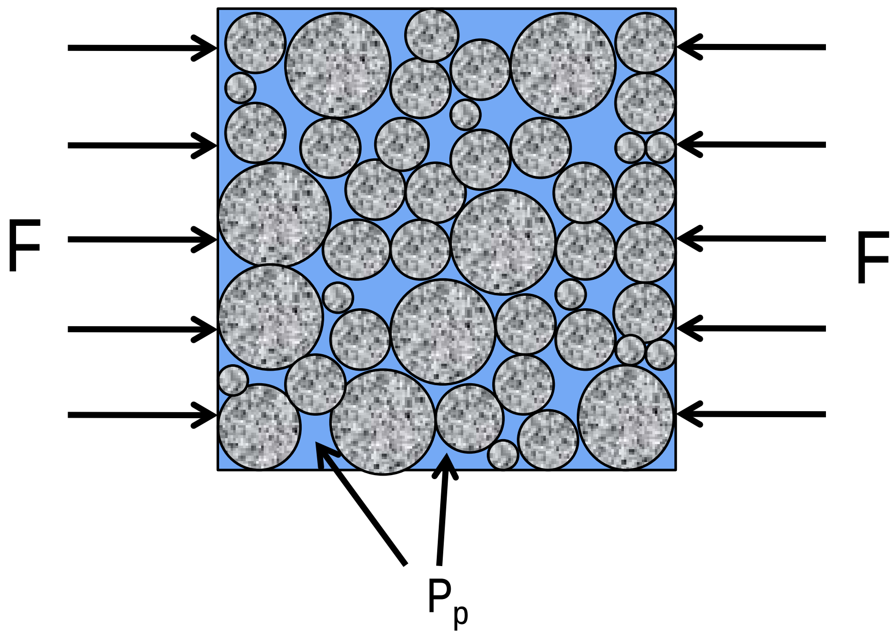

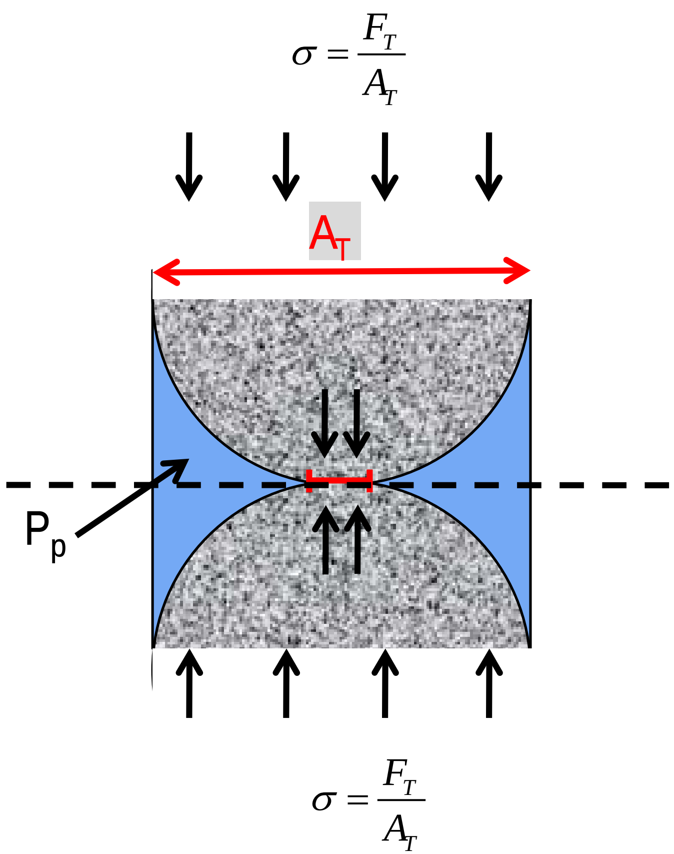

- A_c: contact area of grain
- A_T: diameter (area) of grain
- p_p: pore pressure

$$\sigma' = \sigma - p_p$$

Fluid flow in rock
Implication for mechanics: Effective stress

– Effective stress

$$\sigma' = \sigma - p$$

– Exact effective stress law (more general)

$$\sigma' = \sigma - \alpha p$$

$$\alpha = 1 - \frac{K}{K_s}$$

– α: Biot coefficient ($0 < \alpha < 1$)

– K: bulk modulus of rock

– Ks: bulk modulus of individual grain

– For nearly solid rock with no interconnected pores (such as quartzite): $\alpha = 0$

– For highly porous rock (such as uncemented sands): $\alpha = 1$

Fluid flow in rock
Implication for mechanics: Effective stress

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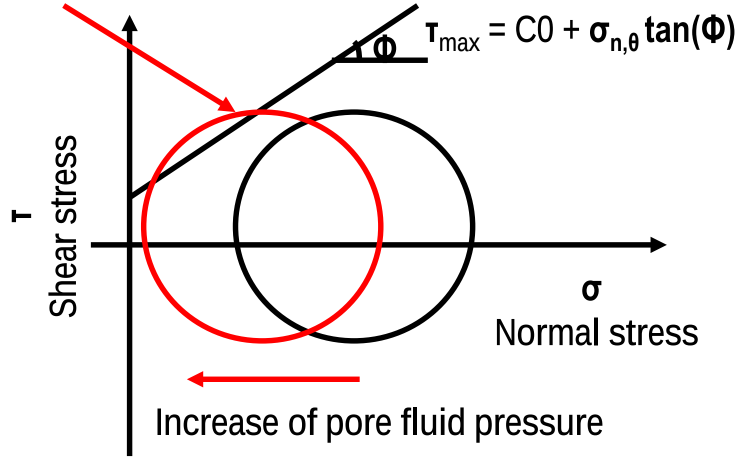
- Physically, this means that the solid framework carries the part σ' of the total external stress σ while the remaining part αp is carried by the fluid.
- Two important mechanism explained by the concept of effective stress
  - Deformation due to the change of pore pressure – subsidence and heaving of rock
  - Rock or fracture failure due to the increased pore pressure

Fluid flow in rock
Implication for mechanics: Effective stress



- Increase of pore pressure induce failure of intact rock

Fluid flow in rock

Implication for mechanics – stress dependent permeability

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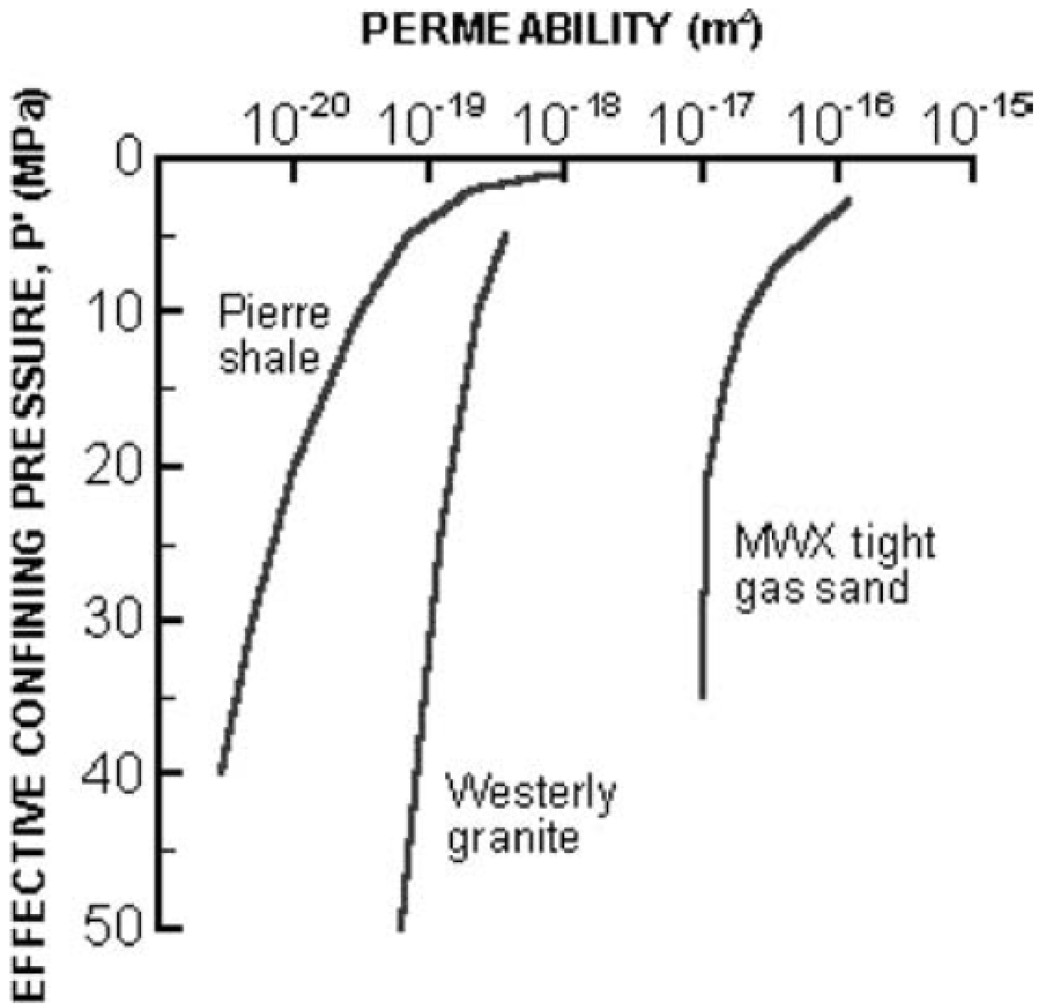
- Permeability with stress (depth)



Rutqvist, J., O. Stephansson. The Role of Hydromechanical Coupling in Fractured Rock Engineering. *Hydrogeology Journal 11(1) 2003: 7-40.*

Implication for mechanics – subsidence(or heaving)

- Reservoir compaction and associated surface subsidence – best-known example of geomechanical effect in reservoir scale

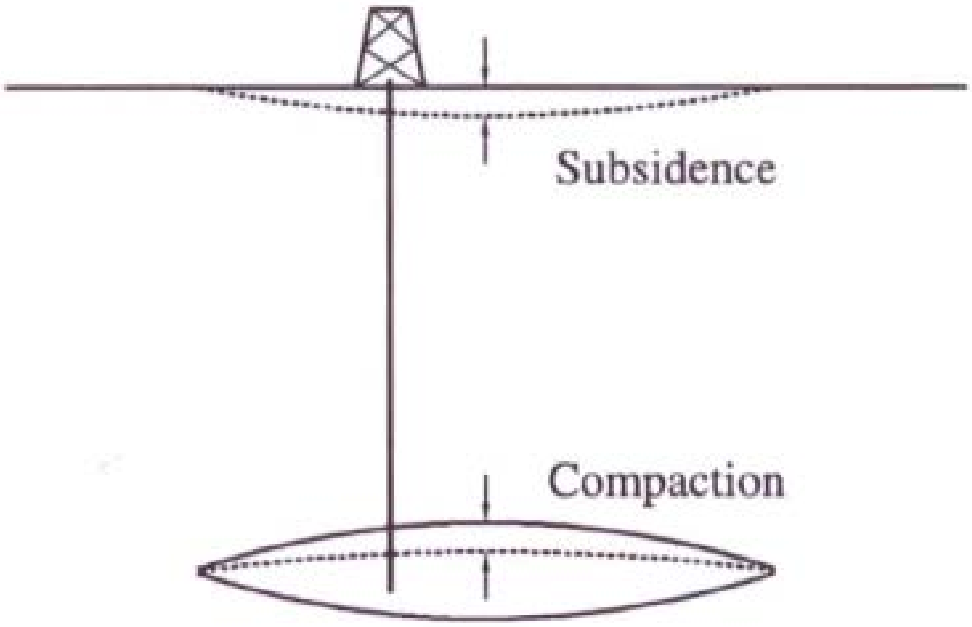


Fig. 12.1. Compaction and subsidence.

Fjaer et al., 2008

Fluid flow in rock
Implication for mechanics – subsidence(or heaving)

- Most reservoir will experience some degree of subsidence.
- For a considerable degree of subsidence;
  - Reservoir pressure drop must be significant (pressure maintenance such as injection may counteract compaction)
  - The reservoir must be highly compressible → More important in soft rock.
  - The reservoir must have a considerable thickness
  - No shielding by the overburden rock

Fluid flow in rock

Implication for mechanics – subsidence(or heaving)

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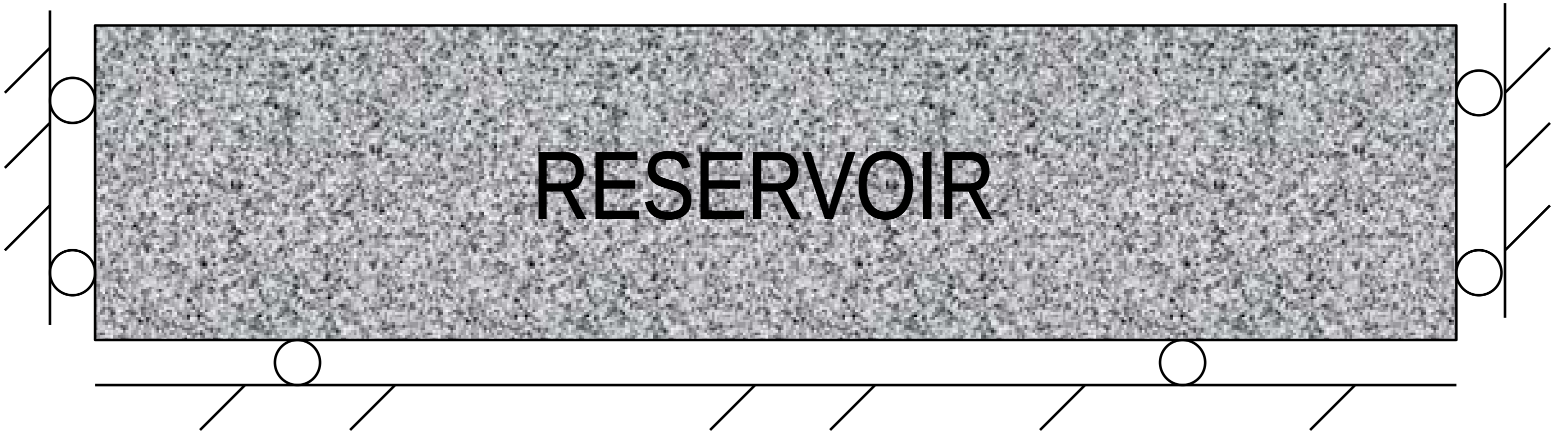
- Uniaxial compaction model
- Compaction coefficient or uniaxial compressibility, Cm;

$$\frac{\Delta h}{h} = -C_m \alpha \Delta p = -\frac{1}{E}\frac{(1+\nu)(1-2\nu)}{1-\nu}\alpha \Delta p$$



Heat transfer in rock
Introduction

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- Heat transfer is thermal energy in transit due to a temperature difference – 열전달은 열에너지가 온도차에 의해 이동하는 현상
- Relevance to Energy Resources and Geo-environmental Engineering
  - Geothermal Energy (100 ~ >300°C)
  - Deep Drilling (25 °C /km)
  - Geological disposal of nuclear waste (up to 100°C)
  - LNG underground storage cavern (< -162°C)

Heat transfer in rock
Introduction

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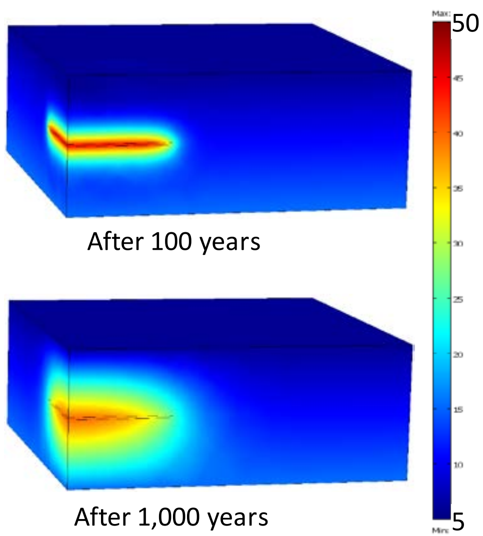
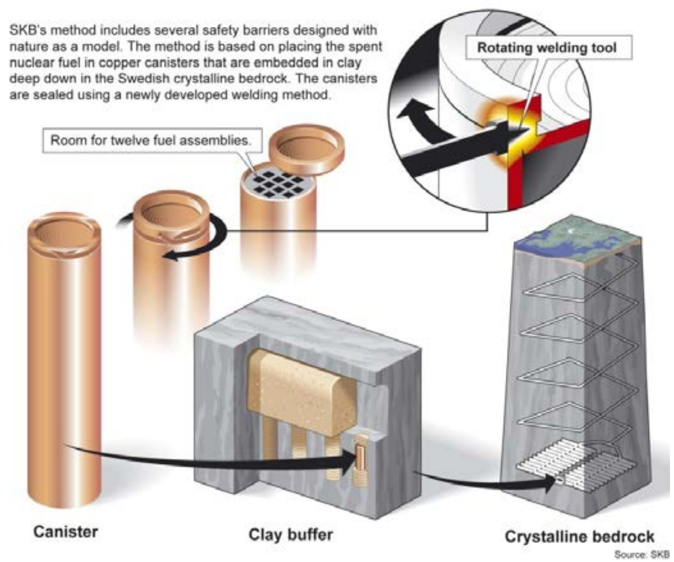
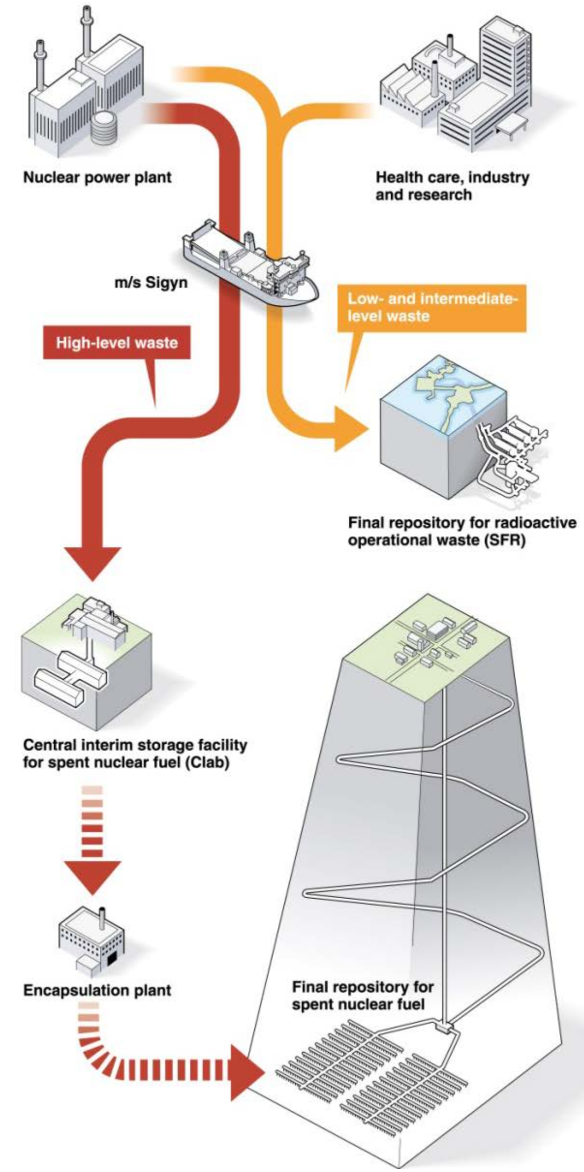
- Geological Disposal of High-Level Radioactive Waste



After 100 years

After 1,000 years





Min & Stephansson, 2009

SKB, 2009, www.skb.se

Heat transfer in rock
Introduction

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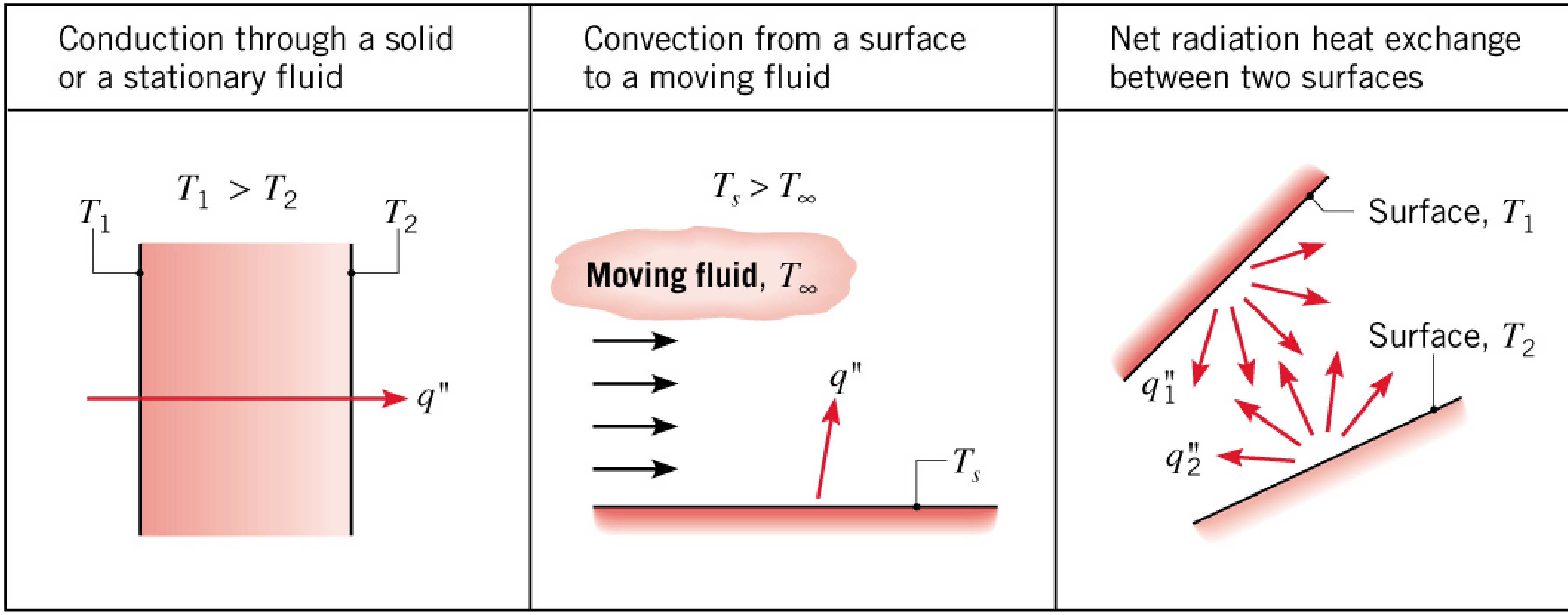
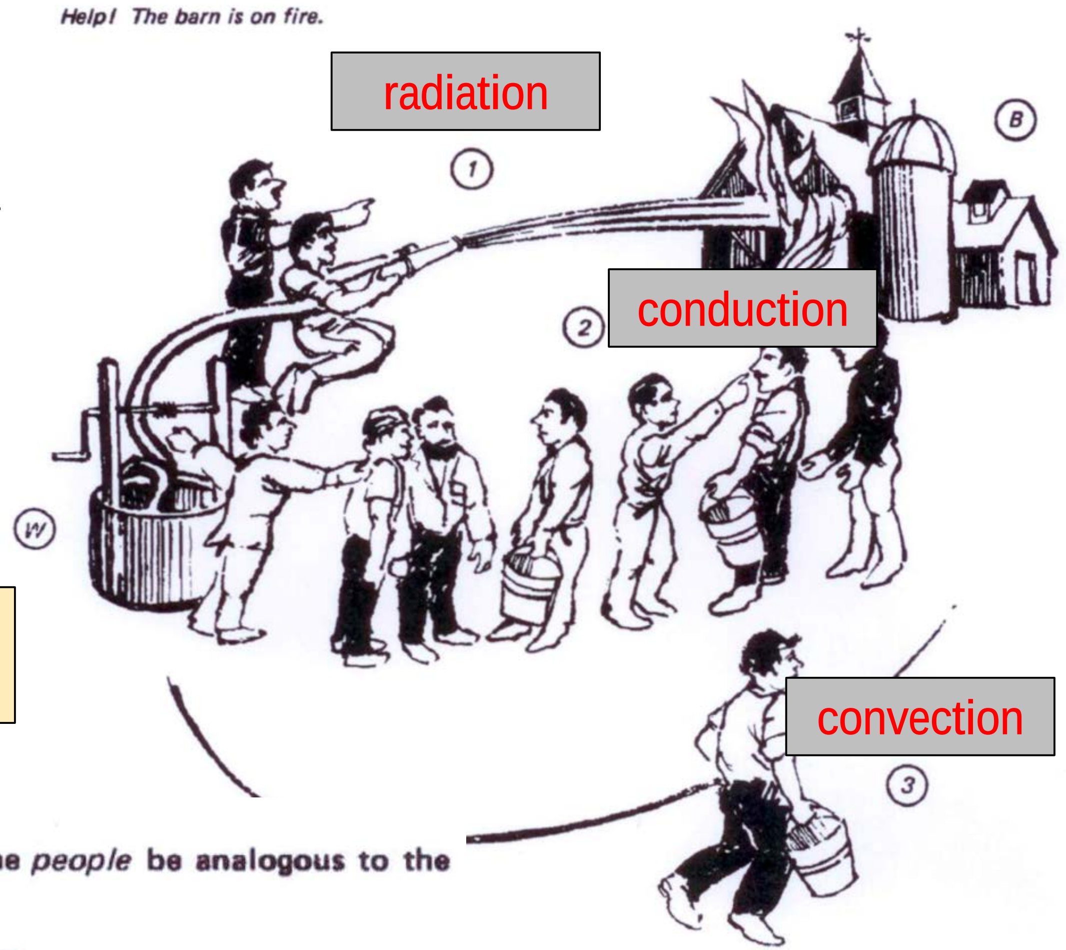


Conduction (전도) Convection (대류) Radiation (복사)

Incropera, 2007

Heat transfer in rock
Introduction



Water → Heat
People → Heat Transfer Medium

Let the *water* be analogous to *heat*, and let the *people* be analogous to the *heat transfer medium*. Then:

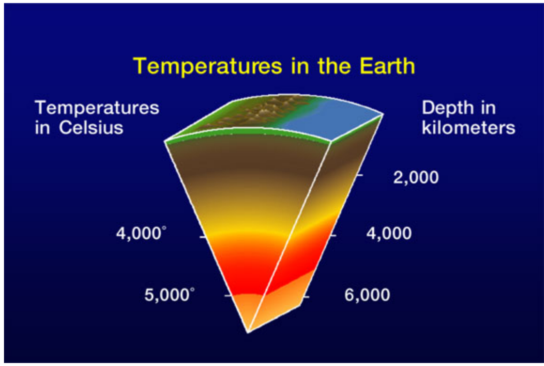
Case 1 The hose directs water from (W) to (B) independently of the medium. This is analogous to *thermal radiation* in a vacuum or in most gases.

Case 2 In the bucket brigade, water goes from (W) to (B) through the medium. This is analogous to *conduction*.

Case 3 A single runner, representing the medium, carries water from (W) to (B). This is analogous to *convection*.

Lienhard JH, 2008, A Heat Transfer Textbook

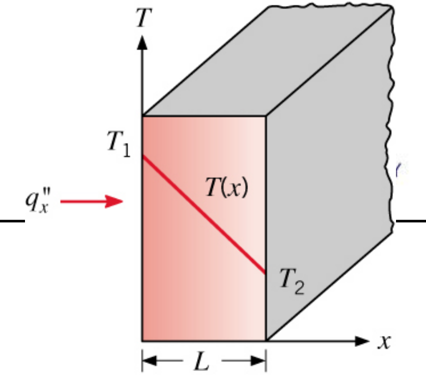
Heat transfer in rock
Introduction



http://geothermal.marin.org

- Geothermal Gradient: the rate at which earth increases with depth, typically: 25 °C/km
- Two mechanism:
  - The interior is hot (the center is ~6000°C);
  - Decay of long-lived radioactive isotopes:
    - Th (Thorium 232), U (Uranium 238), K (potassium 40).
    - Concentrated in upper crystal rock.
    - Account for about 80% of the surface heat flow
- Surface heat flow
  - Global means: 87 mW/m^2 (Pollack et al., 1993), In boundaries between plates: 300 mW/m^2

Heat transfer in rock
Constitutive Equation - Heat conduction



- Fourier's Law
  - The heat flux, q'' (W/m^2) resulting from thermal conduction is proportional to the magnitude of the temperature gradient and opposite to it in sign.

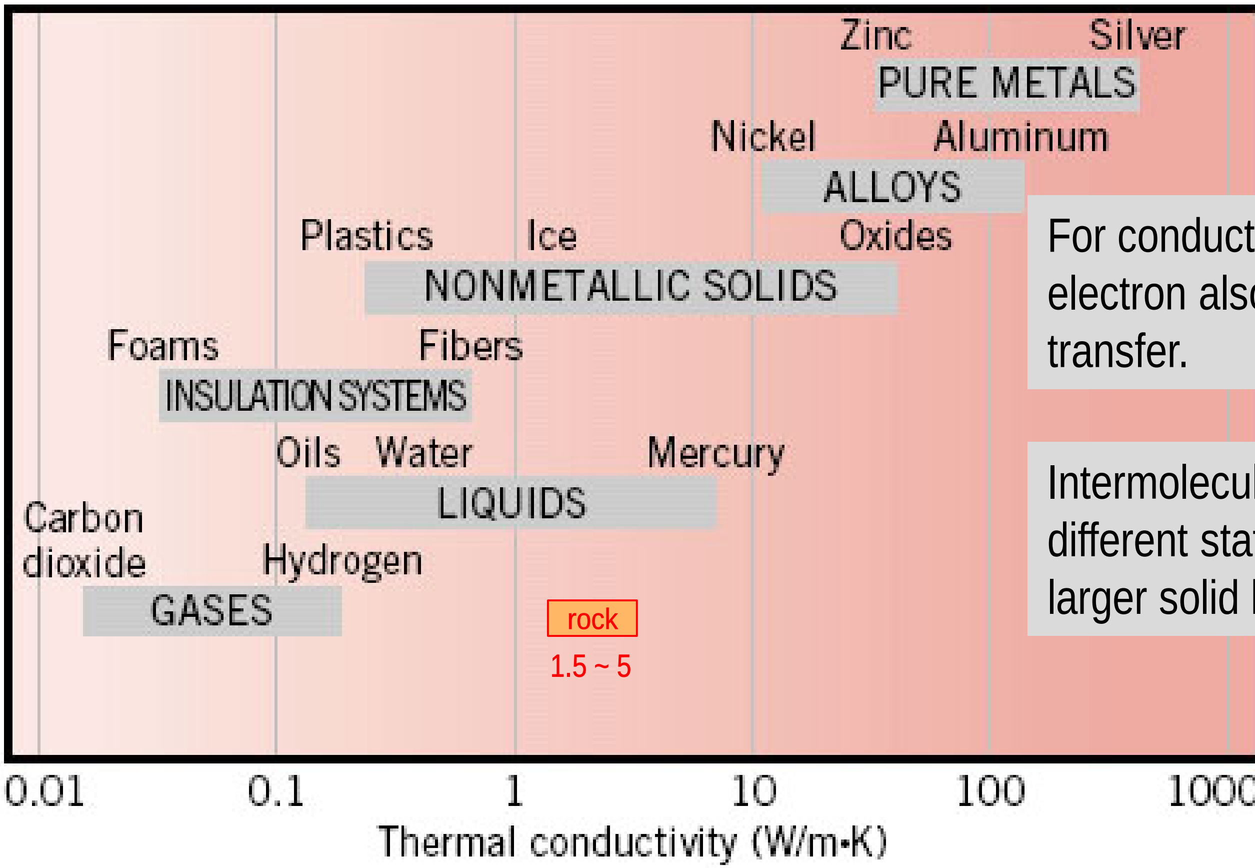
$$q''_x = -k\frac{dT}{dx} \qquad \frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

  - q'' : heat flux (W/m^2), rate of heat transfer per unit area (in the x direction, perpendicular to the direction of transfer)
  - k: thermal conductivity (W/m·K)
  - q: heat rate (W): q = q''·A

Incropera, 2007

Heat transfer in rock
Thermal Properties - Thermal conductivity

- Typical values



For conductor, motion of the free electron also contribute to energy transfer.

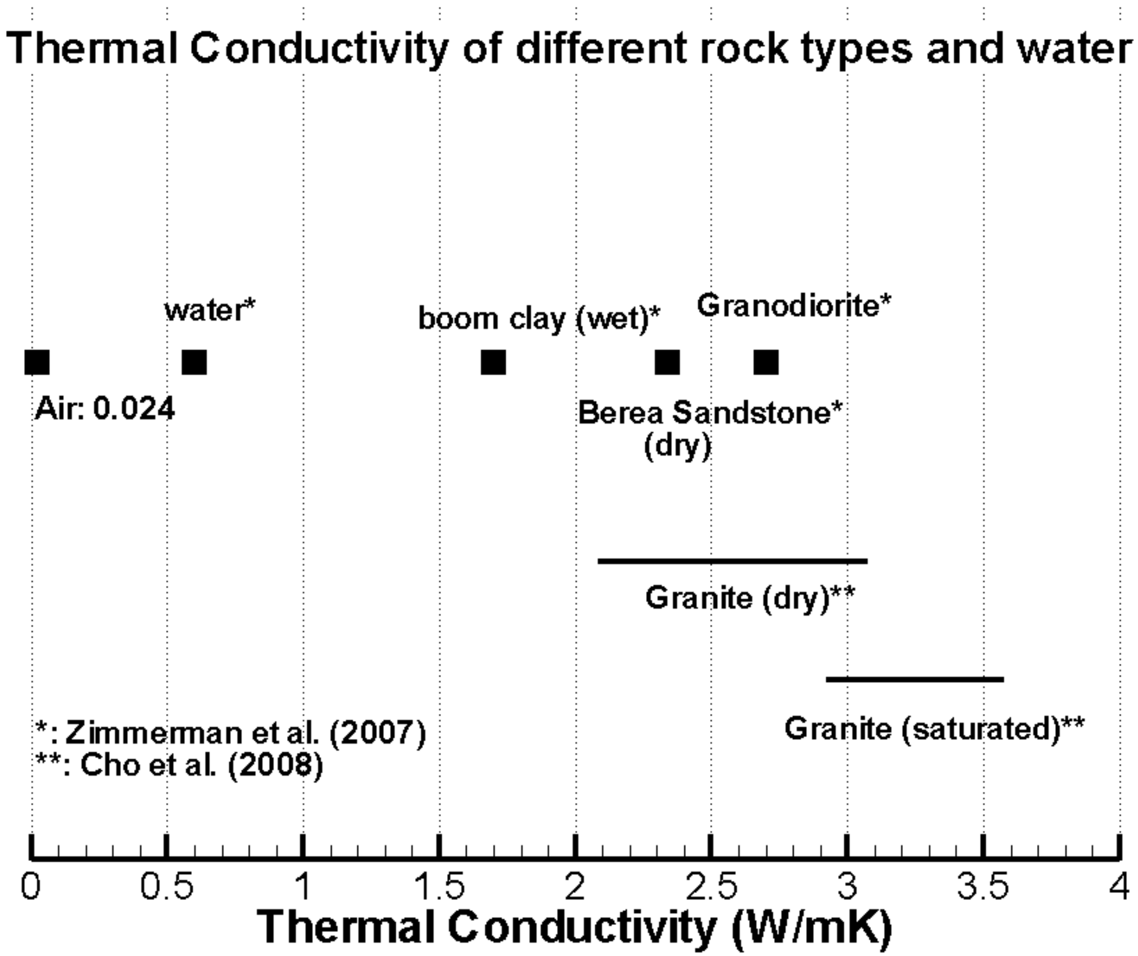
Intermolecular spacing for the different states can explain the larger solid k than liquid.

FIGURE 2.4 Range of thermal conductivity for various states of matter at normal temperatures and pressure.

Incropera, 2007

Heat transfer in rock

Thermal Properties - Thermal conductivity: water, air and Rock



Thermal Properties: Specific heat

- Specific heat (capacity): the measure of the heat energy required to increase the temperature of a unit mass of a substance by a 1°C – ability to store thermal energy.

$$Q = mc_p \Delta T$$

  - Q: heat (J)
  - m: mass (kg)
  - c_p: specific heat capacity (J/kg·K)
  - ΔT: temperature difference

- Volumetric heat capacity =
  - Defined on a volume ρc_p $unit:(J/m^3 \cdot K)$

Heat transfer in rock

Thermal Properties - Thermal diffusivity

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- Thermal diffusivity: the ratio of thermal conductivity to the volumetric heat capacity

$$\alpha = \frac{k}{\rho c_p}$$

  - Materials of large *α* will respond quickly to changes in their thermal environment, while materials of small *α* will respond more sluggishly, taking longer to reach equilibrium.
  - e.g., granite from Forsmark, Sweden,
    - k = 3.58 W/mK, ρ: 2600 kg/m^3, cp: 796 (J/kg·K) → $\alpha = 1.7 \times 10^{-6}$ m^2/sec

Thermal conductivity measurement

- Two groups of methods (Beardsmore & Cull, 2001)
  - Steady-state method
    - Divided-bar apparatus
  - Transient method
    - Needle probe

Heat transfer in rock

Thermal conductivity measurement - Steady State Method

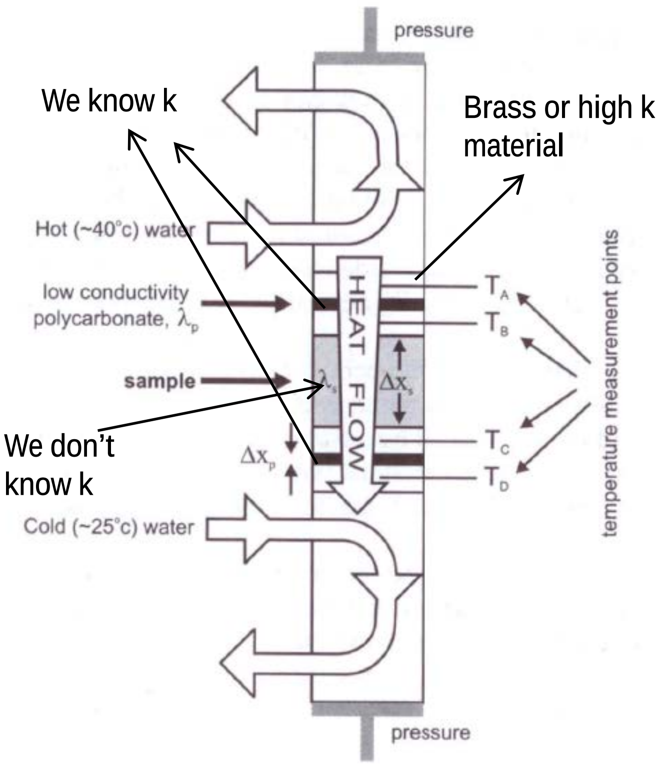

Figure 4.6. A typical divided-bar apparatus.

$$q''_x = k\frac{T_2 - T_1}{L}$$

Beardsmore and Cull, 2001

- Use a 'divided-bar apparatus'
- Measure the 'k' directly
- There are two types of samples: two standard conductivity samples and a sample with unknown k.
- Heat rate is measured from the temperature measurement of known samples.
- Takes long time to achieve thermal equilibrium
- More accurate than 'transient method'
- Rock sample in discs or cylindrical shape with 2-4 cm in diameter
- Top and bottom sections of the bar maintained at constant but different temperatures (warm end at the top! Why?).

Thermal conductivity measurement - Steady State Method

- Three assumptions;
  - Heat conduction along the bar is assumed to be 100% efficient ← no loss of heat through the side
  - Temperature drop across the brass section is negligible compared with the temperature drop in sample.
    - Thermal resistance/Area of brass: 10^{-4} m^2K/W, sample 10^{-2} m^2K/W
  - Two standard conductivity discs must be identical in thickness and thermal conductivity

Heat transfer in rock

Thermal conductivity measurement - Steady State Method

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$\Delta T_s = T_B - T_C$ = temperature drop across the sample

$\Delta T_1 = T_A - T_B$ = temperature drop across the top polycarbonate

$\Delta T_2 = T_C - T_D$ = temperature drop across the bottom polycarbonate

$\Delta x_s, k_s$ = thickness(m), thermal conductivity of sample

$\Delta x_p, k_p$ = thickness(m), thermal conductivity of polycarbonate

q'' = heat flux along bar

$$q'' = q''_{\text{top polycarbonate}} = q''_{\text{sample}} = q''_{\text{bottom polycarbonate}}$$

$$q'' = k_p \frac{\Delta T_1}{\Delta x_p} = k_s \frac{\Delta T_s}{\Delta x_s} = k_p \frac{\Delta T_2}{\Delta x_p}$$

To help compensate for side loss of heat, heat flux across the top and bottom polycarbonate discs is averaged.

$$0.5 \times k_p \frac{\Delta T_1 + \Delta T_2}{\Delta x_p} = k_s \frac{\Delta T_s}{\Delta x_s}$$

Finally, thermal conductivity can be calculated.

$$k_s = \frac{\Delta T_1 + \Delta T_2}{\Delta T_s} \Delta x_s \frac{k_p}{2\Delta x_p}$$

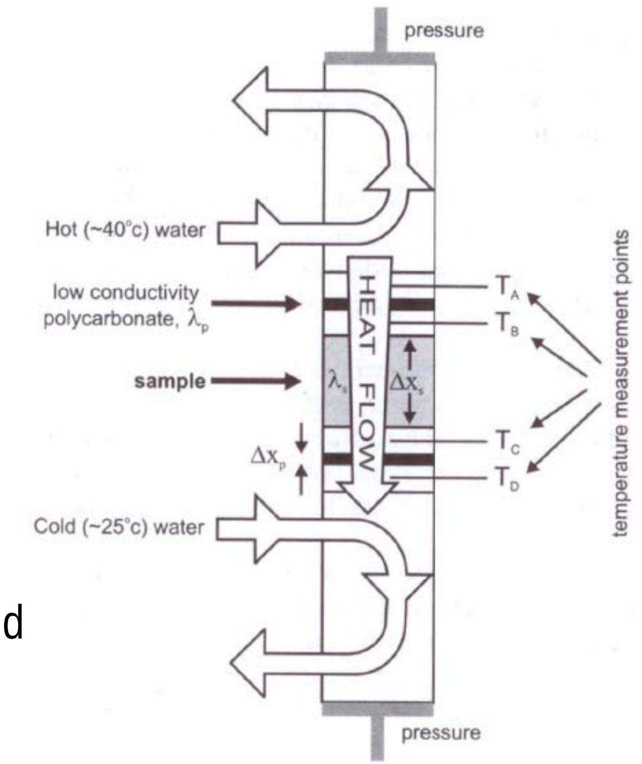
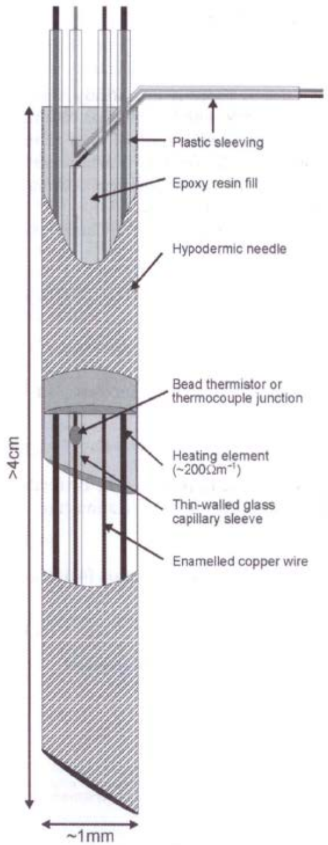


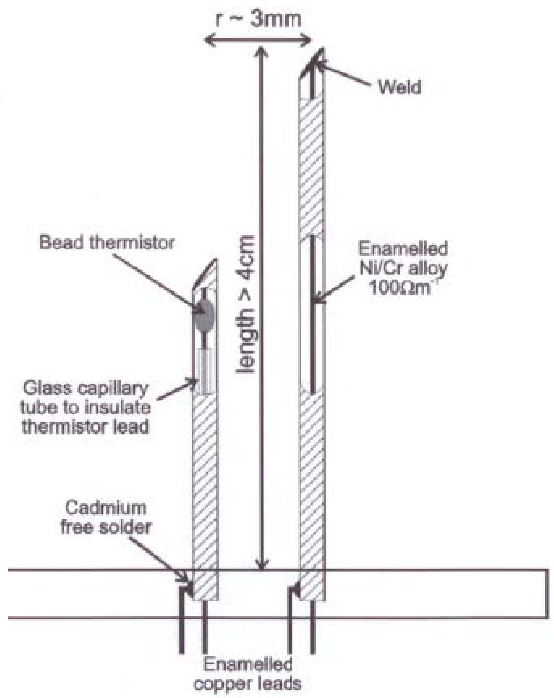
Figure 4.6. A typical divided-bar apparatus.

We assume thermal resistance at the contacts between the bar and the sample is negligible.
Usually, experiment with known sample is involved but it is omitted in this equation for simplicity

Heat transfer in rock

Thermal conductivity measurement - Transient Method





re 4.10. Construction details for a dual-needle probe.

Single needle probe (line source)

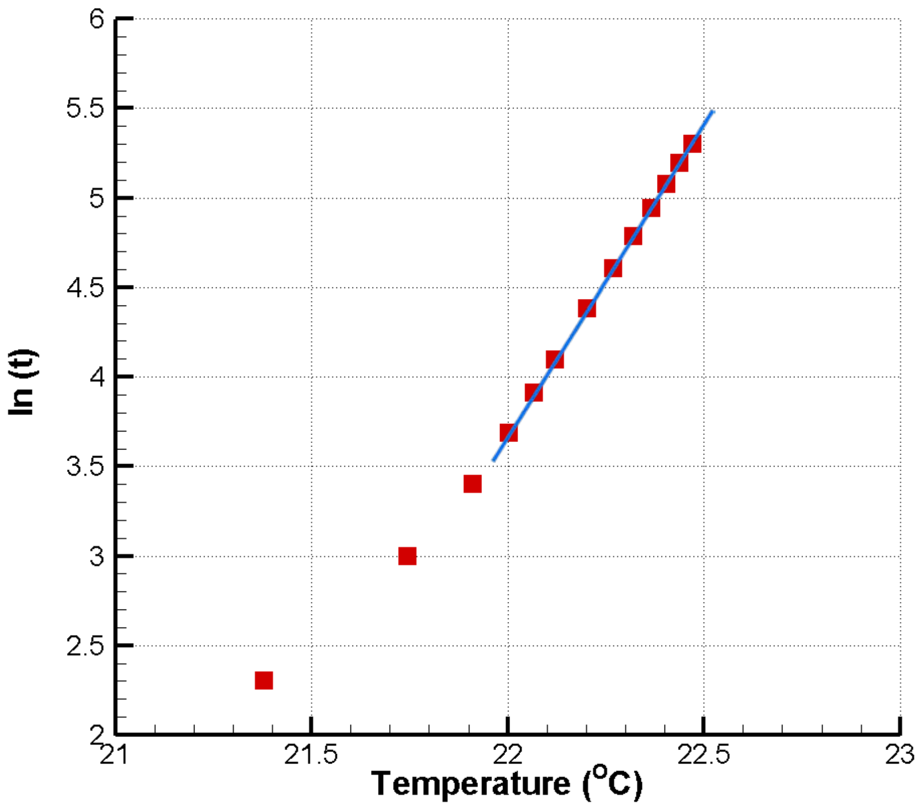
Dual-needle probe

Beardsmore and Cull, 2001

- k can be deducted from the rate at which its T changed in response to an applied heat source.
- Suitable for poorly consolidated sediment or in situ measurement
- "Needle probe method" is the best known method.
- Less accurate than 'steady state' method
- Devices can vary in shape and position of heat source in relation to the temperature measurement point

Heat transfer in rock

Thermal conductivity measurement - Transient Method



– With a line source of heat and a temperature sensor packed closely,

$$k = \left(Q_l / 4\pi\right)\left(\partial \ln(t) / \partial T\right)$$

– Find a linearity and obtain k

P= V x I = 5 x 0.25 = 1.25W
Q=P/0.1(cm)=12.5W/m
Gradient between 60 & 200 sec is ~3.449
K = 12.5/4π x 3.449 = 3.43 W/mK

Data from Beardsmore and Cull, 2001

– Diffusion equation for fluid flow in porous media

$$\frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right)+\dot{q}=S_s\frac{\partial h}{\partial t}$$

$$\frac{k}{\mu}\left[\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}\right]=S\frac{\partial p}{\partial t}$$

– Heat diffusion equation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

Implication to mechanical behavior

- Linear thermal expansion coefficient (unit: /K)

$$\frac{\Delta l}{l} = \alpha\left(T - T_0\right)$$

- Thermal stress ← thermal expansion + mechanical restraint
  - Thermal stress in 1D

$$\sigma_T = \alpha E\left(T - T_0\right)$$

  - Thermal stress when a rock is completely (in all directions) restrained

$$\sigma_T = 3\alpha K\left(T - T_0\right) = \frac{E}{1-2\nu}\alpha\left(T - T_0\right)$$