Chapter 25
All-Pairs Shortest Paths

Introduction to Data Structures

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Outline

- In this chapter, we consider the problem of finding shortest paths between all pairs of vertices in a graph.
- We first present a dynamic-programming algorithm with $\Theta(|V|^3 \lg |V|)$ time based on matrix multiplication to solve the all-pairs shortest-paths problem.
- We next give another dynamic-programming algorithm called the Floyd-Warshall algorithm which runs in $\Theta(|V|^3)$ time.
- We finally cover Johnson's algorithm which solves the allpairs shortest-paths problem in $\Theta(|V|^2 \lg |V| + |V||E|)$ time.

Problem Definition

- Given
  - A directed graph $G=(V,E)$
    - Assume that $v_1, \ldots, v_n$ in V are numbered 1, ...,|V|.
  - A weight function $w: E \rightarrow R$
    - Represented as an adjacency matrix $W=(w_{ij})$
      - $w_{ij} = 0$ if $i = j$
      - $w_{ij} = w(i,j)$ if $i \neq j$ and $(i,j) \in E$
      - $w_{ij} = \infty$ if $i \neq j$ and $(i,j) \notin E$
- For every pair of vertices $i, j \in V$, find a shortest path from i to j.
  - The output of the all-pairs shortest-paths algorithms is an n x n matrix $D = (d_{ij})$, where entry d_{ij} contains the weight of a shortest path from vertex i to vertex j.

A Predecessor Matrix

- $\Pi = (\pi_{ij})$, where
  - π_{ij} is NIL if either i = j or there is no path from i to j.
  - Otherwise, π_{ij} is the predecessor of j on some shortest path from i.
  - The subgraph induced by the i-th row of the Π matrix should be a shortest-path tree with root i.
- The predecessor subgraph of G for i as $G_{\pi,i} = (V_{\pi,i}, E_{\pi,i})$ where
  - $V_{\pi,i}=\{j \in V : \pi_{ij} \neq \text{NIL}\} \cup \{i\}$
  - $E_{\pi,i}=\{(\pi_{ij}, j) : j \in V_{\pi,i} - \{i\}\}$

A Shortest-paths

- If $G_{\pi,i}$ is a shortest-paths tree, then the following procedure prints a shortest path from vertex i to vertex j.

PRINT-ALL-PAIRS-SHORTEST-PATH(Π, i, j)

if i == j
    print i
elseif π_{ij} == NIL
    print "no path from" i "to" j "exists"
else PRINT-ALL-PAIRS-SHORTEST-PATH(Π, i, π_{ij})
    print j

Thomas T. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Lemma 24.1 (recap)

- All subpaths of shortest paths are shortest paths
  - Let $p=<v_1,v_2,\ldots,v_k>$ be a shortest path from vertex v_1 to vertex v_k
  - For any i and j such that $1 \leq i \leq j \leq k$, let $p_{ij}=<v_i,v_{i+1},\ldots,v_j>$ be the subpath of p from vertex v_i to vertex v_j
  - Assume that there is a path p'_{ij} with weight $w(p'_{ij}) < w(p_{ij})$
  - Then $w(p)=w(p_{1i})+w(p_{ij})+w(p_{jk}) > w(p_{1i})+w(p'_{ij})+w(p_{jk})$, which is a contradiction.

The Structure of a Shortest Path

- Let p be a shortest path from vertex i to vertex j with at most m edges.
- If i = j
  - The path p has weight 0 and no edge.
- If i ≠ j
  - We can decompose p into $i \leadsto k \rightarrow j$ ($i \leadsto k$: p′) where p′ has at most m−1 edges.
- By Lemma 24.1, the path p′ is a shortest path from vertex i to vertex k.
  - $\delta(i,j) = \delta(i,k) + w_{kj}$

A Dynamic Programming Algorithm Based On Matrix Multiplication

A Dynamic Programming Algorithm

- Let $\ell_{ij}^{(m)}$ be the minimum weight of any path from vertex i to vertex j that contains at most m edges.
- When $m = 0$, there is a shortest path from i to j with no edges if and only if i = j. Thus,
  - $\ell_{ij}^{(0)} = \begin{cases} 0 & \text{if } i = j \\ \infty & \text{if } i \neq j \end{cases}$

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

A Dynamic Programming Algorithm

- When $m \geq 1$, we compute $\ell_{ij}^{(m)}$ as the minimum of
  - $\ell_{ij}^{(m-1)}$ (the weight of a shortest path from vertex i to vertex j consisting of at most m-1 edges) and
  - the minimum weight of any path from vertex i to vertex j consisting of at most m edges, obtained by looking at all possible predecessors k of vertex j.
- Thus,
  - $$\ell_{ij}^{(m)} = \min(\ell_{ij}^{(m-1)}, \min_{1 \le k \le n}\{\ell_{ik}^{(m-1)} + w_{kj}\})$$
    $$= \min_{1 \le k \le n}\{\ell_{ik}^{(m-1)} + w_{kj}\}$$

A Recursive Solution

- Let $\ell_{ij}^{(m)}$ be the minimum weight of any path from vertex i to vertex j that contains at most m edges.
- When $m = 0$,
  - $\ell_{ij}^{(0)} = \begin{cases} 0 & \text{if } \; i = j \\ \infty & \text{if } \; i \neq j \end{cases}$
- When $m \geq 1$,
  - $\ell_{ij}^{(m)} = \min(\ell_{ij}^{(m-1)}, \min_{1 \leq k \leq n} \{\ell_{ik}^{(m-1)} + w_{kj}\})$

    $= \min_{1 \leq k \leq n} \{\ell_{ik}^{(m-1)} + w_{kj}\}$ (since $w_{jj} = 0$ for all j)

Actual Shortest-path Weights $\delta(i, j)$

- If the graph contains no negative-weight cycles, then for every pair of vertices i and j for which $\delta(i, j) < \infty$, there is a shortest path from i to j that is simple and thus contains at most n−1 edges.
- A path from vertex i to vertex j with more than n−1 edges cannot have lower weight than a shortest path from i to j.

Computing the Shortest-path Weights Bottom Up

- Take the matrix $W=(w_{ij})$ as input.
- Compute a series of matrices $L^{(1)}$, $L^{(2)}$, …, $L^{(n-1)}$ where $L^{(m)} = (\ell_{ij}^{(m)})$ for $m=1,2,\cdots,n-1$.
  - $L^{(1)} = W$.
  - $L^{(m)} = (\ell_{ij}^{(m)})$ is a matrix of weights of shortest-paths with at most m edges.
  - $L^{(n-1)}$ = contains the actual shortest-path weights.
- The heart of the algorithm is the following procedure, which, given matrices $L^{(m-1)}$ and W , returns the matrix $L^{(m)}$.

Computing the Shortest-path Weights Bottom Up

- Pseudocode for computing $L^{(m)}$ from $L^{(m-1)}$ and W with $\Theta(n^3)$ time

```
EXTEND-SHORTEST-PATHS(L, W)
n = L.rows
Let L' = (ℓ_ij' ) be a new n×n matrix
for i=1 to n
    for j=1 to n
        ℓ_ij' = ∞
        for k=1 to n
            ℓ_ij' = min(ℓ_ij' , ℓ_ik+w_kj)
return L'
```

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Relation to Matrix Multiplication

```
EXTEND-SHORTEST-PATHS(L, W)
n = L.rows
Let L′ = (ℓij′) be a new n×n matrix
for i=1 to n
    for j=1 to n
        ℓij′ = ∞
        for k=1 to n
            ℓij′ = min(ℓij′, ℓik+wkj)
return L′
```

Extend-Shortest-Paths can be represented as $L^{(m)} = L^{(m-1)} \cdot W$

$\ell_{ik}^{(m-1)} \rightarrow a_{ik}$

$w_{kj} \rightarrow b_{kj}$

$\ell_{ij}^{(m)} \rightarrow c_{ij}$

$\min \rightarrow +$

$+ \rightarrow \cdot$

```
SQUARE-MATRIX-MULTIPLY(A,B)
n = A.rows
Let C be a new n×n matrix
for i=1 to n
    for j=1 to n
        cij = 0
        for k=1 to n
            cij = cij+aik·bkj
return C
```

Matrix multiplication is represented as $C = A \cdot B$

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Relationship to Matrix Multiplication

```
EXTEND-SHORTEST-PATHS(L, W)
    n = L.rows
    Let L′ = (ℓij′) be a new n×n matrix
    for i=1 to n
        for j=1 to n
            ℓij′ = ∞
            for k=1 to n
                ℓij′ = min(ℓij′, ℓik+wkj)
     return L′
```

Extend-Shortest-Paths can be represented as $L^{(m)} = L^{(m-1)} \cdot W$

$\ell_{ik}^{(m-1)} \rightarrow a_{ik}$

$w_{kj} \rightarrow b_{kj}$

$\ell_{ij}^{(m)} \rightarrow c_{ij}$

$\min \rightarrow +$

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SQUARE-MATRIX-MULTIPLY(A,B)
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                cij = cij+aik·bkj
     return C
```

Matrix multiplication is represented as $C = A \cdot B$

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Computing the shortest-path Weights Bottom Up

- A slow pseudocode for computing $L^{(n-1)}$

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

n = W.rows
$L^{(1)} = W$
for $m = 2$ to $n-1$
    let $L^{(m)}$ be a new $n \times n$ matrix
    $L^{(m)}$ = EXTEND-SHORTEST-PATHS($L^{(m-1)}$, W)
return $L^{(n-1)}$

- In fact, $L^{(n-1)} = L^{(n-2)} \cdot W = L^{(n-3)} \cdot W^2 = \ldots = W^{n-1}$.
- Thus, running time is $\Theta(n^3) \times (n-2) = \Theta(n^4)$.

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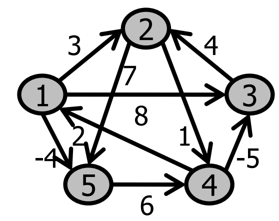
An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L(1) = W
    for m = 2 to n-1
        let L(m) be a new n x n matrix
        L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
    return L(n-1)
```



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An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L(1) = W
    for m = 2 to n-1
        let L(m) be a new n x n matrix
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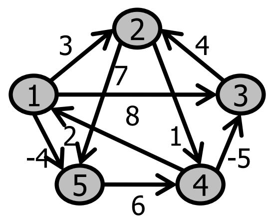
    return L(n-1)
```



$L^{(1)}$ W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

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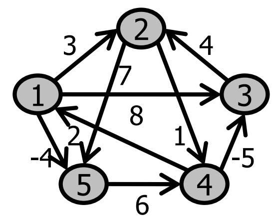
An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L^(1) = W
    for m = 2 to n-1
        let L^(m) be a new n x n matrix
        L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
    return L^(n-1)
```




```
        for i = 1 to n
            for j = 1 to n
                ℓ_ij' = ∞
                for k = 1 to n
                    ℓ_ij' = min(ℓ_ij', ℓ_ik + w_kj)
```

$L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

$$\begin{bmatrix} \; \end{bmatrix}$$

m	i	j
2	1	1

ℓ_{ij}'	∞

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An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L^(1) = W
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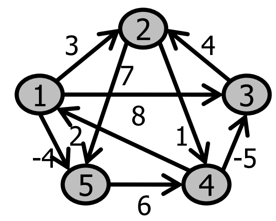
        let L^(m) be a new n x n matrix
        L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
    return L^(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(1)}$ $\quad$ W $\quad$ $L^{(2)}$

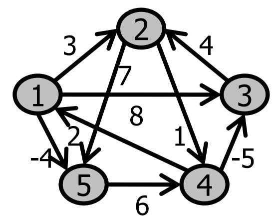
$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} \; \end{bmatrix}$$

m	i	j	k
2	1	1	1

ℓ_{ij}'	0

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L^(1) = W
    for m = 2 to n-1
        let L^(m) be a new n x n matrix
        L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
    return L^(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓ_ij' = ∞
        for k = 1 to n
            ℓ_ij' = min(ℓ_ij', ℓ_ik + w_kj)
```

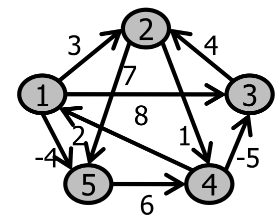
$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	1	1	2

ℓ_{ij}'	0

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
n = W.rows
L(1) = W
for m = 2 to n-1
    let L(m) be a new n x n matrix
    L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
return L(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

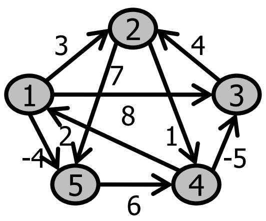
m	i	j	k
2	1	1	3

ℓ_{ij}'	0

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L^(1) = W
    for m = 2 to n-1
        let L^(m) be a new n x n matrix
        L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
    return L^(n-1)
```




```
    for i = 1 to n
        for j = 1 to n
            ℓ_ij' = ∞
            for k = 1 to n
                ℓ_ij' = min(ℓ_ij', ℓ_ik + w_kj)
```

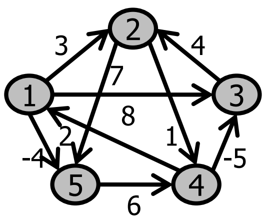
$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} \; \end{bmatrix}$$

m	i	j	k
2	1	1	4

ℓ_{ij}'	0

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
n = W.rows
L(1) = W
for m = 2 to n-1
    let L(m) be a new n x n matrix
    L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
return L(n-1)
```




```
for i = 1 to n
   for j = 1 to n
      ℓij' = ∞
      for k = 1 to n
         ℓij' = min(ℓij' , ℓik + wkj)
```

$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

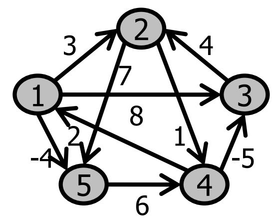
m	i	j	k
2	1	1	5

ℓ_{ij}'	0

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
n = W.rows
L(1) = W
for m = 2 to n-1
    let L(m) be a new n x n matrix
    L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
return L(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij' , ℓik+wkj)
```

$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j
2	1	2

ℓ_{ij}'	∞

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
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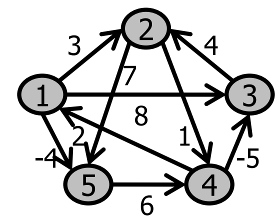
        let L(m) be a new n x n matrix
        L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
    return L(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij' , ℓik+wkj)
```

$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	1	2	1

ℓ_{ij}'	3

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
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    for m = 2 to n-1
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        L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
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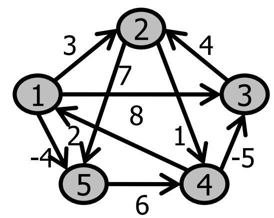


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    for i = 1 to n
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$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	1	2	2

ℓ_{ij}'	3

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An Example of Slow-All-Pairs-Shortest-Paths

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SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
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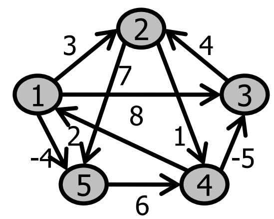


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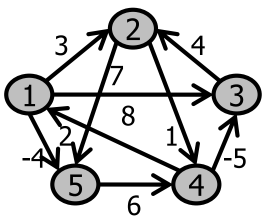


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m	i	j	k
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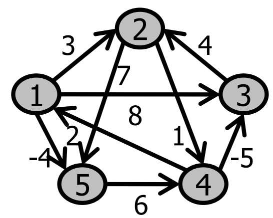


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m	i	j	k
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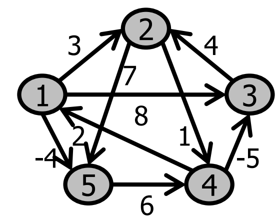


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m	i	j
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ℓ_{ij}'	∞

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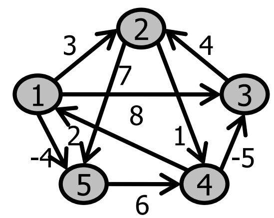

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for i = 1 to n
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m	i	j	k
2	1	3	1

ℓ_{ij}'	8

An Example of Slow-All-Pairs-Shortest-Paths

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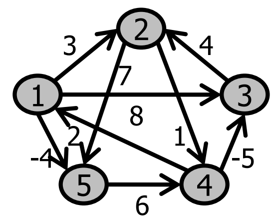


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An Example of Slow-All-Pairs-Shortest-Paths

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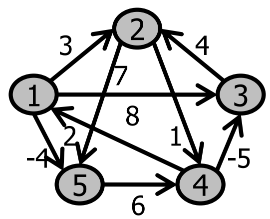

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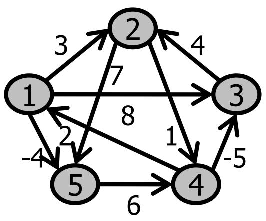


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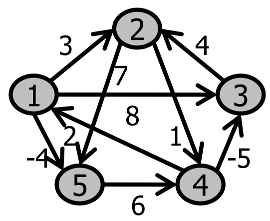

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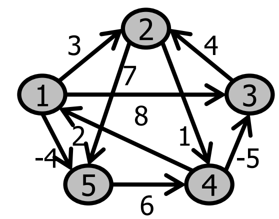


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m	i	j
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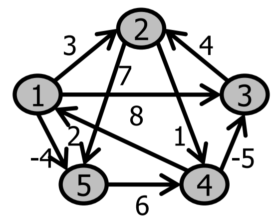


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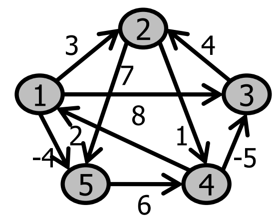


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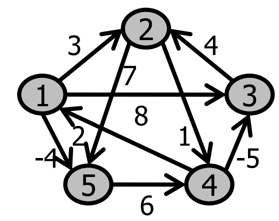


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\begin{bmatrix} 0 & 3 & 8 & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}
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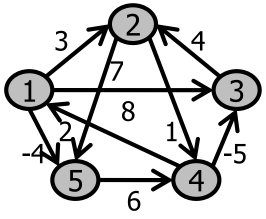

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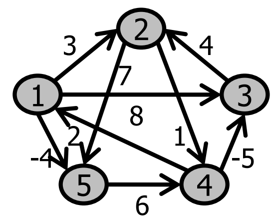


```
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```

$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & 2 & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	1	4	5

ℓ_{ij}'	2

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An Example of Slow-All-Pairs-Shortest-Paths

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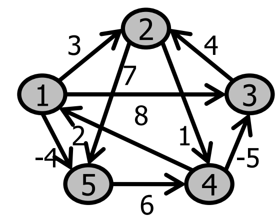


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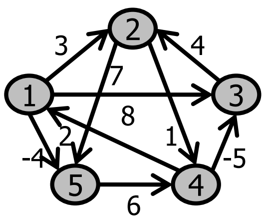
m	i	j
2	1	5

ℓ_{ij}'	∞

An Example of Slow-All-Pairs-Shortest-Paths

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$L^{(1)}$ W $L^{(2)}$

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m	i	j	k
2	1	5	1

ℓ_{ij}'	-4

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An Example of Slow-All-Pairs-Shortest-Paths

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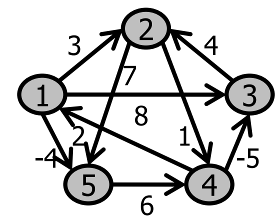


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An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

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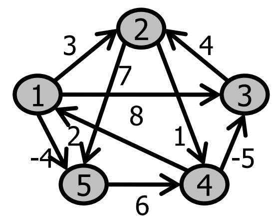

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$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 2 & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	1	5	3

ℓ_{ij}'	−4

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An Example of Slow-All-Pairs-Shortest-Paths

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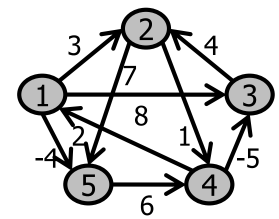


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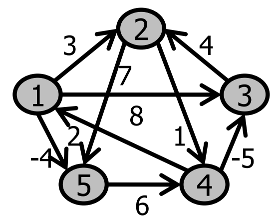


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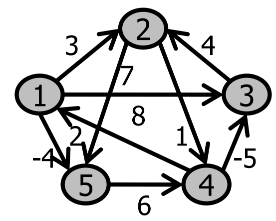


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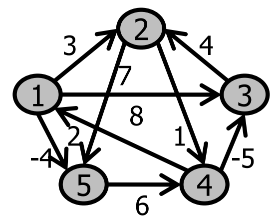


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m	i	j	k
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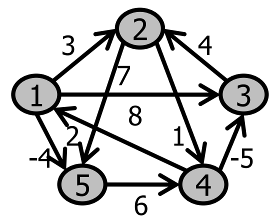


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$L^{(1)}$

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W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
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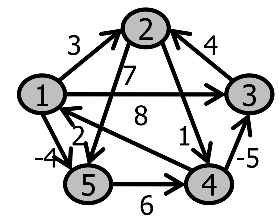


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m	i	j	k
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ℓ_{ij}'	∞

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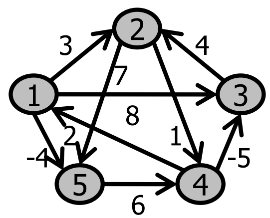

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m	i	j	k
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ℓ_{ij}'	3

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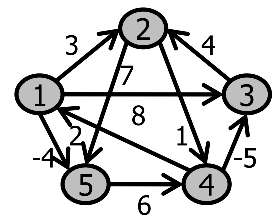


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m	i	j	k
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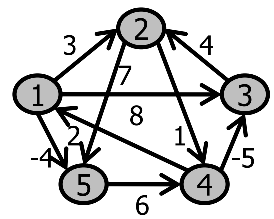

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m	i	j
2	2	2

ℓ_{ij}'	∞

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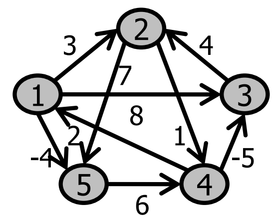


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$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	2	2	1

ℓ_{ij}'	∞

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

1. n = W.rows
2. $L^{(1)}$ = W
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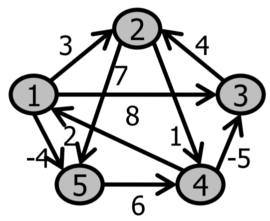

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$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
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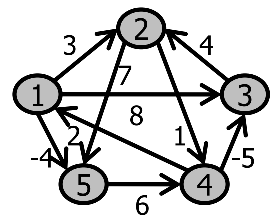


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m	i	j	k
2	2	2	3

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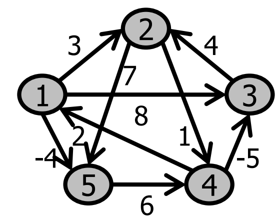


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3.    for i = 1 to n
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$L^{(1)}$ W $L^{(2)}$

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m	i	j	k
2	2	2	4

ℓ_{ij}'	0

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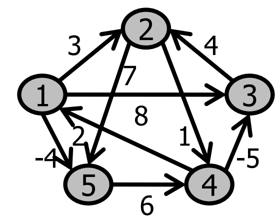


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$L^{(1)}$ $\qquad$ W $\qquad$ $L^{(2)}$

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m	i	j	k
2	2	2	5

ℓ_{ij}'	0

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An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

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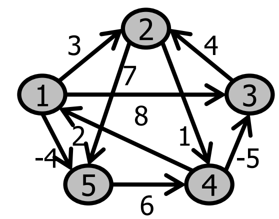


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$L^{(1)}$

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W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j
2	2	3

ℓ_{ij}'	∞

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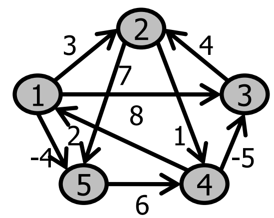


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W
$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$
$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
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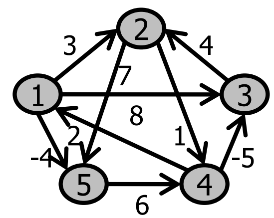

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m	i	j	k
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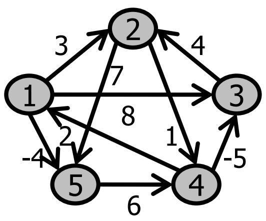


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$L^{(1)}$ W $L^{(2)}$

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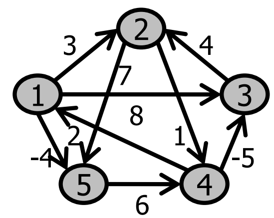


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$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & \color{red}{1} & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & \color{red}{-5} & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	2	3	4

ℓ_{ij}'	-4

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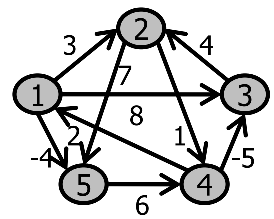

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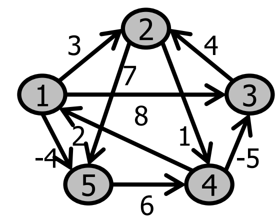

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m	i	j
2	2	4

ℓ_{ij}'	∞

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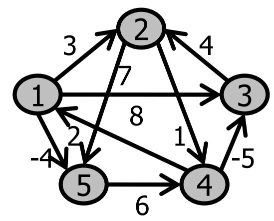


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W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	2	4	1

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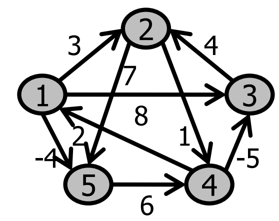

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W

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$L^{(2)}$

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m	i	j	k
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ℓ_{ij}'	1

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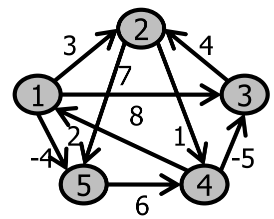


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W

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$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

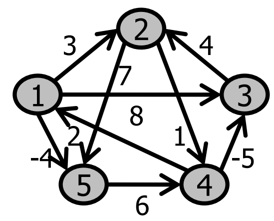
m	i	j	k
2	2	4	3

ℓ_{ij}'	1

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L^(1) = W
    for m = 2 to n-1
        let L^(m) be a new n x n matrix
        L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
    return L^(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(1)}$ W $L^{(2)}$

$$
\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}
$$

m	i	j	k
2	2	4	4

ℓ_{ij}'	1

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An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
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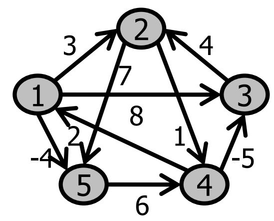
        let L(m) be a new n x n matrix
        L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
    return L(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

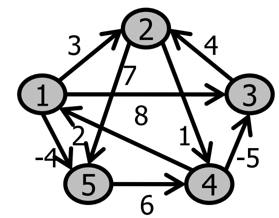
$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	2	4	5

ℓ_{ij}'	1

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L^(1) = W
    for m = 2 to n-1
        let L^(m) be a new n x n matrix
        L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
    return L^(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓ_ij' = ∞
        for k = 1 to n
            ℓ_ij' = min(ℓ_ij' , ℓ_ik + w_kj)
```

$L^{(1)}$
$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W
$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$
$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

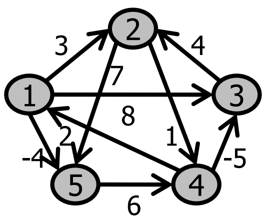
m	i	j
2	2	5

ℓ_{ij}'	∞

An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

1. n = W.rows
2. $L^{(1)} = W$
3. **for** m = 2 to n-1
4. let $L^{(m)}$ be a new n x n matrix
5. $L^{(m)}$ = EXTEND-SHORTEST-PATHS($L^{(m-1)}$, W)
6. **return** $L^{(n-1)}$



3. for i = 1 to n
4. for j = 1 to n
5. $\ell_{ij}' = \infty$
6. for k = 1 to n
7. $\ell_{ij}' = \min(\ell_{ij}', \ell_{ik} + w_{kj})$

$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	2	5	1

ℓ_{ij}'	∞

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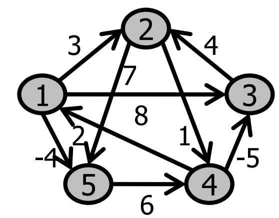
An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L(1) = W
    for m = 2 to n-1
        let L(m) be a new n x n matrix
        L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
    return L(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	2	5	2

ℓ_{ij}'	7

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An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
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    return L^(n-1)
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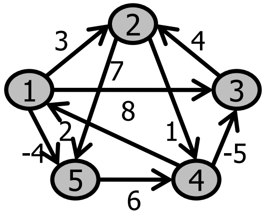


```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij' , ℓik + wkj)
```

$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	2	5	3

ℓ_{ij}'	7

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An Example of Slow-All-Pairs-Shortest-Paths

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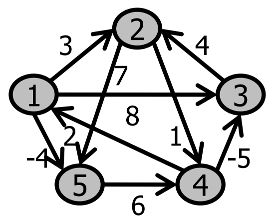


```
3.    for i = 1 to n
4.       for j = 1 to n
5.          ℓ_ij' = ∞
6.          for k = 1 to n
7.             ℓ_ij' = min(ℓ_ij' , ℓ_ik + w_kj)
```

$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
2	2	5	4

ℓ_{ij}'	7

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An Example of Slow-All-Pairs-Shortest-Paths

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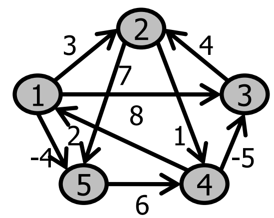


```
for i = 1 to n
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```

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m	i	j	k
2	2	5	5

ℓ_{ij}'	7

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An Example of Slow-All-Pairs-Shortest-Paths

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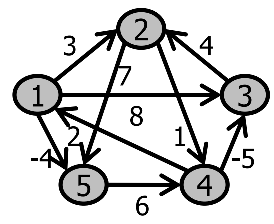


```
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        for k = 1 to n
            ℓ_ij' = min(ℓ_ij', ℓ_ik + w_kj)
```

$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j
2	3	1

ℓ_{ij}'	∞

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An Example of Slow-All-Pairs-Shortest-Paths

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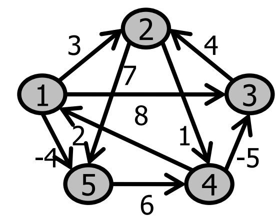


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```

$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j
2	3	2

ℓ_{ij}'	4

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An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
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    L^(1) = W
    for m = 2 to n-1
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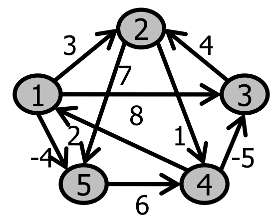
        L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
    return L^(n-1)
```




```
for i = 1 to n
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        ℓ_ij' = ∞
        for k = 1 to n
            ℓ_ij' = min(ℓ_ij', ℓ_ik + w_kj)
```

$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j
2	3	3

ℓ_{ij}'	0

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An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
n = W.rows
L^(1) = W
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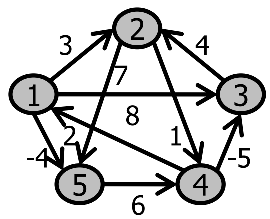
return L^(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j
2	3	4

ℓ_{ij}'	5

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An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
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        L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
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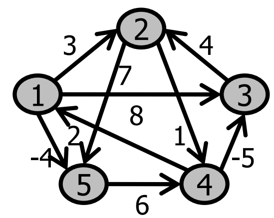
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j
2	3	5

ℓ_{ij}'	11

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An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

```
n = W.rows
L(1) = W
for m = 2 to n-1
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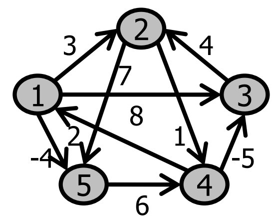
    L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
return L(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & & & & \\ & & & & \end{bmatrix}$$

m	i	j
2	4	1

ℓ_{ij}'	2

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An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

1. n = W.rows
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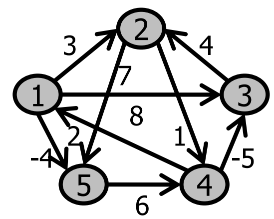
5. $L^{(m)}$ = EXTEND-SHORTEST-PATHS($L^{(m-1)}$, W)
6. **return** $L^{(n-1)}$



3. for i = 1 to n
4. for j = 1 to n
5. $\ell_{ij}' = \infty$
6. for k = 1 to n
7. $\ell_{ij}' = \min(\ell_{ij}', \ell_{ik} + w_{kj})$

$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & & & \\ & & & & \end{bmatrix}$$

m	i	j
2	4	2

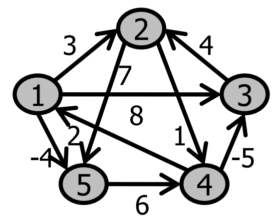
ℓ_{ij}'	−1

Thomas T. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

```
n = W.rows
L^(1) = W
for m = 2 to n-1
    let L^(m) be a new n x n matrix
    L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
return L^(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

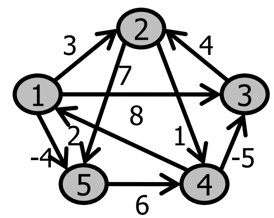
$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & & \\ & & & & \end{bmatrix}$$

m	i	j
2	4	3

ℓ_{ij}'	-5

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths




```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
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return L^(n-1)
```

```
for i = 1 to n
    for j = 1 to n
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        for k = 1 to n
            ℓ_ij' = min(ℓ_ij', ℓ_ik + w_kj)
```

$L^{(1)}$ W $L^{(2)}$

$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & \\ & & & & \end{bmatrix}$$

m	i	j
2	4	4

ℓ_{ij}'	0

Thomas T. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

1. n = W.rows
2. $L^{(1)}$ = W
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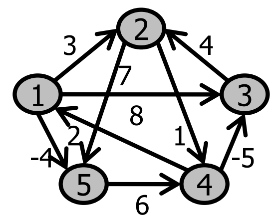

3. for i = 1 to n
4. for j = 1 to n
5. $\ell_{ij}' = \infty$
6. for k = 1 to n
7. $\ell_{ij}' = \min(\ell_{ij}', \ell_{ik} + w_{kj})$

$L^{(1)}$ W $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ & & & & \end{bmatrix}$$

m	i	j
2	4	5

ℓ_{ij}'	−2

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

1. n = W.rows
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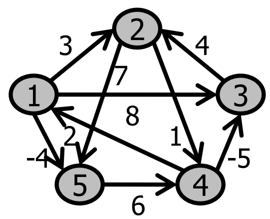


3. for i = 1 to n
4. for j = 1 to n
5. $\ell_{ij}' = \infty$
6. for k = 1 to n
7. $\ell_{ij}' = \min(\ell_{ij}', \ell_{ik} + w_{kj})$

$L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & & & & \end{bmatrix}$$

m	i	j
2	5	1

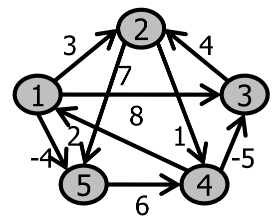
ℓ_{ij}'	8

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

1. n = W.rows
2. $L^{(1)}$ = W
3. **for** m = 2 to n-1
4. let $L^{(m)}$ be a new n x n matrix
5. $L^{(m)}$ = EXTEND-SHORTEST-PATHS($L^{(m-1)}$, W)
6. **return** $L^{(n-1)}$



3. for i = 1 to n
4. for j = 1 to n
5. $\ell_{ij}' = \infty$
6. for k = 1 to n
7. $\ell_{ij}' = \min(\ell_{ij}', \ell_{ik} + w_{kj})$

$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & \infty & & & \end{bmatrix}$$

m	i	j
2	5	2

ℓ_{ij}'	∞

Thomas T. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

SLOW-ALL-PAIRS-SHORTEST-PATHS(W)

```
n = W.rows
L^(1) = W
for m = 2 to n-1
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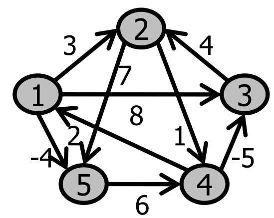
    L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
return L^(n-1)
```




```
for i = 1 to n
   for j = 1 to n
      ℓ_ij' = ∞
      for k = 1 to n
          ℓ_ij' = min(ℓ_ij' , ℓ_ik + w_kj)
```

$L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & \infty & 1 & & \end{bmatrix}$$

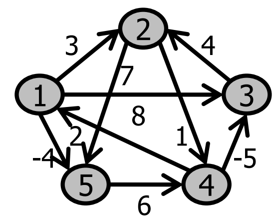
m	i	j
2	5	3

ℓ_{ij}'	1

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L^(1) = W
    for m = 2 to n-1
        let L^(m) be a new n x n matrix
        L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
    return L^(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓ_ij' = ∞
        for k = 1 to n
            ℓ_ij' = min(ℓ_ij' , ℓ_ik + w_kj)
```

$L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & \infty & 1 & 6 & \end{bmatrix}$$

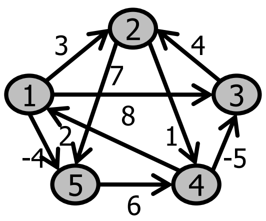
m	i	j
2	5	4

ℓ_{ij}'	6

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L(1) = W
    for m = 2 to n-1
        let L(m) be a new n x n matrix
        L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
    return L(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij' , ℓik+wkj)
```

$L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

W

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$L^{(2)}$

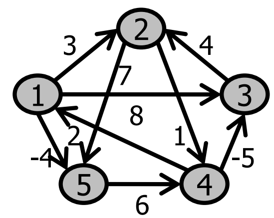
$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & \infty & 1 & 6 & 0 \end{bmatrix}$$

m	i	j
2	5	5

ℓ_{ij}'	0

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L^(1) = W
    for m = 2 to n-1
        let L^(m) be a new n x n matrix
        L^(m) = EXTEND-SHORTEST-PATHS(L^(m-1), W)
    return L^(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓ_ij' = ∞
        for k = 1 to n
            ℓ_ij' = min(ℓ_ij', ℓ_ik + w_kj)
```

$L^{(2)}$ W $L^{(3)}$

$$\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & \infty & 1 & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$$

m
3

Thomas T. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

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```
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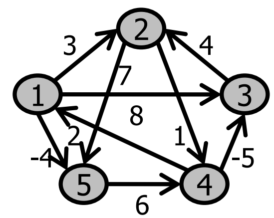
    n = W.rows
    L(1) = W
    for m = 2 to n-1
        let L(m) be a new n x n matrix
        L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
    return L(n-1)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(3)}$ W $L^{(4)}$

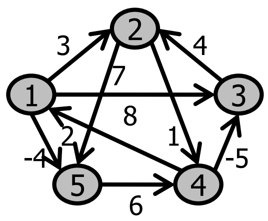
$$\begin{bmatrix} 0 & 3 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$$

m
4

Thomas T. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Slow-All-Pairs-Shortest-Paths

```
SLOW-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L(1) = W
    for m = 2 to n-1
        let L(m) be a new n x n matrix
        L(m) = EXTEND-SHORTEST-PATHS(L(m-1), W)
    return L(n-1)
```



$L^{(4)}$

$$\begin{bmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$$

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Improving the Running Time: Repeated Squaring

- Our goal is to get only $L^{(n-1)}$, not all $L^{(m)}$ matrices
- Note that in the absence of negative-weight cycles, $L^{(m)} = L^{(n-1)}$ for $m \geq n-1$.
  - $\delta(i,j) = \ell_{ij}^{(n-1)} = \ell_{ij}^{(n)} = \ell_{ij}^{(n+1)} = \ldots$
  - $L^{(n-1)} = L^{(n)} = L^{(n+1)} = L^{(n+2)} = \ldots$
- Matrix multiplication defined by EXTEND-SHORTEST-PATHS is associative. (Exercise 25.1-4).
- Thus, we compute the sequence below
  - $L^{(1)} = W$
  - $L^{(2)} = W^2 = W \cdot W$
  - $L^{(4)} = W^4 = W^2 \cdot W^2$
  - $L^{(8)} = W^8 = W^4 \cdot W^4$
  - …
  - $L^{(2^{\lceil \lg(n-1) \rceil})} = W^{2^{\lceil \lg(n-1) \rceil}} = W^{2^{\lceil \lg(n-1) \rceil - 1}} \cdot W^{2^{\lceil \lg(n-1) \rceil - 1}}$

If $2^{\lceil \lg(n-1) \rceil} \geq n-1$,
$L^{(2^{\lceil \lg(n-1) \rceil})} = L^{(2^{\lceil \lg(n-1) \rceil - 1})} = \cdots = L^{(n-1)}$

Improving the Running Time: Repeated Squaring

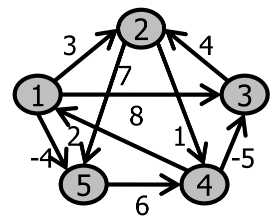
- Pseudocode for computing $L^{(n-1)}$ by repeated squaring

```
FASTER-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L^(1) = W
    m = 1
    for m < n-1
        let L^(2m) be a new n x n matrix
        L^(2m) = EXTEND-SHORTEST-PATHS(L^(m), L^(m))
        m = 2m
    return L^(m)
```

- Running time is $\Theta(n^3) \times \lceil \log(n-1) \rceil = \Theta(n^3 \log n)$.

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Faster-All-Pairs-Shortest-Paths




```
FASTER-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L^(1) = W
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    for m < n-1
        let L^(2m) be a new n x n matrix
        L^(2m) = EXTEND-SHORTEST-PATHS(L^(m), L^(m))
        m = 2m
    return L^(m)
```

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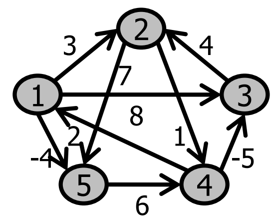
An Example of Faster-All-Pairs-Shortest-Paths




```
FASTER-ALL-PAIRS-SHORTEST-PATHS(W)
    n = W.rows
    L(1) = W
    m = 1
    for m < n-1
        let L(2m) be a new n x n matrix
        L(2m) = EXTEND-SHORTEST-PATHS(L(m), L(m))
        m = 2m
    return L(m)
```

$L^{(1)}$ $L^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

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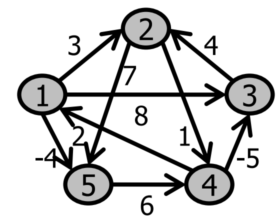
An Example of Faster-All-Pairs-Shortest-Paths

FASTER-ALL-PAIRS-SHORTEST-PATHS(W)

1. n = W.rows
2. $L^{(1)}$ = W
3. m = 1
4. **for** m < n-1
5. let $L^{(2m)}$ be a new n x n matrix
6. $L^{(2m)}$ = EXTEND-SHORTEST-PATHS($L^{(m)}$, $L^{(m)}$)
7. m = 2m
8. **return** $L^{(m)}$



3. for i = 1 to n
4. for j = 1 to n
5. $\ell_{ij}' = \infty$
6. for k = 1 to n
7. $\ell_{ij}' = \min(\ell_{ij}', \ell_{ik} + w_{kj})$

$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} \; \end{bmatrix}$$

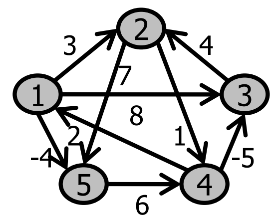
m	i	j
1	1	1

ℓ_{ij}'	∞

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Faster-All-Pairs-Shortest-Paths

```
FASTER-ALL-PAIRS-SHORTEST-PATHS(W)
n = W.rows
L(1) = W
m = 1
for m < n-1
    let L(2m) be a new n x n matrix
    L(2m) = EXTEND-SHORTEST-PATHS(L(m), L(m))
    m = 2m
return L(m)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik + wkj)
```

$L^{(1)}$ $L^{(1)}$ $L^{(2)}$

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m	i	j	k
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ℓ_{ij}'	0

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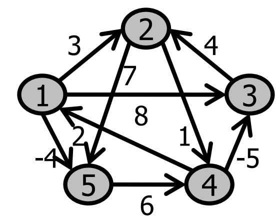


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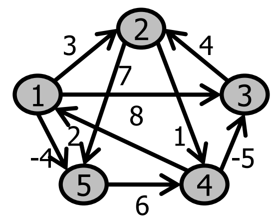


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for i = 1 to n
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```

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m	i	j	k
1	1	1	3

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An Example of Faster-All-Pairs-Shortest-Paths

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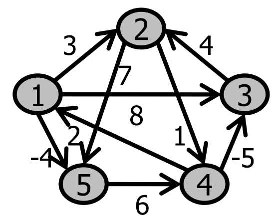


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m	i	j	k
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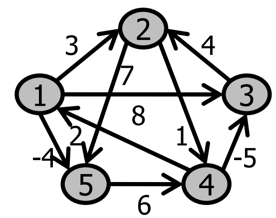


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m	i	j	k
1	1	1	5

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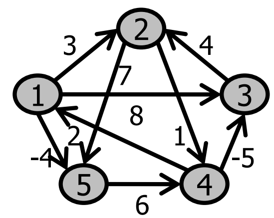


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m	i	j
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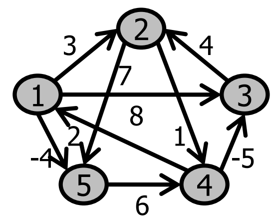


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m	i	j	k
1	1	2	1

ℓ_{ij}'	3

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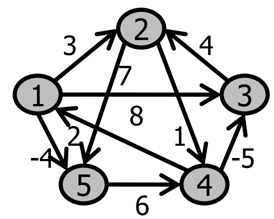


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m	i	j	k
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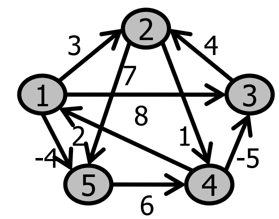


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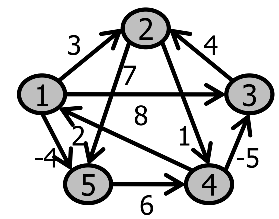


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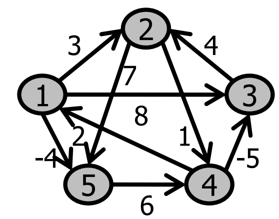


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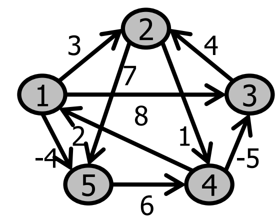


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m	i	j
1	1	3

ℓ_{ij}'	∞

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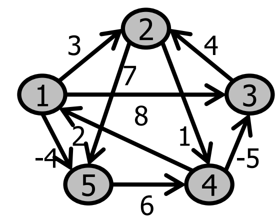


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m	i	j	k
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ℓ_{ij}'	8

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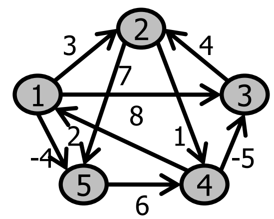


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        L^(2m) = EXTEND-SHORTEST-PATHS(L^(m), L^(m))
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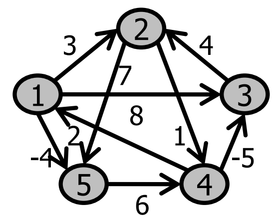


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for i = 1 to n
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      ℓij' = ∞
      for k = 1 to n
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```

$$L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad L^{(2)} = \begin{bmatrix} 0 & 3 & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
1	1	3	3

ℓ_{ij}'	8

Thomas T. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Faster-All-Pairs-Shortest-Paths

```
FASTER-ALL-PAIRS-SHORTEST-PATHS(W)
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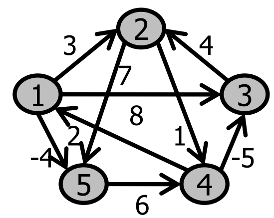


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m	i	j	k
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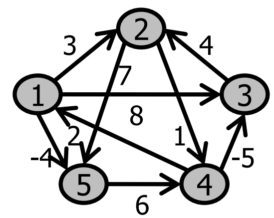


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m	i	j	k
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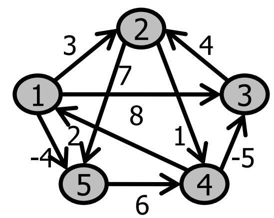


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m	i	j
1	1	4

ℓ_{ij}'	∞

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An Example of Faster-All-Pairs-Shortest-Paths

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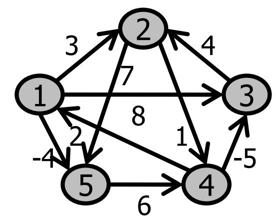


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m	i	j	k
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ℓ_{ij}'	∞

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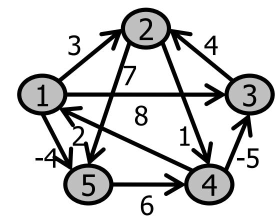


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m	i	j	k
1	1	4	2

ℓ_{ij}'	4

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An Example of Faster-All-Pairs-Shortest-Paths

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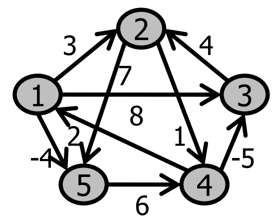


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m	i	j	k
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ℓ_{ij}'	4

An Example of Faster-All-Pairs-Shortest-Paths

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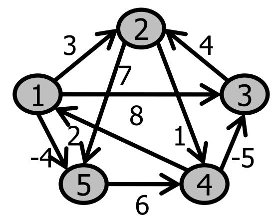


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```

$L^{(1)}$ $L^{(1)}$ $L^{(2)}$

$$
\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}
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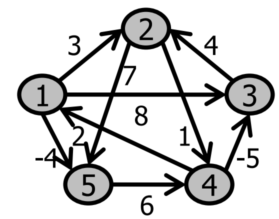
$$

m	i	j	k
1	1	4	4

ℓ_{ij}'	4

An Example of Faster-All-Pairs-Shortest-Paths

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```

```
3.    for i = 1 to n
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$L^{(1)}$ $L^{(1)}$ $L^{(2)}$

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m	i	j	k
1	1	4	5

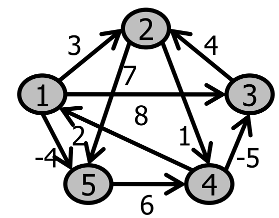
ℓ_{ij}'	2

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Faster-All-Pairs-Shortest-Paths

FASTER-ALL-PAIRS-SHORTEST-PATHS(W)

1. n = W.rows
2. $L^{(1)} = W$
3. m = 1
4. **for** m < n-1
5. let $L^{(2m)}$ be a new n x n matrix
6. $L^{(2m)}$ = EXTEND-SHORTEST-PATHS($L^{(m)}$, $L^{(m)}$)
7. m = 2m
8. **return** $L^{(m)}$



3. for i = 1 to n
4. for j = 1 to n
5. $\ell_{ij}' = \infty$
6. for k = 1 to n
7. $\ell_{ij}' = \min(\ell_{ij}', \ell_{ik} + w_{kj})$

$$L^{(1)} \quad L^{(1)} \quad L^{(2)}$$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & 2 & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j
1	1	5

ℓ_{ij}'	∞

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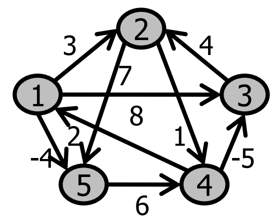


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m	i	j	k
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ℓ_{ij}'	−4

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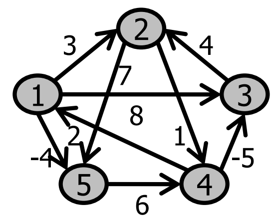
An Example of Faster-All-Pairs-Shortest-Paths

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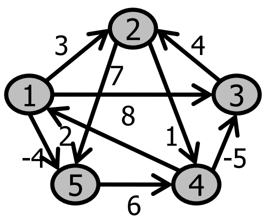


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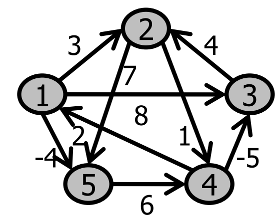


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$L^{(1)}$ $L^{(1)}$ $L^{(2)}$

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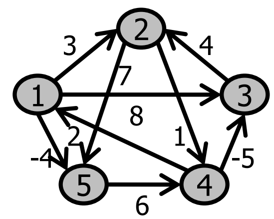


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```

$L^{(1)}$ $L^{(1)}$ $L^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

m	i	j	k
1	1	5	5

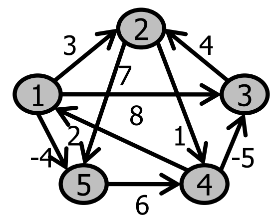
ℓ_{ij}'	−4

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Faster-All-Pairs-Shortest-Paths

FASTER-ALL-PAIRS-SHORTEST-PATHS(W)

1. n = W.rows
2. $L^{(1)} = W$
3. m = 1
4. **for** m < n-1
5. let $L^{(2m)}$ be a new n x n matrix
6. $L^{(2m)}$ = EXTEND-SHORTEST-PATHS($L^{(m)}$, $L^{(m)}$)
7. m = 2m
8. **return** $L^{(m)}$



3. for i = 1 to n
4. for j = 1 to n
5. $\ell_{ij}' = \infty$
6. for k = 1 to n
7. $\ell_{ij}' = \min(\ell_{ij}', \ell_{ik} + w_{kj})$

$L^{(1)}$ $L^{(1)}$ $L^{(2)}$

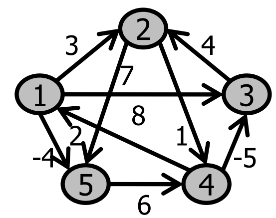
$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & \infty & 1 & 6 & 0 \end{bmatrix}$$

m
1

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

An Example of Faster-All-Pairs-Shortest-Paths

```
FASTER-ALL-PAIRS-SHORTEST-PATHS(W)
n = W.rows
L(1) = W
m = 1
for m < n-1
    let L(2m) be a new n x n matrix
    L(2m) = EXTEND-SHORTEST-PATHS(L(m), L(m))
    m = 2m
return L(m)
```




```
for i = 1 to n
    for j = 1 to n
        ℓij' = ∞
        for k = 1 to n
            ℓij' = min(ℓij', ℓik+wkj)
```

$L^{(2)}$ $\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & \infty & 1 & 6 & 0 \end{bmatrix}$

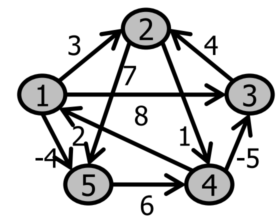
$L^{(2)}$ $\begin{bmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ \infty & \infty & 1 & 6 & 0 \end{bmatrix}$

$L^{(4)}$ $\begin{bmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$

m
2

An Example of Faster-All-Pairs-Shortest-Paths

```
FASTER-ALL-PAIRS-SHORTEST-PATHS(W)
n = W.rows
L^(1) = W
m = 1
for m < n-1
    let L^(2m) be a new n x n matrix
    L^(2m) = EXTEND-SHORTEST-PATHS(L^(m), L^(m))
return L^(m)
```



$L^{(4)}$

$$\begin{bmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$$

m
4

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Floyd-Warshall Algorithm

Floyd-Warshall Algorithm

- We shall use a different dynamic-programming formulation to solve the all-pairs shortest-paths problem on a directed graph G=(V,E).
- As before, negative-weight edges may be present, but we assume that there are no negative-weight cycles.
- It runs in $\Theta(|V|^3)$ time.

Floyd-Warshall Algorithm

- The intermediate vertices of a shortest path.
  - An intermediate vertex of a simple path $p=<v_1,v_2,\ldots,v_l>$ is any vertex of p other than v_1 and v_l, that is, any vertex in the set $\{v_2,v_3,\ldots,v_{l-1}\}$.

Floyd-Warshall Algorithm

- For the vertices V = {1,2,…,n} of G, consider a subset {1,2,…,k} of vertices for some k.
- For any pair of vertices i, j$\in$V,
  - Consider all paths from i to j whose intermediate vertices are all drawn from {1,2,…,k}
  - Let p be a minimum-weight path from among them. (p is a simple path.)
- It exploits a relationship between path p and shortest paths from i to j with all intermediate vertices in the set {1,2,…,k-1}.

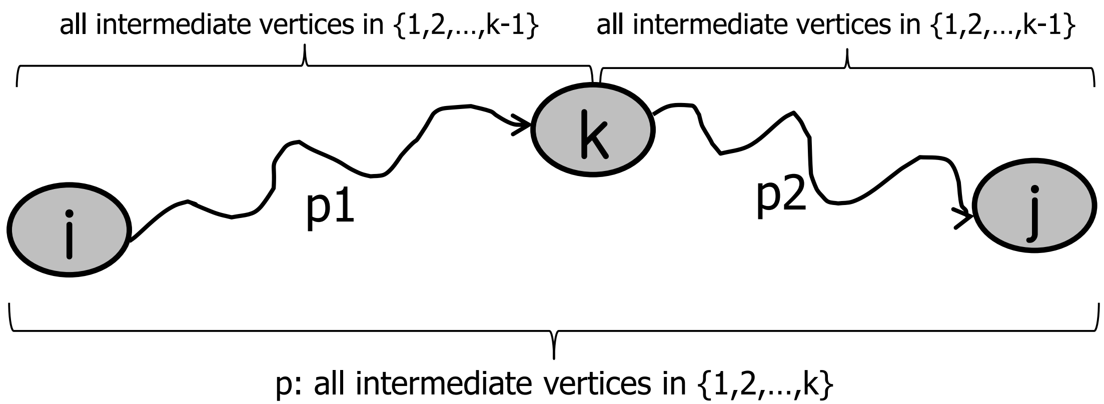
Floyd-Warshall Algorithm

- The relationship depends on whether or not k is an intermediate vertex of path p.
  - If k is not an intermediate vertex of path p, then all intermediate vertices of path p are in the set {1,2,…,k-1}.
  - If k is intermediate vertex of path p, we decompose p into $i \leadsto k \leadsto j$, ($i \leadsto k$: p1, $k \leadsto j$: p2)
    - By Lemma 24.1 (Subpaths of shortest paths are shortest paths), p_1 is a shortest path from i to k with all intermediate vertices in the set {1,2,…,k}.
    - In fact,

  p_1 is a shortest path from i to k with all intermediate vertices in the set {1,2,…,k-1}

  p_2 is a shortest path from k to j with all intermediate vertices in the set {1,2,…,k-1}.

The Structure of a Shortest Path



- Path p is a shortest path from vertex i to vertex j, and k is the highest-numbered intermediate vertex of p
- Path p_1, the portion of path p from vertex i to vertex k, has all intermediate vertices in the set {1,2,…,k-1}
- The same holds for path p_2 from vertex k to vertex j

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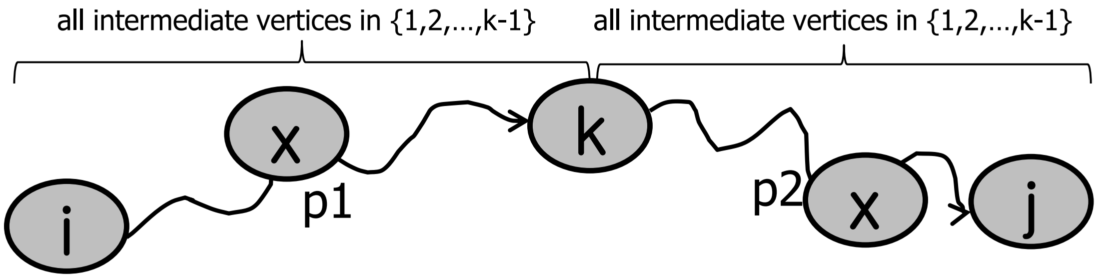
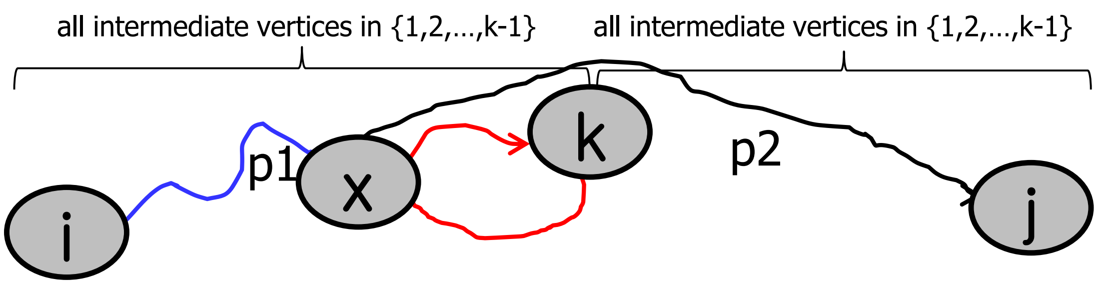
The Structure of a Shortest Path



- What if p_1 and p_2 share the same node x?

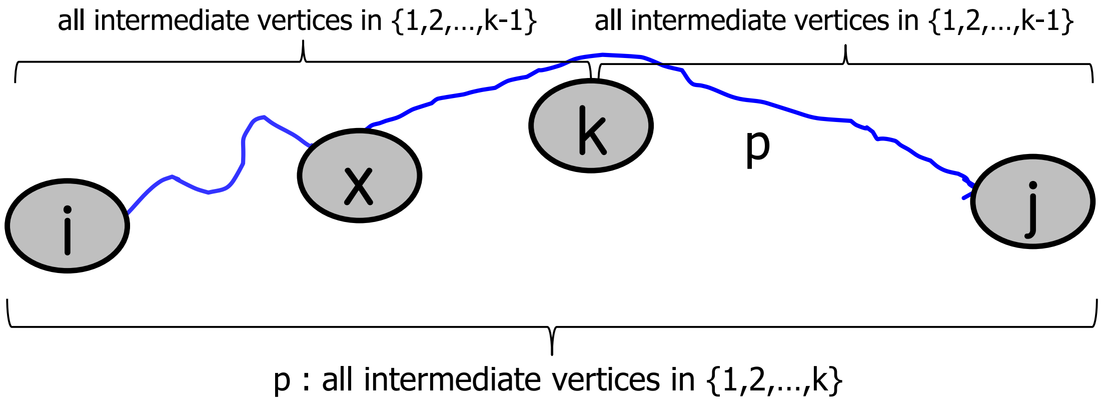
Thomas T. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

The Structure of a Shortest Path



- What if p_1 and p_2 share the same node x?
  - Node k becomes in a cycle.

The Structure of a Shortest Path



- What if p_1 and p_2 share the same node x?
  - Node k becomes in a cycle.
  - The shortest path from i to j with intermediate vertices {1, 2, ..., k} is equivalent to the shortest path from i to j with intermediate vertices {1, 2, ..., k-1}.

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

A Recursive Solution

- $d_{ij}^{(k)}$: weight of a shortest path from i to j for which all intermediate vertices are in the set {1,2,…k}
- $d_{ij}^{(0)}=w_{ij}$
  - When k = 0, a path from i to j with no intermediate vertex numbered higher than 0 has no intermediate vertices at all (such a path has at most 1 edge).
- Thus,
  - $$d_{ij}^{(k)} = \begin{cases} w_{ij} & \text{if } k = 0 \\ \min(d_{ij}{}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}) & \text{if } k \geq 1 \end{cases}$$
- The matrix $D^{(n)}=(d_{ij}^{(n)})$ gives the final answer.
  - $d_{ij}^{(n)}=\delta(i,j)$ for all i, $j \in V$.

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Computing the Shortest-path

FLOYD-WARSHALL(W)

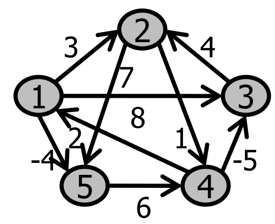
1. n = W.rows
2. $D^{(0)} = W$
3. **for** k = 1 to n
4. let $D^{(k)} = (d_{ij}^{(k)})$ be a new n x n matrix
5. **for** i = 1 to n
6. **for** j = 1 to n
7. $d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)}+d_{kj}^{(k-1)})$
8. **return** $D^{(n)}$

- Running time is $\Theta(n^3)$ since line 7 takes $O(1)$.

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
    n = W.rows
    D^(0) = W
    for k = 1 to n
        let D^(k) = (d_ij^(k)) be a new n x n matrix
        for i = 1 to n
            for j = 1 to n
                d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1)+d_kj^(k-1))
    return D^(n)
```



Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Floyd-Warshall Algorithm

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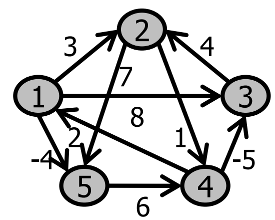
FLOYD-WARSHALL(W)
    n = W.rows
    D^(0) = W
    for k = 1 to n
        let D^(k) = (d_ij^(k)) be a new n x n matrix
        for i = 1 to n
            for j = 1 to n
                d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1)+d_kj^(k-1))
    return D^(n)
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

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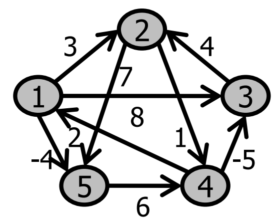
Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
    n = W.rows
    D^(0) = W
    for k = 1 to n
        let D^(k) = (d_ij^(k)) be a new n x n matrix
        for i = 1 to n
            for j = 1 to n
                d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1) + d_kj^(k-1))
    return D^(n)
```



$$D^{(0)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \qquad D^{(1)} = \begin{bmatrix} 0 & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	1	1

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
0	0+0

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Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
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D^(0) = W
for k = 1 to n
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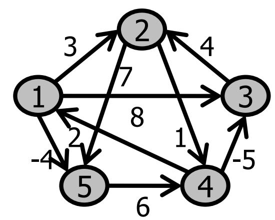
    for i = 1 to n
        for j = 1 to n
            d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1)+d_kj^(k-1))
return D^(n)
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	1	2

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
3	0+3

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Floyd-Warshall Algorithm

FLOYD-WARSHALL(W)

n = W.rows

$D^{(0)} = W$

for k = 1 to n

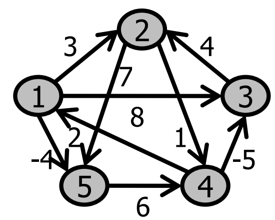
  let $D^{(k)} = (d_{ij}^{(k)})$ be a new n x n matrix

  for i = 1 to n

    for j = 1 to n

      $d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$

return $D^{(n)}$



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

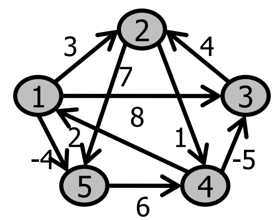
k	i	j
1	1	3

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$
8	0+8

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
n = W.rows
D(0) = W
for k = 1 to n
    let D(k) = (dij(k)) be a new n x n matrix
    for i = 1 to n
        for j = 1 to n
            dij(k) = min(dij(k-1), dik(k-1)+dkj(k-1))
return D(n)
```



$$D^{(0)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \quad D^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	1	4

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	0+∞

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Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
    n = W.rows
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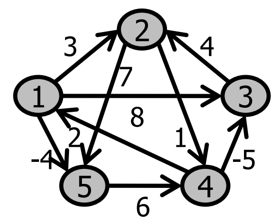
        for i = 1 to n
            for j = 1 to n
                d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1) + d_kj^(k-1))
    return D^(n)
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	1	5

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
-4	$0+(-4)$

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Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
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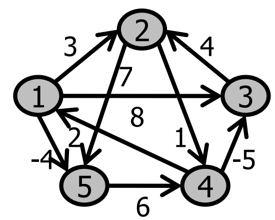
```



$$D^{(0)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$$D^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	2	1

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	$\infty+0$

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Floyd-Warshall Algorithm

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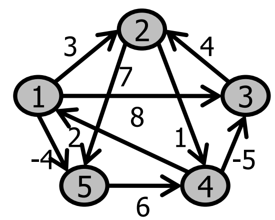

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$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	2	2

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
0	$\infty+3$

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Floyd-Warshall Algorithm

```
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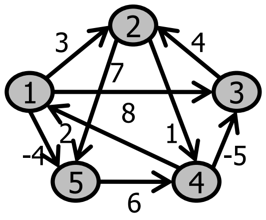
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	2	3

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	$\infty+8$

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Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
n = W.rows
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for k = 1 to n
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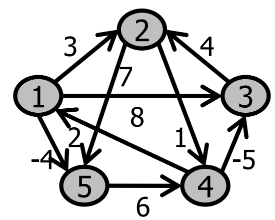
    for i = 1 to n
        for j = 1 to n
            d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1)+d_kj^(k-1))
return D^(n)
```



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$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

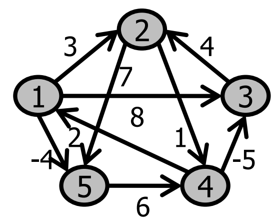
k	i	j
1	2	4

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
1	$\infty+\infty$

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
    n = W.rows
    D^(0) = W
    for k = 1 to n
        let D^(k) = (d_ij^(k)) be a new n x n matrix
        for i = 1 to n
            for j = 1 to n
                d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1)+d_kj^(k-1))
    return D^(n)
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	2	5

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
7	$\infty+(-4)$

Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
n = W.rows
D^(0) = W
for k = 1 to n
    let D^(k) = (d_ij^(k)) be a new n x n matrix
    for i = 1 to n
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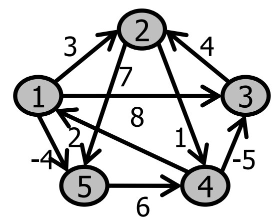

$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	3	1

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	∞+0

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Floyd-Warshall Algorithm

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    n = W.rows
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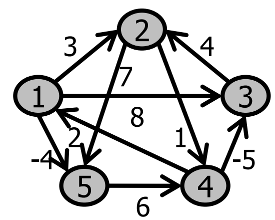
                d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1) + d_kj^(k-1))
    return D^(n)
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	3	2

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
4	$\infty+3$

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Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
    n = W.rows
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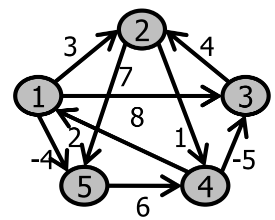
    return D(n)
```



$D^{(0)}$

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$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	3	3

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
0	∞+8

Floyd-Warshall Algorithm

```
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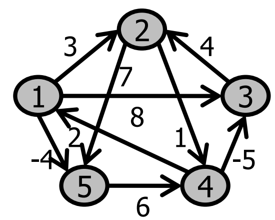

$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	3	4

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	$\infty+\infty$

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Floyd-Warshall Algorithm

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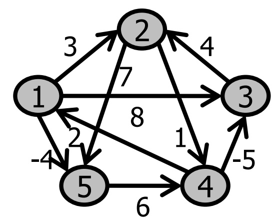

$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	3	5

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	$\infty+(-4)$

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Floyd-Warshall Algorithm

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FLOYD-WARSHALL(W)
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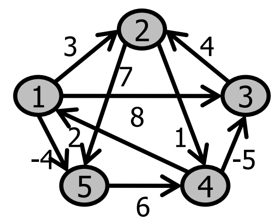
            for j = 1 to n
                dij(k) = min(dij(k-1), dik(k-1)+dkj(k-1))
    return D(n)
```



$$D^{(0)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix} \qquad D^{(1)} = \begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	4	1

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
2	2+0

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Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
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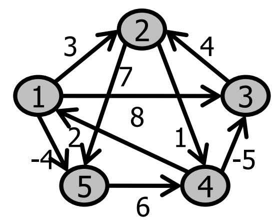
            for j = 1 to n
                d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1) + d_kj^(k-1))
    return D^(n)
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & & & \\ & & & & \end{bmatrix}$$

k	i	j
1	4	2

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	2+3

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Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
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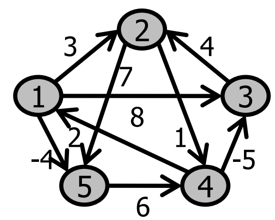
        for i = 1 to n
            for j = 1 to n
                dij(k) = min(dij(k-1), dik(k-1)+dkj(k-1))
    return D(n)
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & & \\ & & & & \end{bmatrix}$$

k	i	j
1	4	3

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
−5	2+8

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Floyd-Warshall Algorithm

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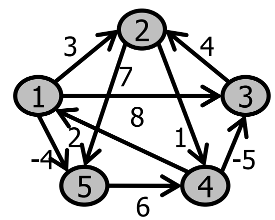
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & \\ & & & & \end{bmatrix}$$

k	i	j
1	4	4

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
0	$2+\infty$

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Floyd-Warshall Algorithm

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FLOYD-WARSHALL(W)
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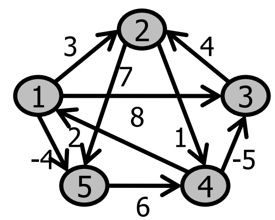
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ & & & & \end{bmatrix}$$

k	i	j
1	4	5

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	2+(−4)

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Floyd-Warshall Algorithm

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FLOYD-WARSHALL(W)
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        for i = 1 to n
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                dij(k) = min(dij(k-1), dik(k-1)+dkj(k-1))
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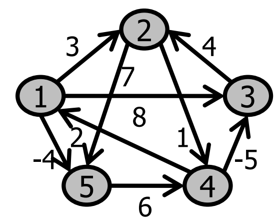

$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & & & & \end{bmatrix}$$

k	i	j
1	5	1

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	$\infty+0$

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Floyd-Warshall Algorithm

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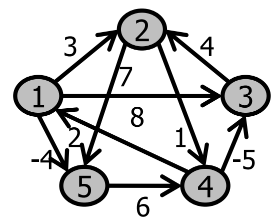
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & & & \end{bmatrix}$$

k	i	j
1	5	2

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	$\infty+3$

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Floyd-Warshall Algorithm

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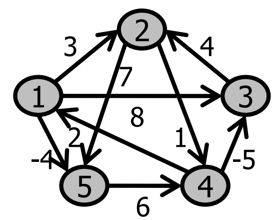
```



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$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & & \end{bmatrix}$$

k	i	j
1	5	3

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
∞	$\infty+8$

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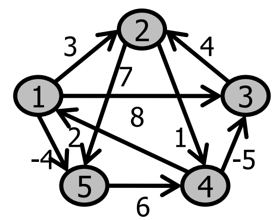
```



$D^{(0)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & \end{bmatrix}$$

k	i	j
1	5	4

$d_{ij}^{(k-1)}$	$d_{ik}^{(k-1)}+d_{kj}^{(k-1)}$
6	$\infty+\infty$

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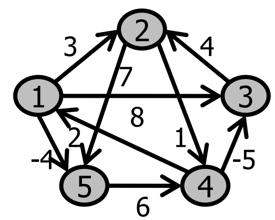
```



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$D^{(1)}$

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1	5	5

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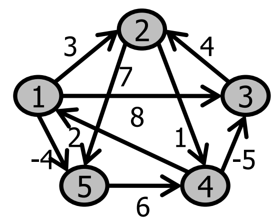
return D^(n)
```



$D^{(1)}$

$$\begin{bmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(2)}$

$$\begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

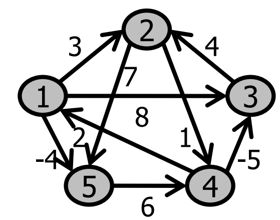
k
2

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Floyd-Warshall Algorithm

FLOYD-WARSHALL(W)

1. n = W.rows
2. $D^{(0)} = W$
3. **for** k = 1 to n
4. let $D^{(k)} = (d_{ij}^{(k)})$ be a new n x n matrix
5. **for** i = 1 to n
6. **for** j = 1 to n
7. $d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$
8. **return** $D^{(n)}$



$$D^{(2)} = \begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

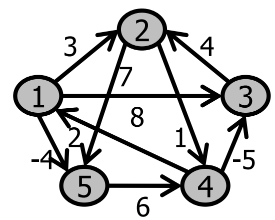
$$D^{(3)} = \begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

k
3

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
n = W.rows
D^(0) = W
for k = 1 to n
    let D^(k) = (d_ij^(k)) be a new n x n matrix
    for i = 1 to n
        for j = 1 to n
            d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1)+d_kj^(k-1))
return D^(n)
```



$D^{(3)}$

$$\begin{bmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$D^{(4)}$

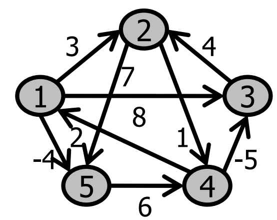
$$\begin{bmatrix} 0 & 3 & -1 & 4 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$$

k
4

Thomas T. Cormen , Charles E. Leiserson , Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Floyd-Warshall Algorithm

```
FLOYD-WARSHALL(W)
    n = W.rows
    D^(0) = W
    for k = 1 to n
        let D^(k) = (d_ij^(k)) be a new n x n matrix
        for i = 1 to n
            for j = 1 to n
                d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1)+d_kj^(k-1))
    return D^(n)
```



$$D^{(4)} = \begin{bmatrix} 0 & 3 & -1 & 4 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix} \qquad D^{(5)} = \begin{bmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{bmatrix}$$

k
5

Thomas T. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, MA, 2009

Constructing a Shortest Path

- Constructing shortest paths in the Floyd-Warshall algorithm
  - Construct the predecessor matrix Π while the algorithm computes the matrices $D^{(k)}$.
  - $\pi_{ij}^{(k)}$: Predecessor of vertex j on a shortest path from vertex i with all intermediate vertices in the set {1,2,…,k}.
- When k=0, a shortest path from i to j has no intermediate vertices at all.
  - $\pi_{ij}^{(0)} = \begin{cases} NIL & \text{if } i = j \text{ or } w_{ij} = \infty \\ i & \text{if } i \neq j \text{ and } w_{ij} < \infty \end{cases}$

Constructing a Shortest Path

- When $k \geq 1$,
  - If we take the path $i \rightsquigarrow k \rightsquigarrow j$, where $k \neq j$, then the predecessor of j we choose is the same as the predecessor of j we chose on a shortest path from k with all intermediate vertices in the set {1,2,…,k-1}.
  - Otherwise, we choose the same predecessor of j that we chose on a shortest path from i with all intermediate vertices in the set {1,2,…,k-1}.
- In other words,
  - $$\pi_{ij}^{(k)} = \begin{cases} \pi_{ij}^{(k-1)} & \text{if } d_{ij}^{(k-1)} \leq d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \\ \pi_{kj}^{(k-1)} & \text{if } d_{ij}^{(k-1)} > d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \end{cases}$$