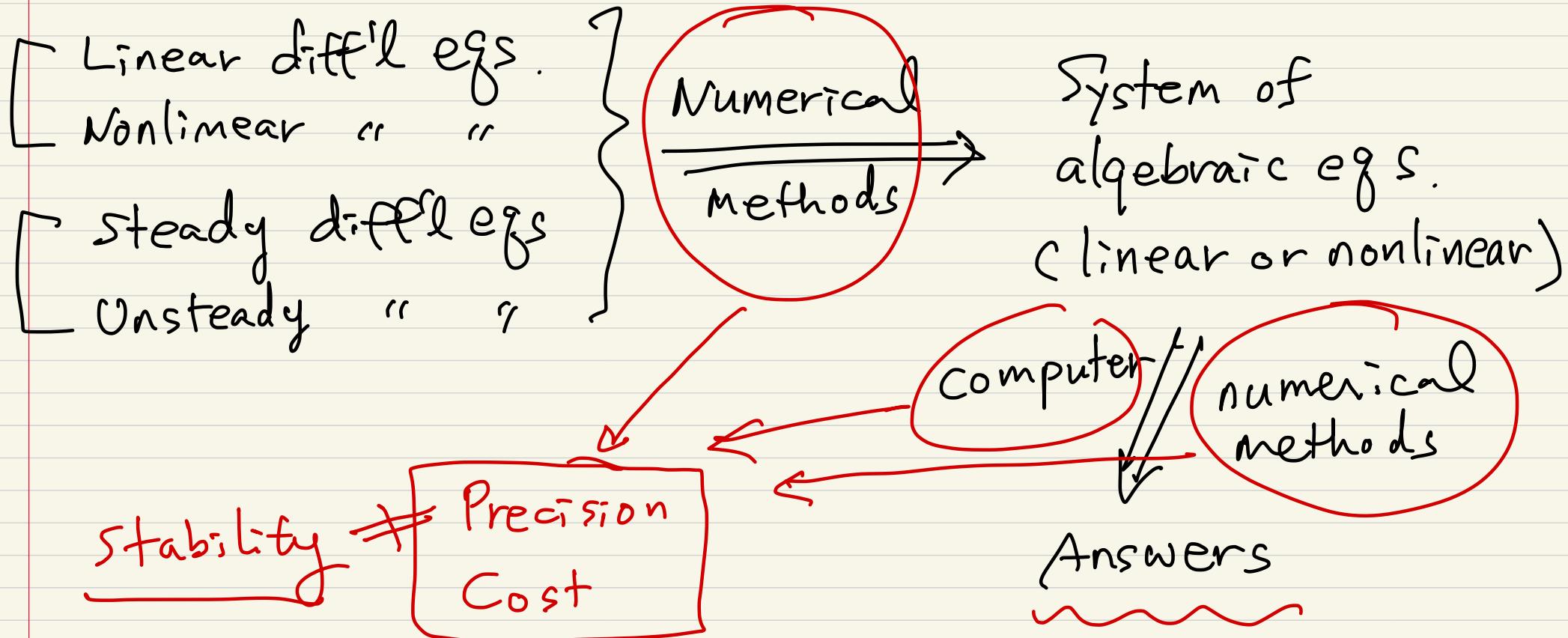


# Numerical analysis in Mechanical Engineering for Engineering Applications.



# Chapter 0, Linear Algebra

- Matrix-matrix multiplication

$$\begin{matrix} A & B \\ m \times n & n \times l \end{matrix} \longrightarrow C = AB \quad \begin{matrix} m \times l & m \times n & n \times l \end{matrix}$$

matrix elements  $c_{ij} = a_{ik} b_{kj}$   $i = 1, 2, \dots, m$   
 $j = 1, 2, \dots, l$

$k = 1, 2, \dots, n$

let the columns of  $B$  be denoted

$$\underline{b^1}, \underline{b^2}, \underline{b^3}, \dots, \underline{b^l}$$

$$AB = [A\underline{b^1}, A\underline{b^2}, \dots, A\underline{b^l}] \quad n \times 1$$

Operation counts

$$\begin{matrix} A & \underline{b} \\ m \times n & n \times 1 \end{matrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

multiplications:  
 $n \times m$

additions:  
 $(n-1) \times m$

if  $m=n$ ,  $\Theta(n^2)$  operations are required.

A B  $\longrightarrow$   $m \times n \times l$  multiplications  
 $m \times n \times l$  additions

if  $m=n=l$ ,  $\Theta(n^3)$  operations.

ex)  $n=500 \rightarrow n^3 = 1.25 \times 10^8$

Intel Itanium 2  $\rightarrow$  300 MFlops

300 MFlops  
floating-point operations  
per second

CRAY C90 Speed

$3 \times 10^8$  operations/sec

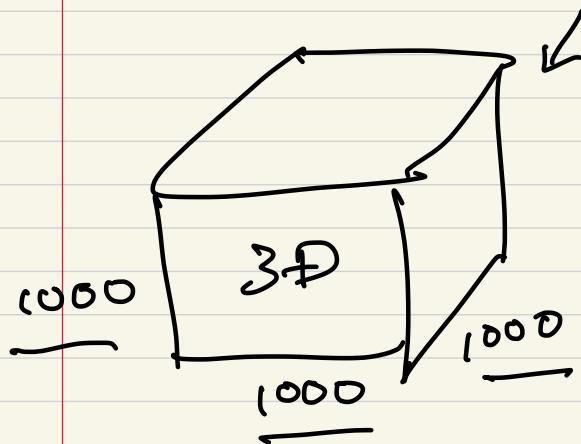
For  $AB$ ,  $t = \frac{1.25 \times 10^8}{3 \times 10^8} \approx 0.3 \text{ sec}$

In mid 1980's, IBM PC  $\rightarrow$  8000 Flops  $\rightarrow t \approx 4 \text{ hrs.}$

NEC SX5  $\rightarrow$  3 G FLOPs  $\rightarrow t \approx 0.03 \text{ sec}$

GAIA 1536 cores  $\rightarrow$  23.370 TFlops  $\rightarrow t \approx 0.5 \times 10^{-5} \text{ sec}$

How about  $n = 10^6$ ?  $\rightarrow n^3 = 10^{18}$



Intel Itanium 2  $\rightarrow t = \frac{10^{18}}{3 \times 10^8} \approx 10^6$  hrs  
(PC)

NEC SX5  $\rightarrow t = 10^5$  hrs  $\rightarrow 10^9$  kw.

$\hookrightarrow$  1M kw for 100 CPU hrs

GAIA  $t = \frac{10^{18}}{23.37 \times 10^2} \approx 42$  sec.

$\Rightarrow$  Matrix - matrix multiplication is  
a very expensive job.

$$Ax = b$$
$$x = A^{-1}b$$