

"Phase Transformation in Materials"

09.04.2023 Eun Soo Park

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Introduction

- Web lecture assistance: https://myetl.snu.ac.kr/courses/249368
 - All materials will be posted at the webpage.
 - text message will be sent for the important and urgent notice.

Text: "Phase Transformations in Metals and Alloys",
D.A. Porter and K.E. Eastering, Chapman & Hall
Prerequisite coursework: Materials Science and Engineering, Thermodynamics

References: 1) "Diffusion in Solid," Paul G. Shewman, TMS (1989)

2) "Physical Metallurgy Principles," Reed-Hill, PWS–Kent (1992)

Additional reading materials will be provided.

Course Goals

presents a unified treatment of the This course thermodynamics and kinetics of phase transformations from phenomenological and atomistic viewpoints. Phase transformations in condensed metal and nonmetal systems will be discussed. This course begins with reviewing the principles of thermodynamics, phase equilibriums, diffusion, and crystal interfaces. The topics include absolute reaction rate theory, thermodynamics of irreversible processes, thermodynamics of surfaces and interfaces, chemical kinetics, nucleation and spinodal decomposition, order-disorder growth, transformations, diffusional transformations, martensitic transformations, coarsening, and glass transition. By the end of the semester, you will be able to understand key concepts, experimental techniques, and open questions in the transformation phenomena of various materials.

Contents of this course_Phase transformation

Background to understand phase transformation (Ch1) Thermodynamics and Phase Diagrams (Ch2) Diffusion: Kinetics

(Ch3) Crystal Interface and Microstructure

Representative Phase transformation (Ch4) Solidification: Liquid \rightarrow Solid

(Ch5) Diffusional Transformations in Solid: Solid \rightarrow Solid

(Ch6) Diffusionless Transformations: Solid → Solid

Schedule

- week 1 Introduction to phase transformation
- week 2 Equilibrium single component system/ Binary solutions
- week 3 Binary phase diagram
- week 4 Ternary phase diagram
- week 5 Atomic mechanism of diffusion/ Interstitial diffusion
- week 6 Substitutional diffusion/ Atomic mobility/ Diffusion in alloy, 1st Exam
- week 7 Interfacial free energy/ Solid/vapor interfaces
- week 8 Boundaries in single-phase solids/ Interphase interfaces in solids
- week 9 Interface migration, Mid-term
- week 10 Nucleation in pure metals
- week 11 Growth a pure solid/ Alloy solidification
- week 12 Solidification of ingots and casting, 2nd Exam
- week 13 Nucleation/ Precipitate growth/ Precipitation in age-hardening alloys
- week 14 Various diffusional transformation in solids
- week 15 Diffusionless transformation Martensite transformation
- week 16 Team project presentation & Final exam

Components of Your Grade:

1) Exams (Mid-term: 35%, Final exam: 45%)(1st: 25%; 2nd 25%; final: 30%) There will be two exams, each of which will take 2-3 hours. I will not use class time for the exams and instead will reserve separate time slots.. The exams will be conceptual and difficult.

2) Team project (10%)

Course participants will organize into small groups. Topics for phase transformation will be discussed, and each group will submit a proposal and reports as well as give a final presentation.

3) Homework (5%) (+Incentive Homework 5%) and Attendance (5%) Assignments handed in after the start of class lose credit depending on the timing. If you wish, you may work together on homework assignments. But, you must hand in your own work, in your own words.

Remarks:

- 1) The weight of each component above could be adjusted up to 5% based on students' performance. 2) Student who retakes this course will have their final scores adjusted downward by 5% in order to ensure fairness with other students.
- * 청강 학생들은 숙제는 안내도 되지만 반드시 시험에 참여해야 함을 미리 고지합니다! 6

Policies and Procedures

- All homework are due by the start of class on the stated deadline.
 - Late assignments go to my office. If I'm not around, slide it under my door and leave me an email so that I know when you turned it in.
 - You lose 20% of the full assignment value per day late. Since homework are due on Wednesday, you can get 80% credit if you turn it in on Friday, 50% on next Wednesday, nothing thereafter.
- If you wish, you may work together on homework assignments. BUT, you must hand in your own work, in <u>your own words.</u>
- IMPORTANT: you MUST reference your sources appropriately, including texts, journals web sites, etc.
 - Article authors, title, journal, volume, year, pages
 - Book authors, title, publisher, year, pages
 - Web address
 - etc.



Materials Science and Engineering

Chapter 1 Introduction

- Chapter 2 Atomic Structure and Interatomic Bonding
- Chapter 3 Fundamentals of Crystallography
- Chapter 4 The Structure of Crystalline Solids
- Chapter 5 Structure of Polymers
- Chapter 6 Imperfections in Solids
- Chapter 7 Diffusion
- Chapter 8 Mechanical properties of Metals
- Chapter 9 Dislocations and Strengthening Mechanism Chapter 10 Failure
- Chapter 11 Phase Diagrams
- Chapter 12 Phase Transformations
- Chapter 13 Properties and Applications of Metals Chapter 14 Properties and Applications of Ceramics Chapter 15 Properties and Applications of Polymers Chapter 16 Composites Materials
- Chapter 17 Fabrication and Processing of Engineering Materials

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Materials Science and Engineering



Microstructure-Properties Relationships

Alloy design & Processing

Performance



Properties

down to atomic scale

"Tailor-made Materials Design"

Alloy design + Process control *Microstructure Control of Materials*



Better Material Properties

Q1: What is alloy design? "Materials selection" - Composition 1) Alloy design: a. Materials selection

One of the Most Popular Structural Materials ; Iron-Carbon Alloy (or Steel)



Steel frame of building

1) Alloy design: Materials selection b. Change of alloy compositions

Application of Iron-Carbon Alloy

K1 – main battle tank of Korea army



Need of the strongest materials

1) Alloy design: Materials selection \implies c. Change of alloy system

Dominant Material for Airplanes ; Aluminum Alloy

B737-800 of Korean Air



Need of light, strong and tough material

1) Alloy design: Materials selection \implies c. Change of alloy system



Q2: How to classify material depending on the structure?

2) Classification of Material depending on the structure *Atomic*

Structure of crystals, liquids and glasses

Crystals

Liquids, glasses





- periodic
- grain boundaries

amorphous = non-periodic no grain boundaries



Q3: What are phase, phase diagram and phase transformation?

3) Microstructure control

: Equilibrium phase → Only consider Thermodynamics ① What is Phase?

A phase is a chemically and structurally homogeneous portion of the microstructure. (structure or composition or order) Temperature Gas Steam 100Water Liquid Solid Ice l atm Phase diagram ; **Pressure** (log scale) equilibrium phase of material

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Microstructure control

: Equilibrium phase

Only consider Thermodynamics

② Phase Diagram of Temperature – Composition ;

most useful in materials science & engineering



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Microstructure control

Phase transformation ➡ Atomic Migration ③ Phase Transformation of Iron and Atomic Migration



Body-Centered Cubic

Q4: What is microstructure?

4) What is microstructure?

Microstructure originally meant the structure inside a material that could be observed with the aid of a microscope.

In contrast to the crystals that make up materials, which can be approximated as collections of atoms in specific packing arrangements (crystal structure), microstructure is the collection of defects in the materials.

What defects are we interested in?
 Interfaces (both grain boundaries and interphase boundaries), which are planar defects,
 Dislocations (and other line defects), and
 Point defects (such as interstititals and vacancies as well as solute atoms in solution)

* Imperfection: Grain Boundaries (Planer defect)



Q5: How to control the microstructure by process control?

- 5) Microstructure control
- : process control after materials selection **—** property optimization

A Example of ① Grain Boundary Engineering ; Turbine blade in Aircraft Engine



F100-PW-229 in F-16 fighting falcon

Turbine Blade

Grain boundaries at high Temperature ; Diffusion path of atoms



Reducing grain boundaries



5) Microstructure control : 2) Secondary phase control

a. Phase Diagram of Iron–Carbon Alloy



5) Microstructure control : 2) Secondary phase control b. Equilibrium Phases of Iron-Carbon Alloy



5) Microstructure control : 2 Secondary phase control

c. Mechanism of Precipitation



5) Microstructure control : 2 Secondary phase control

d. Effect of Second Phase Particle on Mechanical Property



5) Microstructure control : 2 Secondary phase control

e. Control of Microstructures by Precipitation Transformation in Aluminum Alloy

Boeing 767 by AA7150 T651 alloy



Q6: How to optimize microstructure by process control?

6) Microstructure optimization by process control

① Cold Work_압력을 가해 성형하고 인성을 증가시키는 과정

김홍도 "Smithy_대장간"



Joseon Dynasty



Modern forging machine
6) Microstructure optimization by process control

Hardening Mechanism by Cold Working



6) Microstructure optimization by process control

Changes of Strength and Ductility by Cold Working



6) Microstructure optimization by process control (2) Changes of Microstructure & Mechanical Properties

during Annealing



Q7: How to optimize material property? "Alloy design + Process control"

7) Alloy design + Process control Property optimization

e.g. Production and Application of Electrical Steel

Hot rolling - cold rolling – 1st annealing – 2nd annealing





Stacked transformer core

Coils

Transformer Motor Etc.

Soft magnetization property



Microstructural Parameters vs Properties

Microstructural Parameters

- Grain size
- Grain shape
- Phase structure
- Composite structure
- Chemical composition (alloying)
- Crystal structure
- Defect structure (e.g. porosity)

Alloy design + Process control

Microstructure Control of Materials

Properties

- Strength
- Toughness
- Formability
- Conductivity
- Corrosion Resistance
- Piezoelectric strain
- Dielectric constant
- Magnetic Permeability



Important!!! 8) *Understanding* and *Controlling* Phase Transformation *of* Materials

Q8: How to control the phase transformation? "Thermodynamics + Kinetics"



How does thermodynamics different from kinetics? Thermodynamics → There is no time variable. says which process is possible or not and never says how long it will take. The existence of a thermodynamic driving force does not mean that the reaction will necessarily occur!!!





Allotrope (同質異像): any of two or more physical forms in which an element can exist 화학성분 같고 결정구조 다름

There is a driving force for diamond to convert to graphite but there is (huge) nucleation barrier.

How long it will take is the problem of kinetics. The time variable is a key parameter.



➡ Governed by Thermodynamics & Kinetics

Q9: What are the representative PTs?

9) Representative Phase Transformation

- Solidification: Liquid \implies Solid
- Phase transformation in Solids
 - 1 Diffusion-controlled phase transformation ; Generally long-distance atomic migration
 - Precipitation transformation
 - Eutectoid transformation (**S** => **S**₁ + **S**₂)
 - etc.
 - ② Diffusionless transformation ; Short-distance atomic migration
 - Martensitic transformation

Melting and Crystallization are Thermodynamic Transitions



In general, wetting angle = 0 No superheating required!

1 Diffusion-controlled phase transformation

Diffusion- Controlled Phase Transformation time dependency



Non-Equilibrium Phases \rightarrow **Equilibrium phase**



a. Need of Controlling not only *Temperature & Composition* but *Process conditions* (Cooling Rate)

Transformation Kinetics : b. Isothermal Transformation Diagram



TTT diagram Isothermal transformation diagram

c. Isothermal Transformation Diagram of a Eutectoid Iron-Carbon Alloy



Microstructure control by phase transformation

d. Control of Phases by Heat Treatment



e. Control of Mechanical Properties by Proper Heat Treatment in Iron-Carbon Alloy



Martensite

Brittle

Tip of needle shape grain

Nucleation site of fracture

Process control



Proper heat treatment (tempering)



Tempered martensite



Very small & spherical shape grain

Good strength, ductility, toughness

② Diffusionless Transformation

Individual atomic movements are less than one interatomic spacing.

e.g. a Martensitic transformation in iron-carbon alloy

e.g. b Martensitic transformation in Ni-Ti alloy;

55~55.5 wt%Ni - 44.5~45 wt%Ti ("Nitinol")



Principles - Shape memory process



" γ
M transformation: Short-distance atomic migration "

Glass Formation is Controlled by Kinetics

- Glass-forming liquids are those that are able to "by-pass" the melting point, T_m
- Liquid may have a "high viscosity" that makes it difficult for atoms of the liquid to diffuse (rearrange) into the crystalline structure
- Liquid maybe cooled so fast that it does not have enough time to crystallize
- Two time scales are present
 - "Internal" time scale controlled by the viscosity (bonding) of the liquid
 - "External" timescale controlled by the cooling rate of the liquid



Temperature

Microstructure-Properties Relationships



Q10: What are the contents of this course?

10) Contents of this course_Phase transformation

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"Phase Transformation in Materials"

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Office: 33-313 Telephone: 880-7221 Email: espark@snu.ac.kr Office hours: by an appointment **Contents for previous class**

Microstructure-Properties Relationships



Contents for previous class

Contents of this course_Phase transformation

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(Ch2) Diffusion: Kinetics

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Contents for today's class I

Chapter 1

Thermondynamics and Phase Diagrams

- Equilibrium
- Single component system

Gibbs Free Energy as a Function of Temp. and Pressure

- Classification of phase transition
- Driving force for solidification

Q1: "thermodynamic equilibrium"? Lowest possible value of Gibb's Free Energy

→ The main use of thermodynamics in metallurgy is to allow the prediction of whether an alloy is in equilibrium.



: total potential energy of the system is a minimum.

Thermal equilibrium

: absence of temperature gradients in the system

Chemical equilibrium

: no further reaction occurs between the reacting substances i.e. the forward and reverse rates of reaction are equal.

Thermodynamic equilibrium

: the system is under mechanical, thermal and chemical equilibrium The properties of the system-P, T, V, concentrations-do not change with time.



Chapter 1.1

Relative Stability of a System \implies <u>Gibbs Free Energy</u>

Gibbs free energy : *G*=*E* +*PV*-*TS*=*H*-*TS*

Useful when P is constrained during thermodynamic process.

H : *Enthalpy* ; Measure of the heat content of the system

Mixture of one or more phases

H = E + PV

 $H \cong E$ for Condensed System

E : Internal Energy, Kinetic + Potential Energy of a atom within the system <u>Kinetic Energy :</u> Atomic Vibration (Solid, Liquid) Translational and Rotational Energy in liquid and gas. <u>Potential Energy :</u> Interactions or Bonds between the atoms within the system

T: The Absolute Temperature

S : *Entropy*, The *Randomness* of the System

Chapter 1.1



Lowest possible value of Gibb's Free Energy No desire to change ad infinitum

Equilibrium



Q2: What is single component system?

Different elements or chemical compounds



CI

Austenit

α+

liquid

1.2.1 Gibbs Free Energy as a Function of Temp. 10

1.2 Single component system Temperature

- How is phase stability measured?

* What is the role of temperature on equilibrium?

Q3: C_v vs. C_p ?

Specific heat (the quantity of heat (in joules) required to raise the temperature of substance by 1K) at constant volume VS. at constant pressure
$H = E + PV \longrightarrow dE = \delta Q - P \cdot dV$

When V is constant,

$$\frac{\partial Q}{\partial T} = \frac{dE}{dT} + P \frac{dV}{dT} \qquad C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial E}{\partial T}\right)_V \qquad C_V = \left(\frac{\partial E}{\partial T}\right)_V \quad \text{or} \quad E = \int C_V dT$$

Experimentally, it is easy to keep constant P (Cp) than constant V because it is difficult to make constant V (Cv) \rightarrow pressure ex) 1 atm,

When pressure is constant,

$$H \equiv E + PV \longrightarrow dH = dE + PdV + VdP$$
$$= \delta Q - \delta w + PdV + VdP$$
$$= \delta Q - PdV + PdV + VdP$$
$$= \delta Q - PdV + PdV + VdP$$
$$= \delta Q - PdV + PdV + VdP$$
$$= \delta Q + VdP$$

 $\frac{dP}{dT} = 0$ when *P* is constant

$$\left(\frac{dH}{dT}\right)_{P} = \left(\frac{\delta Q}{dT}\right)_{P} = C_{P}$$

 $H = \int C_p dT$

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In alloy work one usually consider systems at constant pressure so that enthalpy changes are more important then energy changes. $C_{\rm P}$; tempeature-dependent function

$$C_P = a + bT + CT^{-2}$$

(empirical formula above room temp)





Molecules have <u>internal structure</u> because they are composed of atoms that have different ways of moving within molecules. Kinetic energy stored in these <u>internal degrees of freedom</u> contributes to a substance's specific heat capacity and not to its temperature. 13 * What is the role of temperature on equilibrium?

Q4: How is C_p related with H and S?

Draw the plots of (a) C_p vs. T, (b) H vs. T and (c) S vs. T.

How is C_p related with H and S?

$$C_{P} = \left(\frac{\partial H}{\partial T}\right)_{P} \quad \mathbf{H} = \mathbf{?} \quad H = \int_{298}^{T} C_{P} dT$$

H = 0 at 298K for a pure element

in its most stable state.

: When considering a phase transformation or a chemical reaction, the important thing is not the value of the thermodynamic function, but <u>the amount of change.</u>

Enropy :
$$S = \frac{q}{T}$$

S = ? $\frac{C_P}{T} = \left(\frac{\partial S}{\partial T}\right)_P$ $S = \int_0^T \frac{C_P}{T} dT$



* What is the role of temperature on equilibrium?

Q5: How to draw the plots of H vs.T and G vs. T in single component system?

Compare the plots of H vs.T and G vs. T.



Fig. 1.3

$$dG = VdP - SdT$$

$$G(P,T) = G(P_0, T_0) + \int_{P_0}^{P_1} V(T_0, P)dP - \int_{T_0}^{T_1} S(P,T)dT$$

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* What is the role of temperature on equilibrium?

Q6: G^S vs G^L as a function of temperature?

1.2.1 Gibbs Free Energy as a Function of Temp.

- Which is larger, H^L or H^S?
- H^L > H^s at all temp.
- Which is larger, S^L or S^S?
- S^L > S^S at all temp.
- \rightarrow Gibbs free energy of the liquid <u>decreases more rapidly</u> with increasing temperature than that of the solid.



Fig. 1.4 Variation of enthalpy (H) and free energy (G) with temperature for the Solid and liquid phases of a pure metal. L is the latent heat of melting, Tm the Equilibrium melting temperature.

- Which is larger, G^L or G^S at low T?
- $G^L > G^S$ (at low Temp) and $G^S > G^L$ (at high Temp)

Fig. 1.4



Q7: What is the role of pressure on equilibrium?

* Clausius-Clapeyron Relation : $\left(\frac{dP}{dT}\right)_{eq} = \frac{\Delta H}{T_{eq}\Delta V}$ (applies to all coexistence curves)

1.2.2 Pressure Effects

When the two phases with different molar volumes are in equilibrium, if the pressure changes, the equilibrium temperature T should also change with pressure.



Fig. 1.5 Effect of pressure on the equilibrium phase diagram for pure iron

Q8: How to classify phase transition?

"First order transition" vs "Second order transition"



The First-Order Transition

Latent heat Energy barrier Discontinuous entropy, heat capacity

- First Order Phase Transition at T_T:
 - G is continuous at T_T
 - First derivatives of G (V, S, H) are discontinuous at T_T

$$V = \left(\frac{\partial G}{\partial P}\right)_T \qquad S = -\left(\frac{\partial G}{\partial T}\right)_P \qquad H = G - T\left(\frac{\partial G}{\partial T}\right)_P$$

- Second derivatives of G (α , β , C_p) are <u>discontinuous</u> at T_T

$$C_P = \left(\frac{\partial H}{\partial T}\right)_P \qquad \qquad \alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_P \qquad \qquad \beta = \frac{-1}{V} \left(\frac{\partial V}{\partial P}\right)_T$$

 Examples: Vaporization, Condensation, Fusion, Crystallization, Sublimation.





The Second Order Transition

No Latent heat Continuous entropy

- Second Order Phase Transition at T_T:
 - G is <u>continuous</u> at T_T

- First derivatives of G (V, S, H) are continuous at T_T

$$V = \left(\frac{\partial G}{\partial P}\right)_T \qquad S = \left(\frac{\partial G}{\partial T}\right)_P \qquad H = G - T \left(\frac{\partial G}{\partial T}\right)_P$$

- Second derivatives of G (α , β , C_p) are <u>discontinuous</u> at T_T

$$C_{P} = \left(\frac{\partial H}{\partial T}\right)_{P} \qquad \alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_{P} \qquad \beta = \frac{-1}{V} \left(\frac{\partial V}{\partial P}\right)_{T}$$

 Examples: Order-Disorder Transitions in Metal Alloys, Onset of Ferromagnetism, Ferroelectricity, Superconductivity.



Т

Q9: What is the driving force for "Solidification: Liquid → Solid"?

1.2.3 Driving force for solidification



Temperature Fig. 1.6

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4. Solidification: Liquid — Solid



4 Fold Anisotropic Surface Energy/2 Fold Kinetics, Many Seeds

4.1.1. Homogeneous Nucleation



$$\Delta G = G_2 - G_1 = -V_S (G_V^L - G_V^S) + A_{SL} \gamma_{SL}$$

for spherical nuclei (isotropic) of radius : r

$$\Delta G_r = -\frac{4}{3} \pi r^3 \Delta G_V + 4\pi r^2 \gamma_{SL}$$

4.1.1. Homogeneous Nucleation



Fig. 4.2 The free energy change associated with homogeneous nucleation of a sphere of radius r.

Why r^{*} is not defined by $\Delta G_r = 0$?

 $r < r^*$: unstable (lower free E by reduce size) $r > r^*$: stable (lower free E by increase size) r^* : critical nucleus size



1.2.3 Driving force for solidification



Electrostatic levitation in KRISS



KRISS material : Dr. G.W.Lee



Cyclic cooling curves of $Zr_{41.2}Ti_{13.8}Cu_{12.5}Ni_{10}Be_{22.5}$



* The homogeneous nucleation rate - kinetics

$$I = \frac{nkT}{h} \exp\left(-\frac{G_A}{kT}\right) \exp\left(-\frac{16\pi\sigma_{SL}^3 T_E^2}{3L^2(\Delta T)^2 kT}\right)$$

: insensitive to Temp.



* Relationship between Maximum Supercoolings and T_m



Fig. 3.7. Maximum supercooling as a function of melting point. (From *Thermo*dynamics in *Physical Metallurgy*, American Society for Metals, Cleveland, 1911, p. 11.)

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Solidification: Liquid ---- Solid

- casting & welding
- single crystal growth
- directional solidification
- rapid solidification

Nucleation in Pure Metals

- $T_{\rm m}$: $G_{\rm L}$ = $G_{\rm S}$
- Undercooling (supercooling) for nucleation: 250 K ~ 1 K
- <Types of nucleation>
- Homogeneous nucleation



- Heterogeneous nucleation



Q10: What is the driving force for "Melting: Solid → Liquid"?

* Driving force for melting



Nucleation of melting

Although nucleation during solidification usually requires some undercooling, melting invariably occurs at the equilibrium melting temperature even at relatively high rates of heating.



In general, wetting angle = 0 No superheating required!

Melting and Crystallization are Thermodynamic Transitions



Contents for today's class

Chapter 1

Thermondynamics and Phase Diagrams

- Equilibrium a

Phase Transformation

$$dG = 0$$
 $AG = G_0 - G_1 < 0$

$$\Delta G = G_2 - G_1 < 0$$

Lowest possible value of G

- No desire to change ad infinitum
- Single component system

Gibbs Free Energy as a Function of Temp. and Pressure

$$\left(\frac{\partial G}{\partial T}\right)_P = -S, \quad \left(\frac{\partial G}{\partial P}\right)_T = V \qquad \left(\frac{dP}{dT}\right)_{eq} = \frac{\Delta H}{T_{eq}\Delta V}$$

Clausius-Clapevron Relation

- Classification of phase transition

First order transition: CDD/Second order transition: CCD

- Driving force for solidification

$$\Delta G = \frac{L\Delta T}{T_m}$$