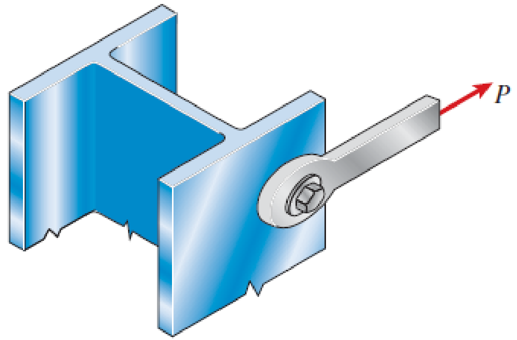
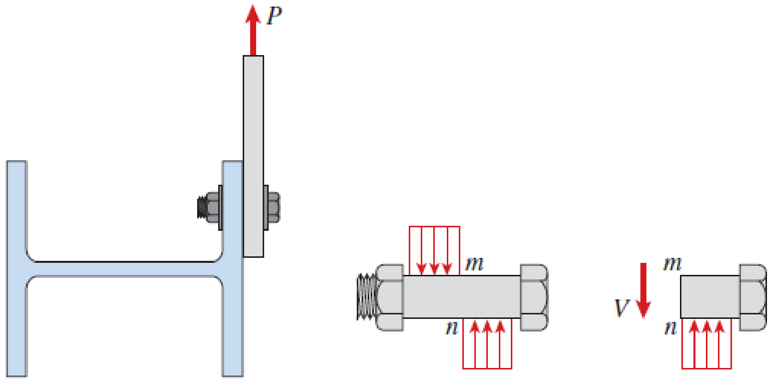
Homework reminder (22nd Mar)

Interesting Question.

- Shear or Moment?



(a)



(b) (c) (d)

$$\tau_{aver} = \frac{V}{A}$$

Kahoot discussion

Truss vs Frame

Static Review

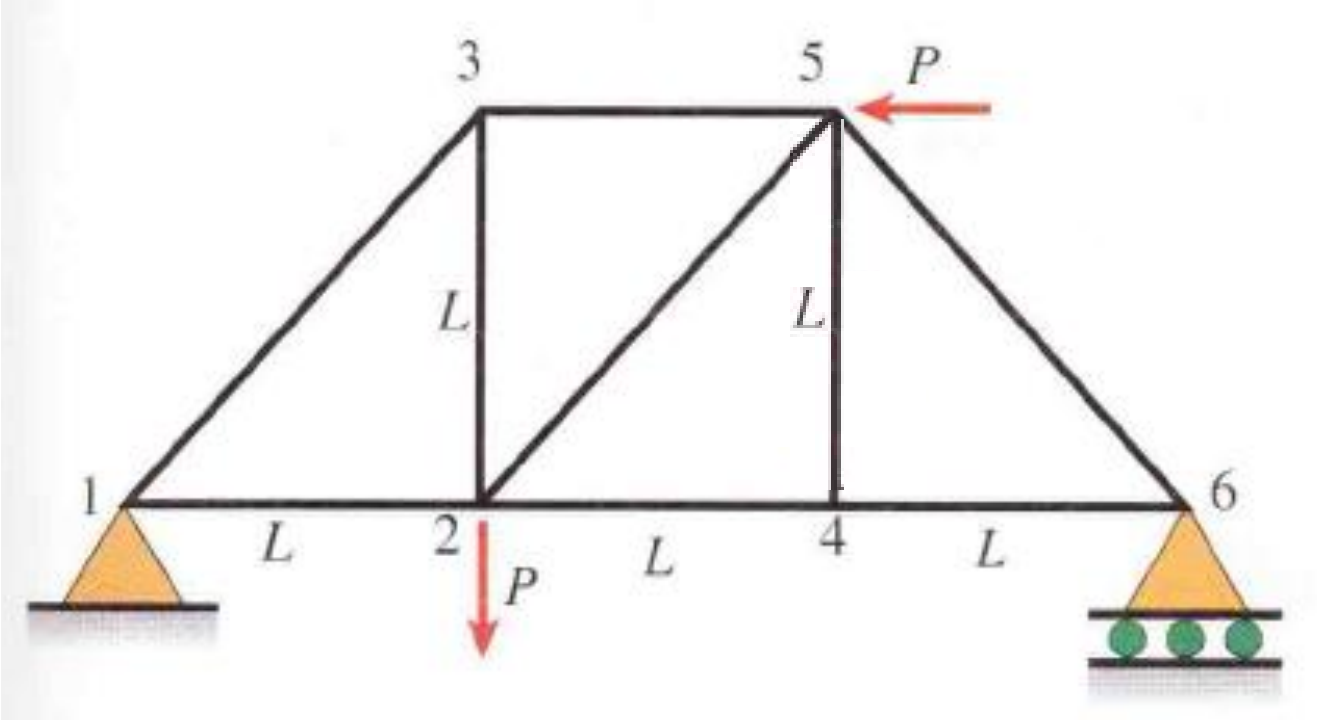
A plane truss has downward applied load P at joint 2 and another load P applied leftward at joint 5. The force in member 4-5 is:

(A) 0

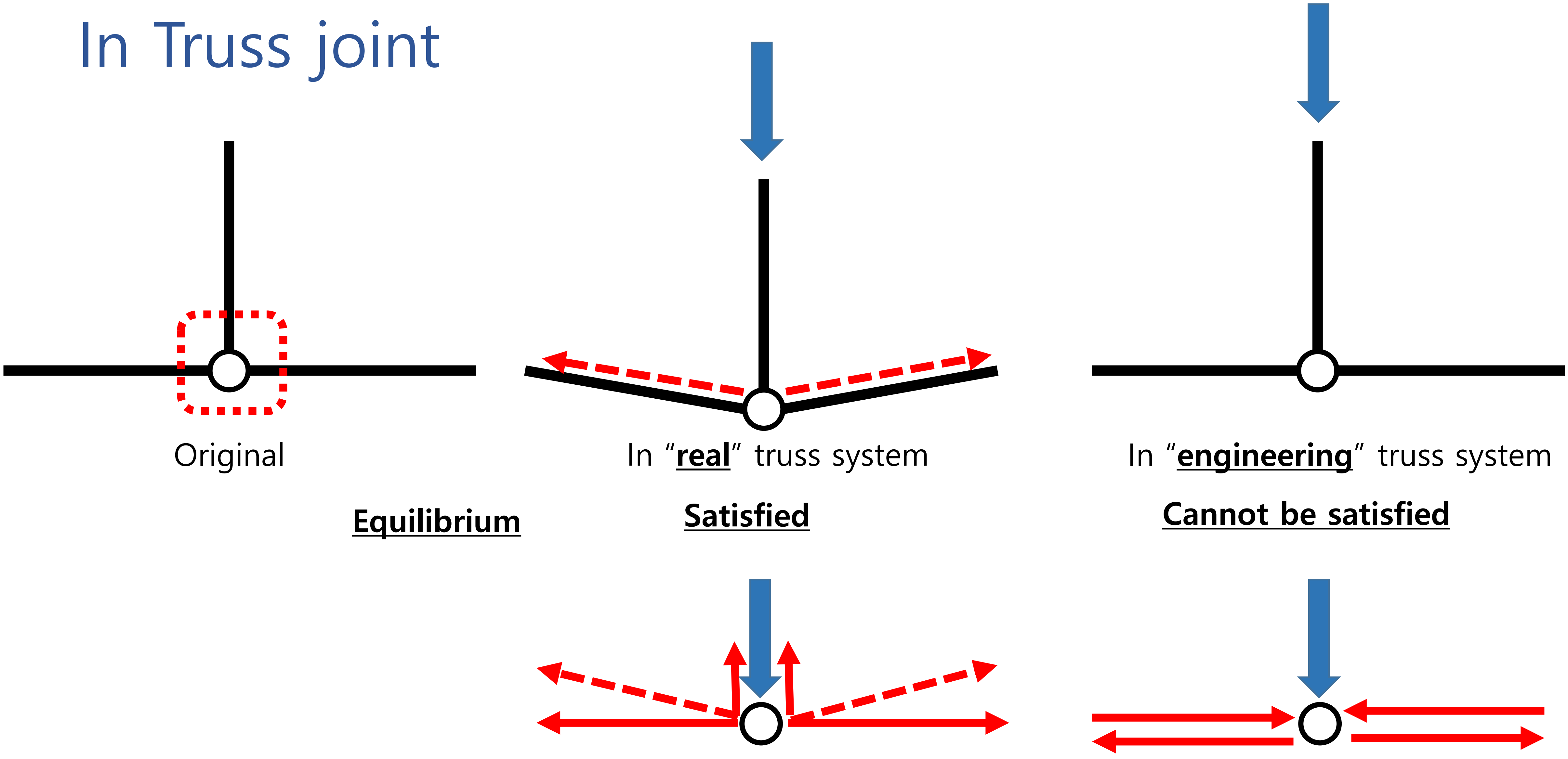
(B) –P/2

(C) -P

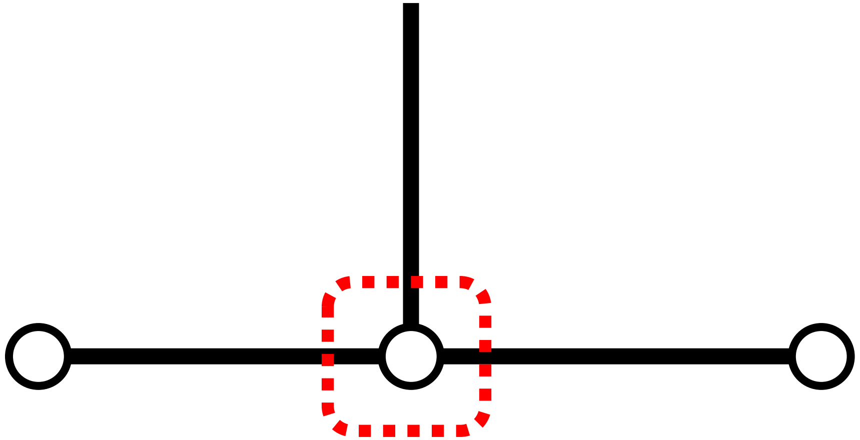
(D) + 1.5 P



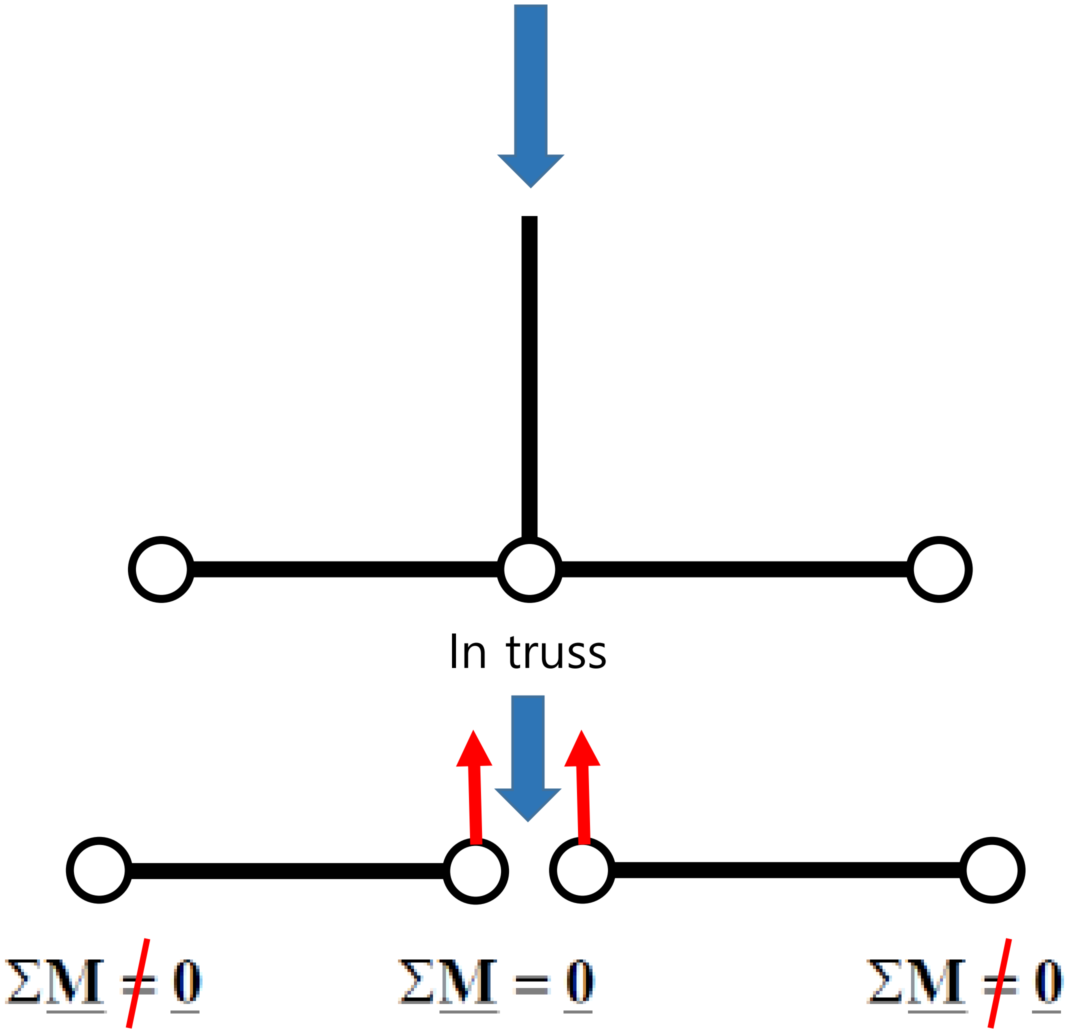
In Truss joint



Another view point



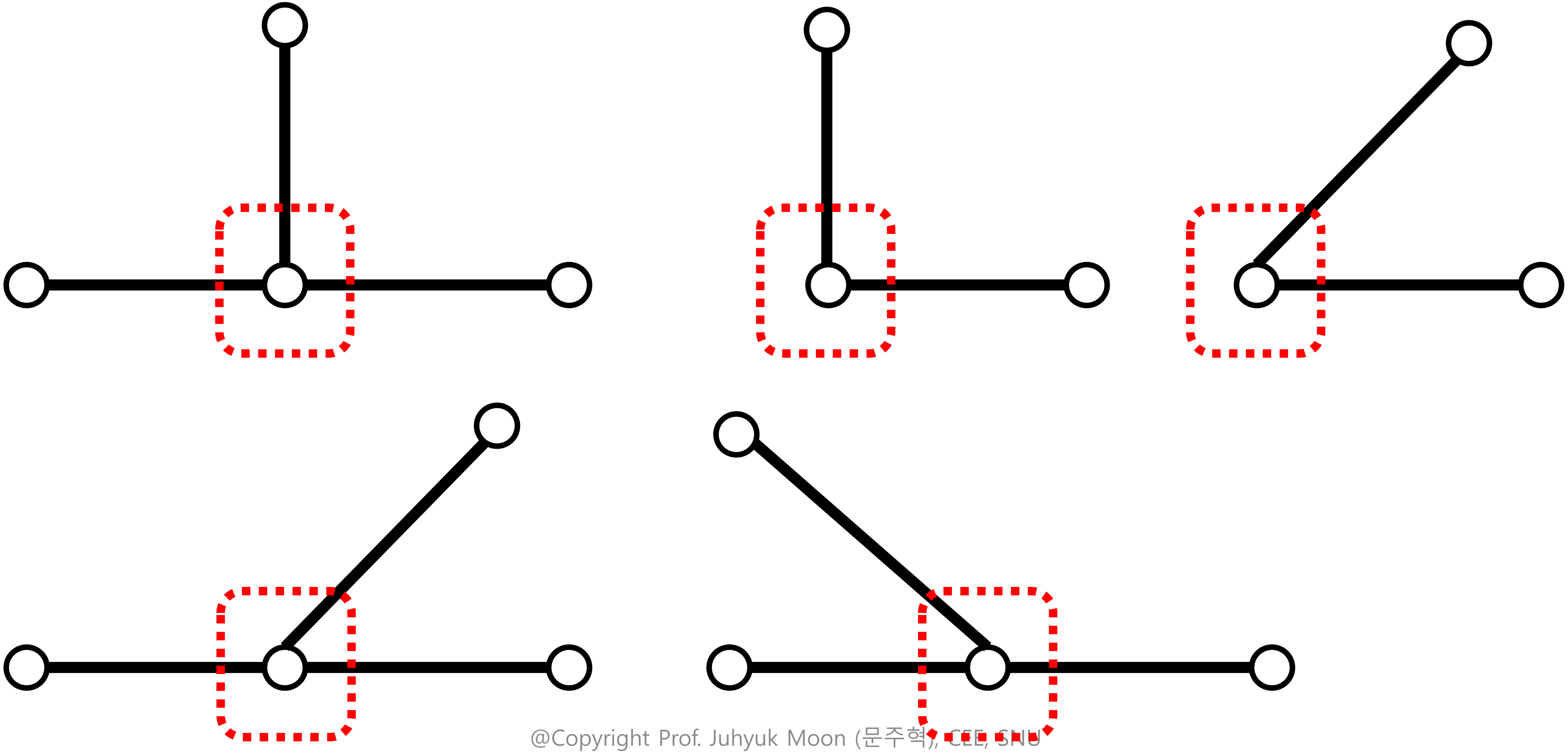
Original



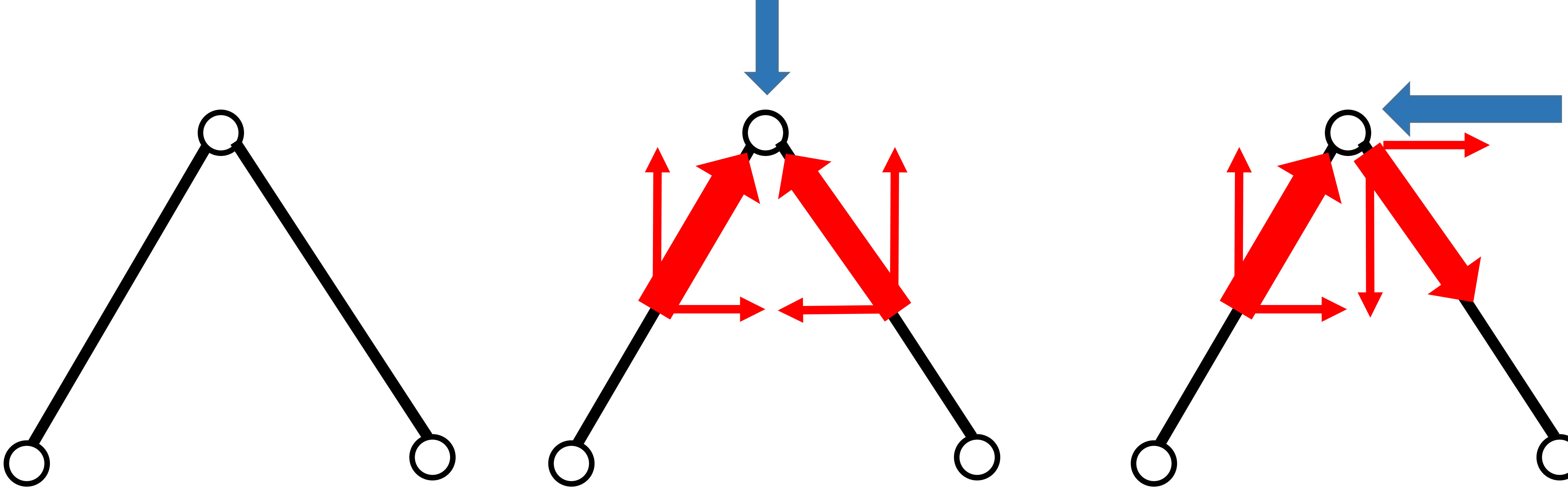
<u>Moment Equilibrium</u>

<u>Cannot be satisfied</u>

Other zero-force members in truss



Other zero-force members in truss



Equilibrium

$\Sigma F_x = 0 \;;\quad \Sigma F_y \neq 0$

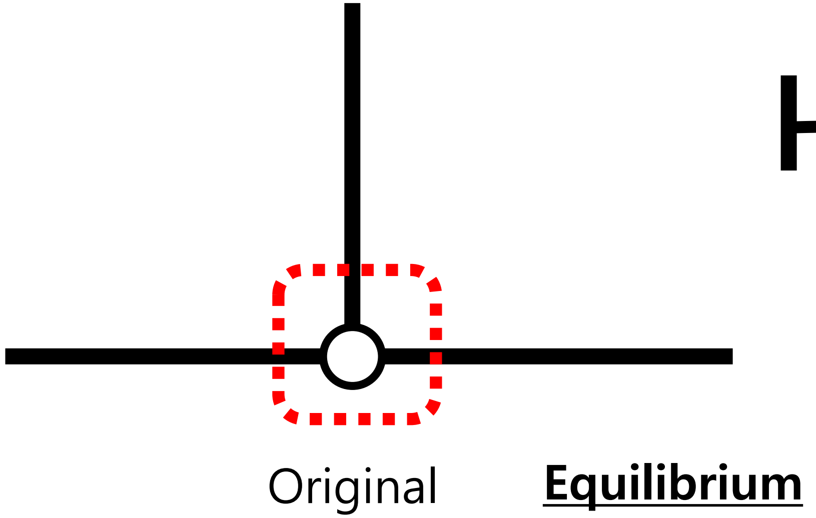
$\Sigma F_x = 0 \;;\quad \Sigma F_y = 0$

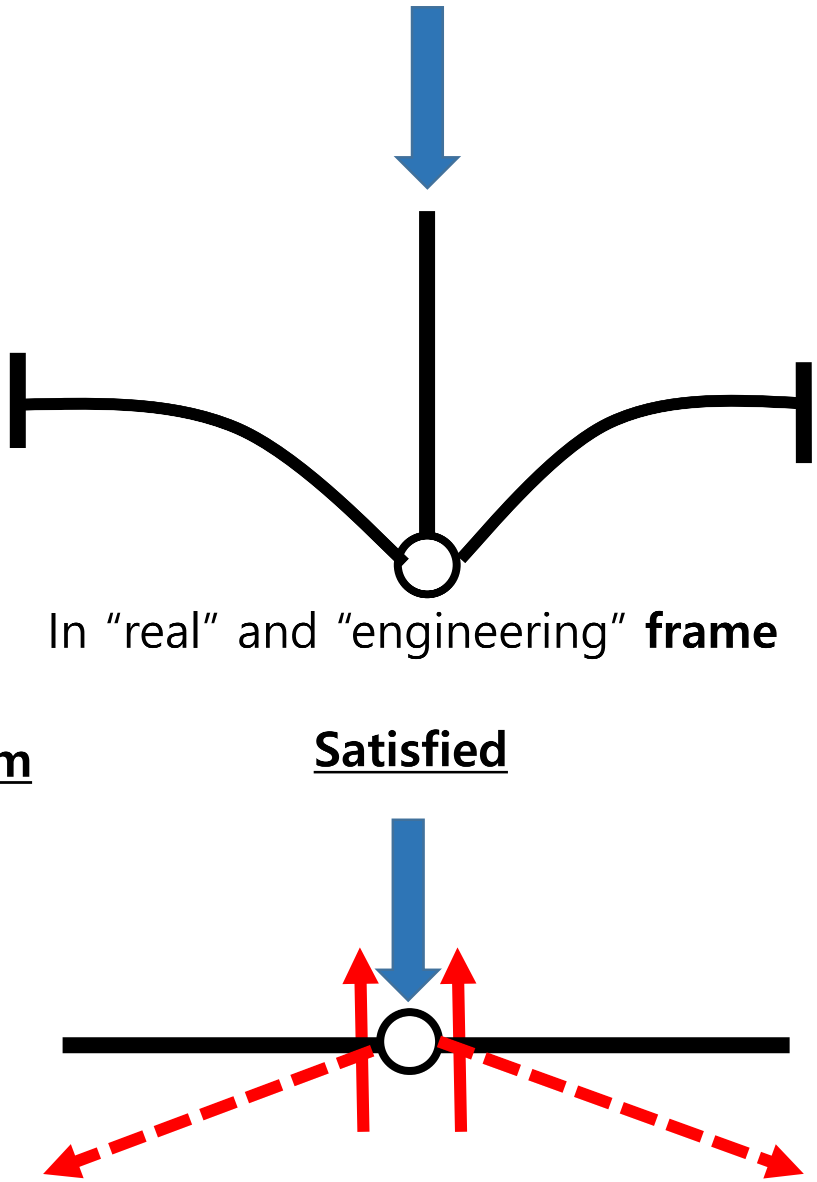
Equilibrium

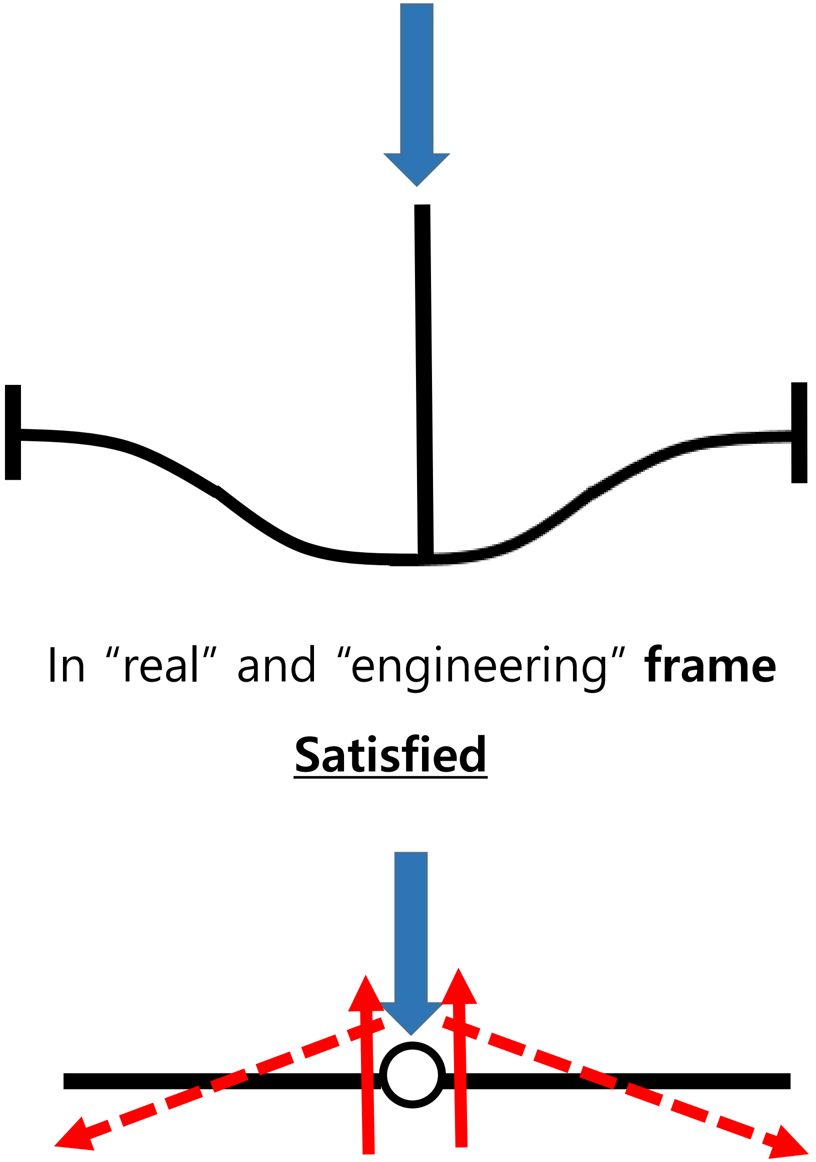
$\Sigma F_x \neq 0 \;;\quad \Sigma F_y = 0$

$\Sigma F_x = 0 \;;\quad \Sigma F_y = 0$

Frame


Original
Equilibrium


In “real” and “engineering” **frame**
Satisfied


In “real” and “engineering” **frame**
Satisfied

Static Review

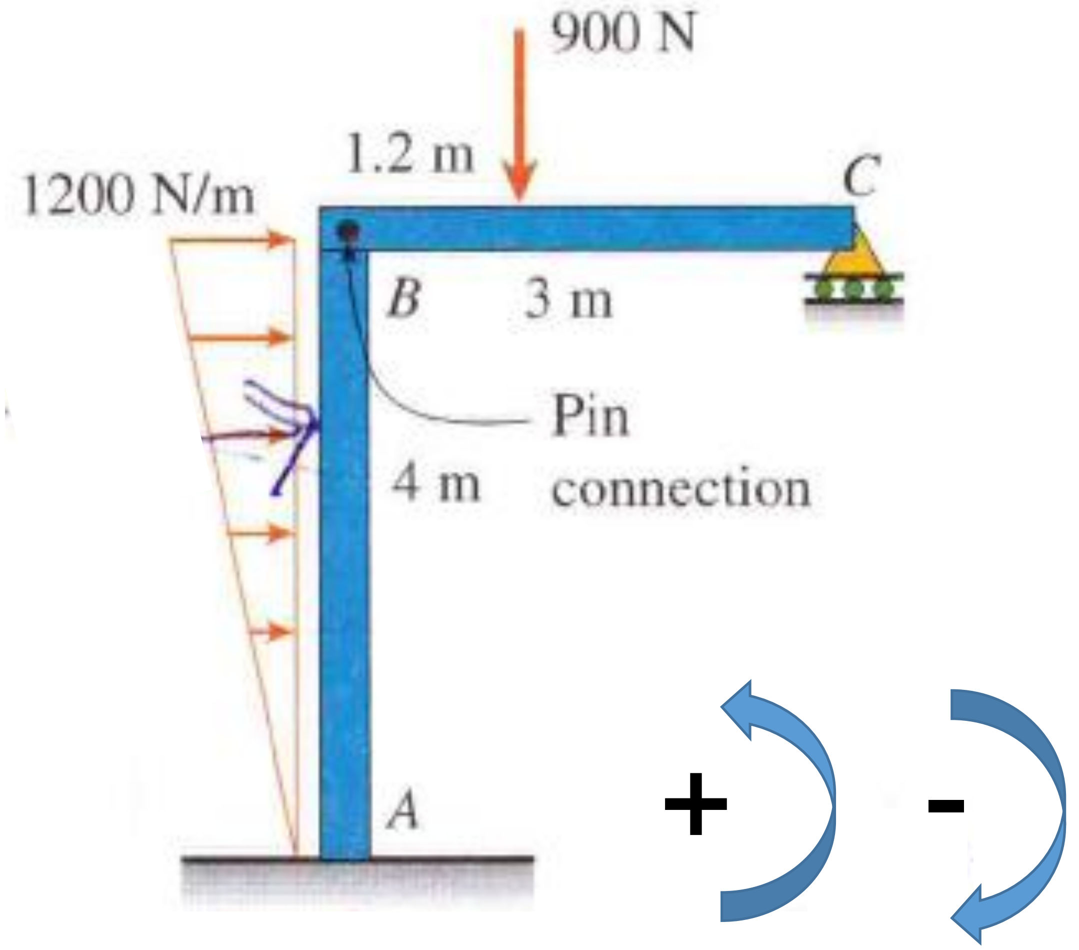
The moment reaction at A in the plane frame below is approximately:

(A) +1400 N.m

(B) -2280 N.m

(C) -3600 N.m

(D) +6400 N.m

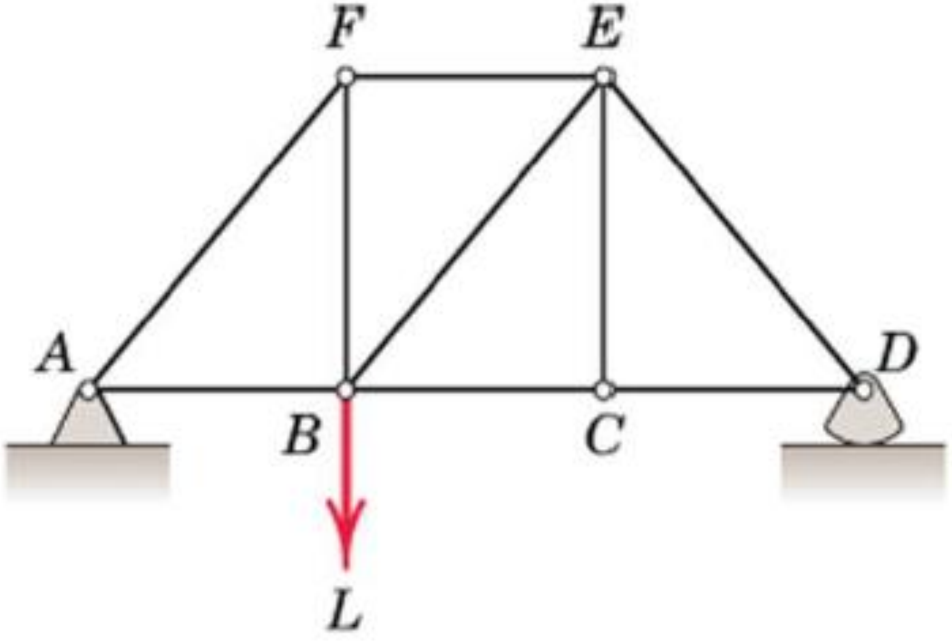


Static Review

- Truss system
- No. of members (m) = 9
- No. of joints (j) = 6
- No. of unknown reactions (R) = 3

$m + R = 2j$

: Statically Determinate Structure

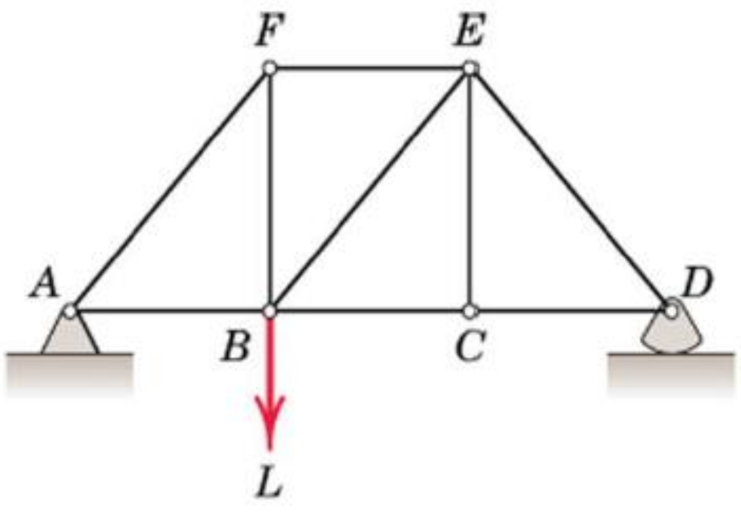


Static Review

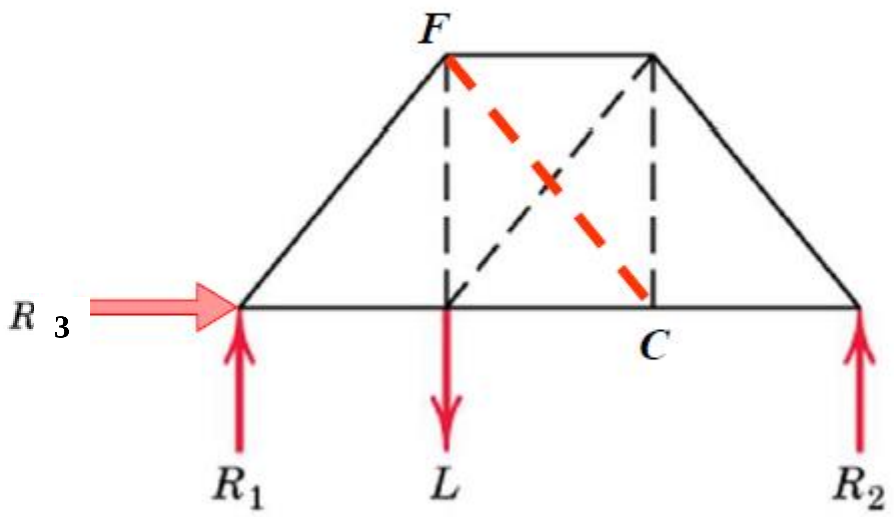
- **Add member FC**
- No. of members (m) = 10
- No. of joints (j) = 6
- No. of unknown reactions (R) = 3

$m + R > 2j$

: Statically Indeterminate Structure

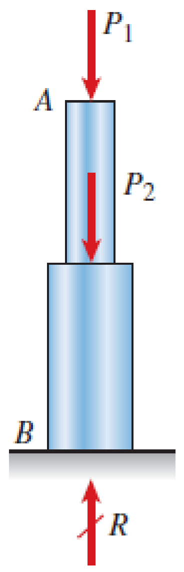


FC = Additional sharing for forces, Additional Stability

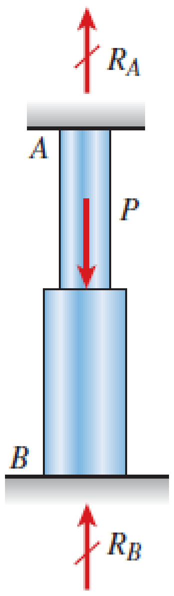


Static Review

- **Other examples.**



Statically Determinate Structure



Statically Indeterminate Structure

Determinacy

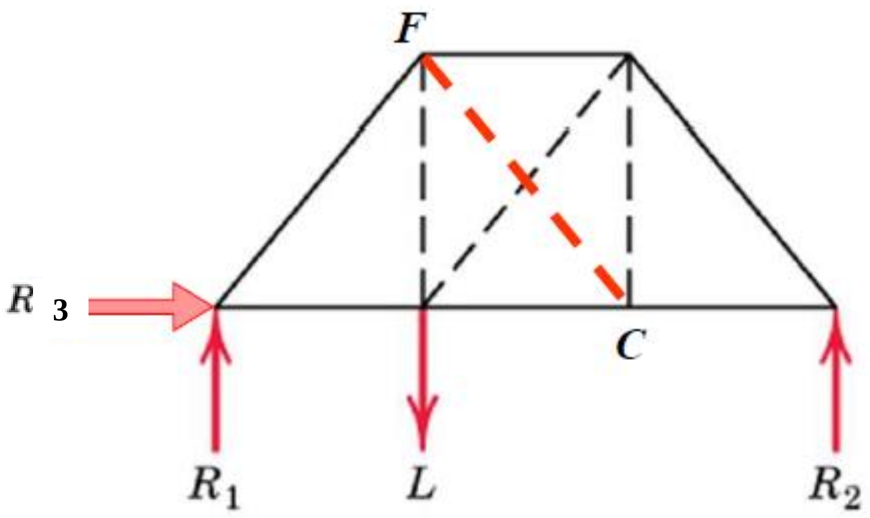
- When more number of members/supports are present than are needed to prevent collapse/stability

  : Statically Indeterminate Structure (truss)

- Cannot be analyzed using equations of equilibrium alone!
- Additional members or supports which are not necessary for maintaining the equilibrium configuration

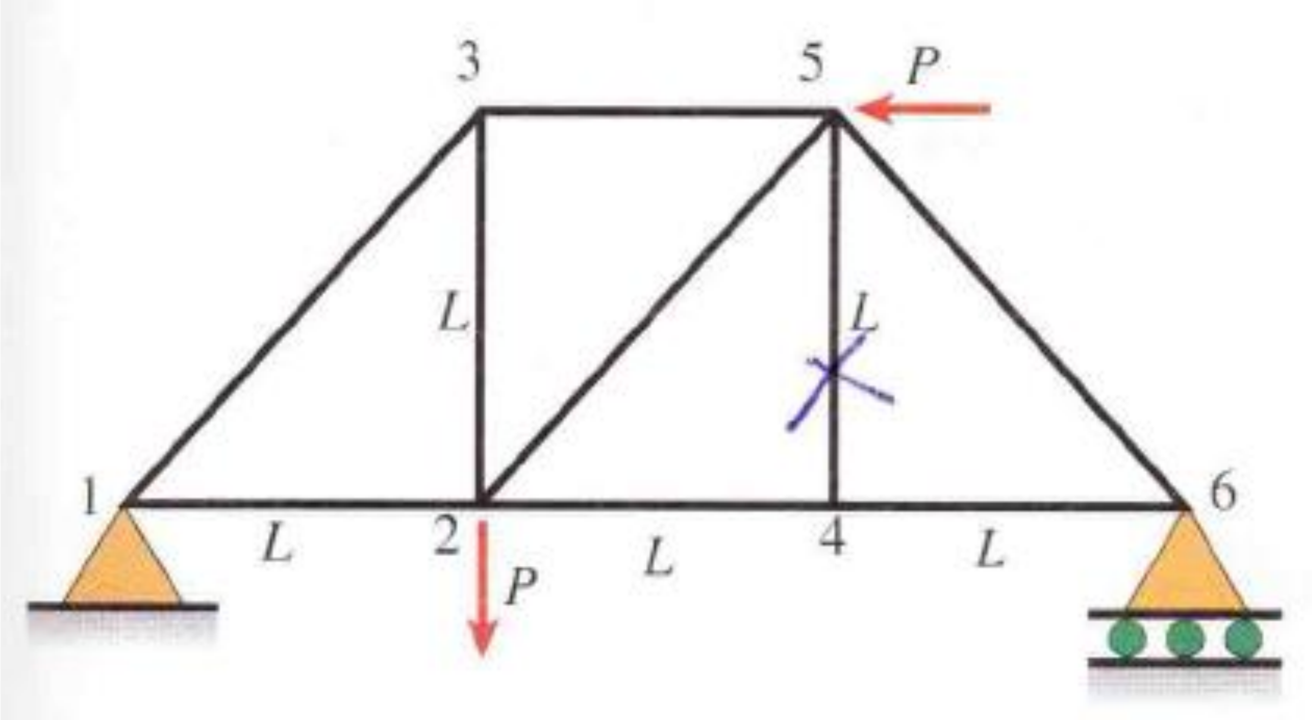
  : Redundant member

- We can solve the statically indeterminate structure but it is more difficult (but possible!)

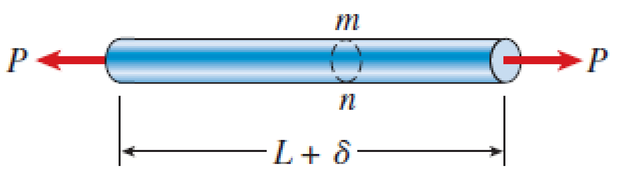


Zero force members in “**Truss (All pin joints)**”

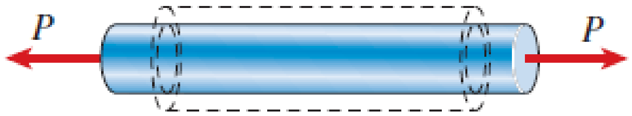
- Element 4-5
- Cannot satisfy “Force Equilibrium (y)” condition at joint 4
- How about in “Frame”?



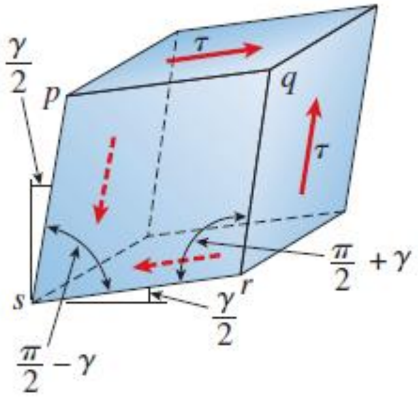
Chapter 2. Axially Loaded Members



$$\sigma = E\epsilon$$



$$\nu = -\frac{\epsilon'}{\epsilon}$$



$$\tau = G\gamma$$

- Stiffness (Flexibility)
- Changes under non-uniform conditions
- Statically Indeterminate Structures
- Thermal effect
- Stress on inclined section
- ~~Strain energy~~
- ~~Impact loading~~
- ~~Stress concentration~~
- ~~Nonlinear behavior~~

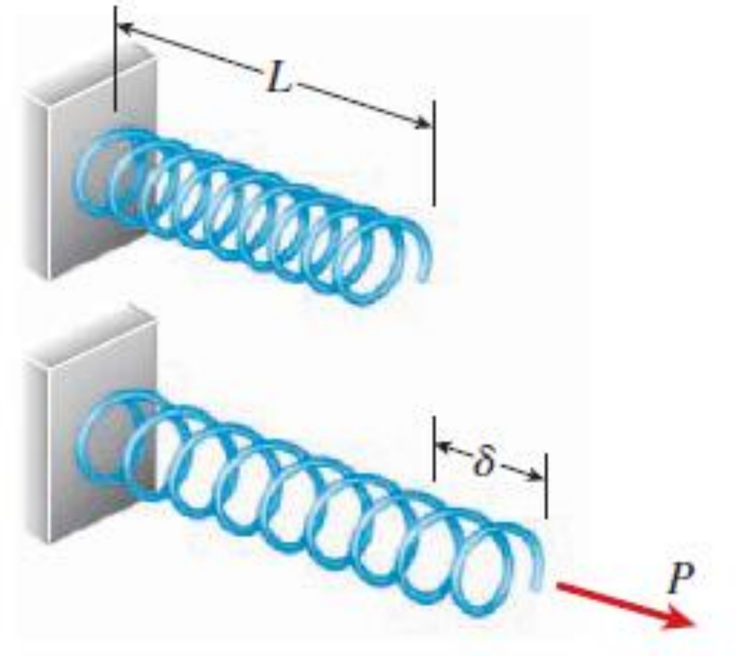
Stiffness (Flexibility)

- Changes in lengths of axially loaded member
- L (unstressed length)
- Load and elongation will be proportional:

$$P = k\delta \qquad \delta = fP$$

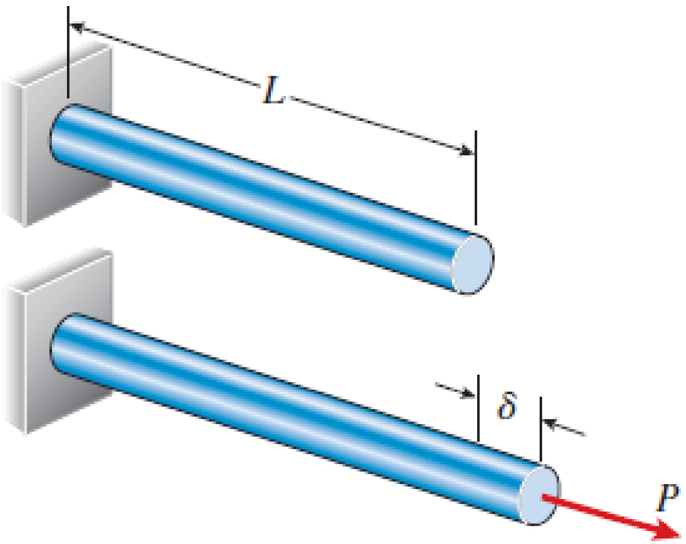
- Stiffness and flexibility are the reciprocal of each other

$$k = \frac{1}{f} \qquad f = \frac{1}{k}$$



Stiffness (Flexibility)

- Prismatic Bars



$$\delta = \frac{PL}{EA}$$

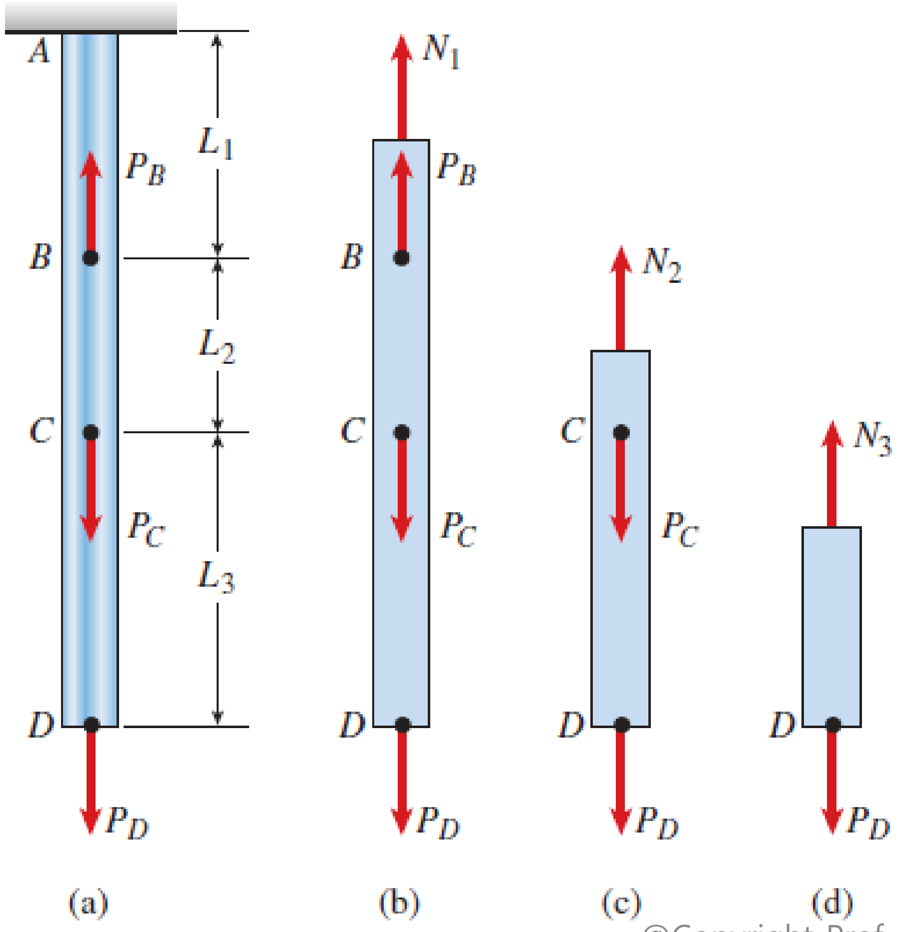
- Stiffness (k) and Flexibility (f) ->

$$k = \frac{EA}{L} \qquad f = \frac{L}{EA}$$

$$P = k\delta \qquad \delta = fP$$

Changes in lengths under non-uniform condition

- Prismatic bar with multiple loading points



$$N_1 = -P_B + P_C + P_D$$

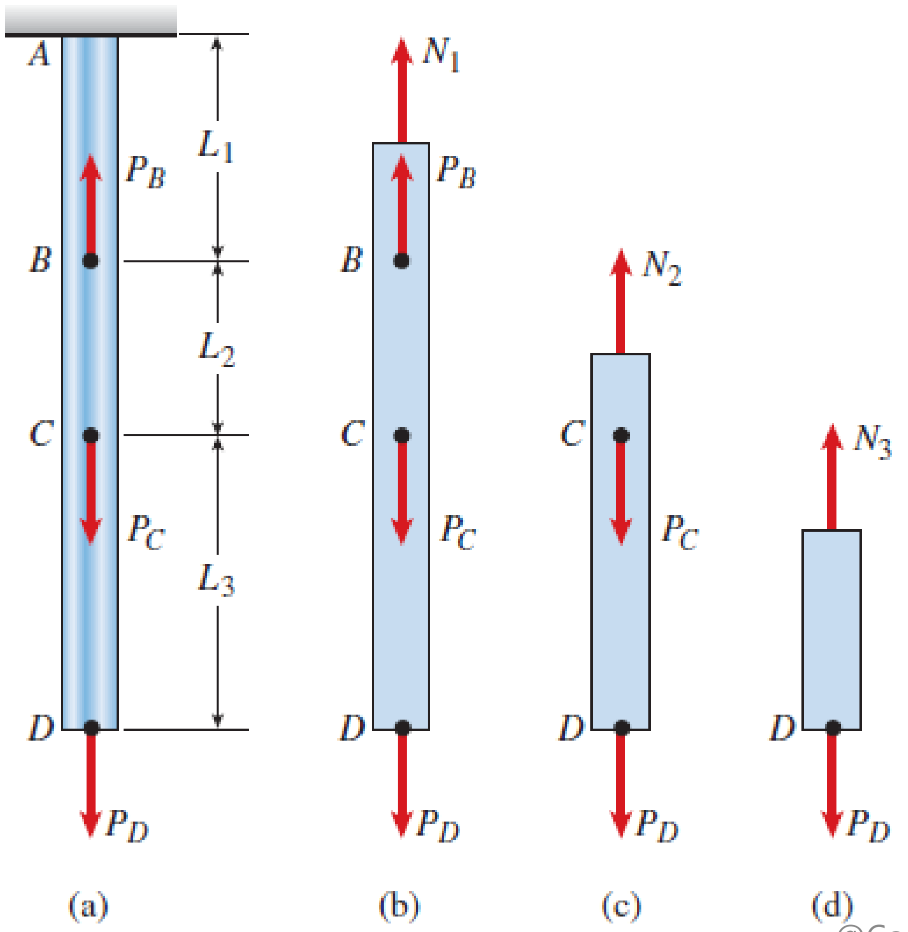
$$N_2 = P_C + P_D$$

$$N_3 = P_D$$

Example: Rubber in tension

Changes in lengths under non-uniform condition

- Prismatic bar with multiple loading points



$$N_1 = -P_B + P_C + P_D$$

$$N_2 = P_C + P_D$$

$$N_3 = P_D$$

$$\delta_1 = \frac{N_1 L_1}{EA}$$

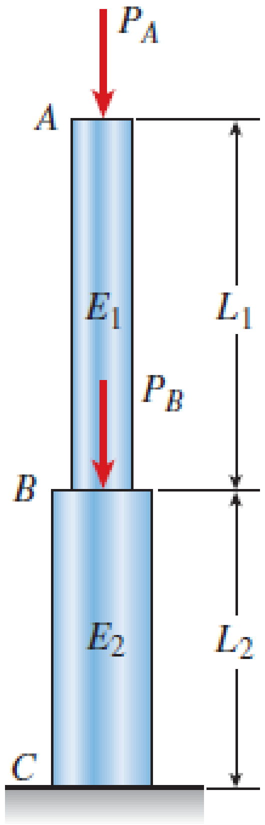
$$\delta_2 = \frac{N_2 L_2}{EA}$$

$$\delta_3 = \frac{N_3 L_3}{EA}$$

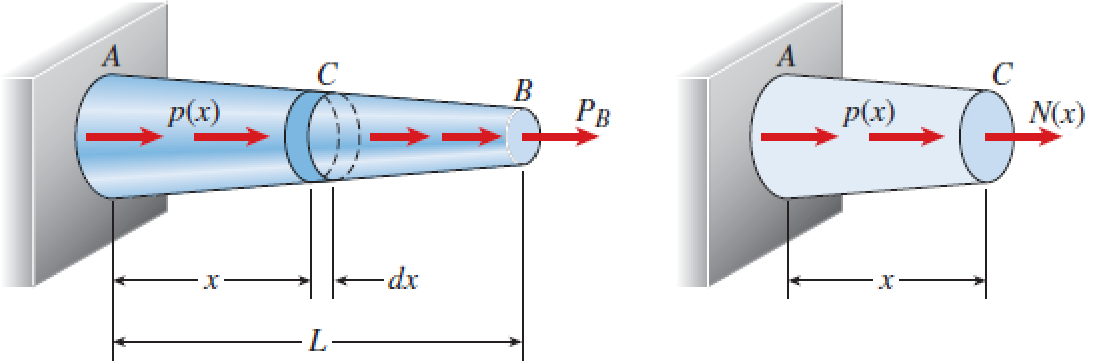
$$\delta = \sum_{i=1}^{3} \delta_i = \delta_1 + \delta_2 + \delta_3$$

Changes in lengths under non-uniform condition

- Prismatic bar with multiple loading points with various sections



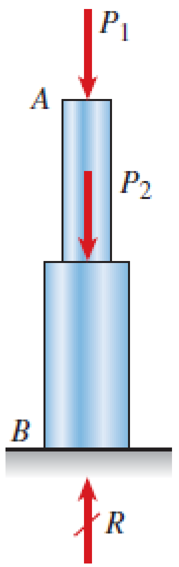
$$\delta = \sum_{i=1}^{n} \frac{N_i L_i}{E_i A_i}$$



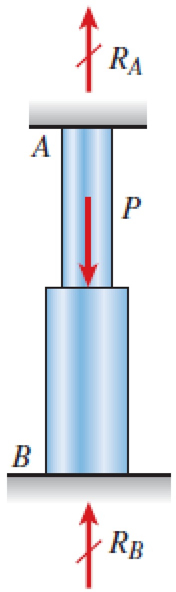
$$\delta = \int_0^L d\delta = \int_0^L \frac{N(x)dx}{EA(x)}$$

Statically indeterminate structures

- Compute net axial force in each cases.



Statically Determinate Structure



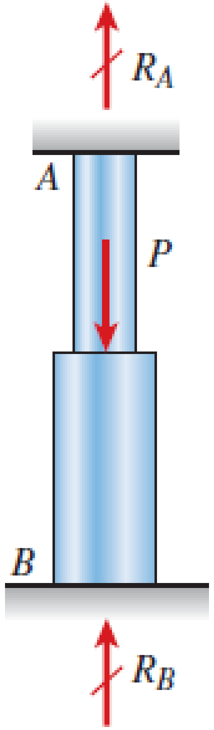
Statically Indeterminate Structure

Statically indeterminate structures

$$\sum F_{vert} = 0 \qquad R_A - P + R_B = 0$$

- What should we calculate?
- There are two vertical reactions but only one useful equation of equilibrium
- We need something else! -> Use an equation of compatibility

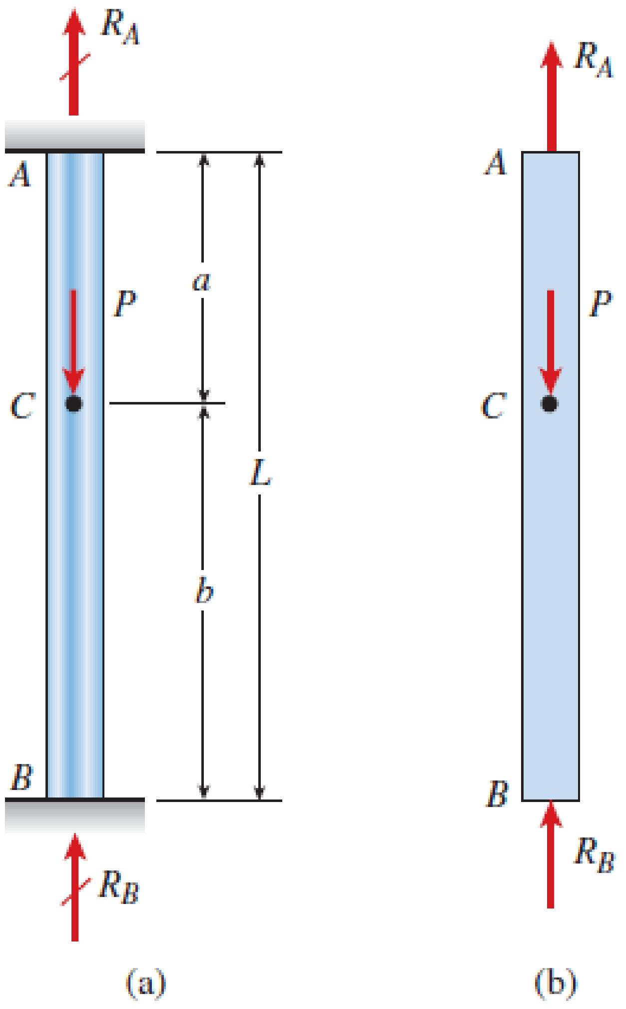
$$\delta_{AB} = 0$$



Statically Indeterminate Structure

Statically indeterminate structures

- Decompose members



$$\delta_{AC} = \frac{R_A a}{EA}$$

$$\delta_{CB} = -\frac{R_B b}{EA}$$

$$\delta_{AB} = \delta_{AC} + \delta_{CB} = \frac{R_A a}{EA} - \frac{R_B b}{EA} = 0$$

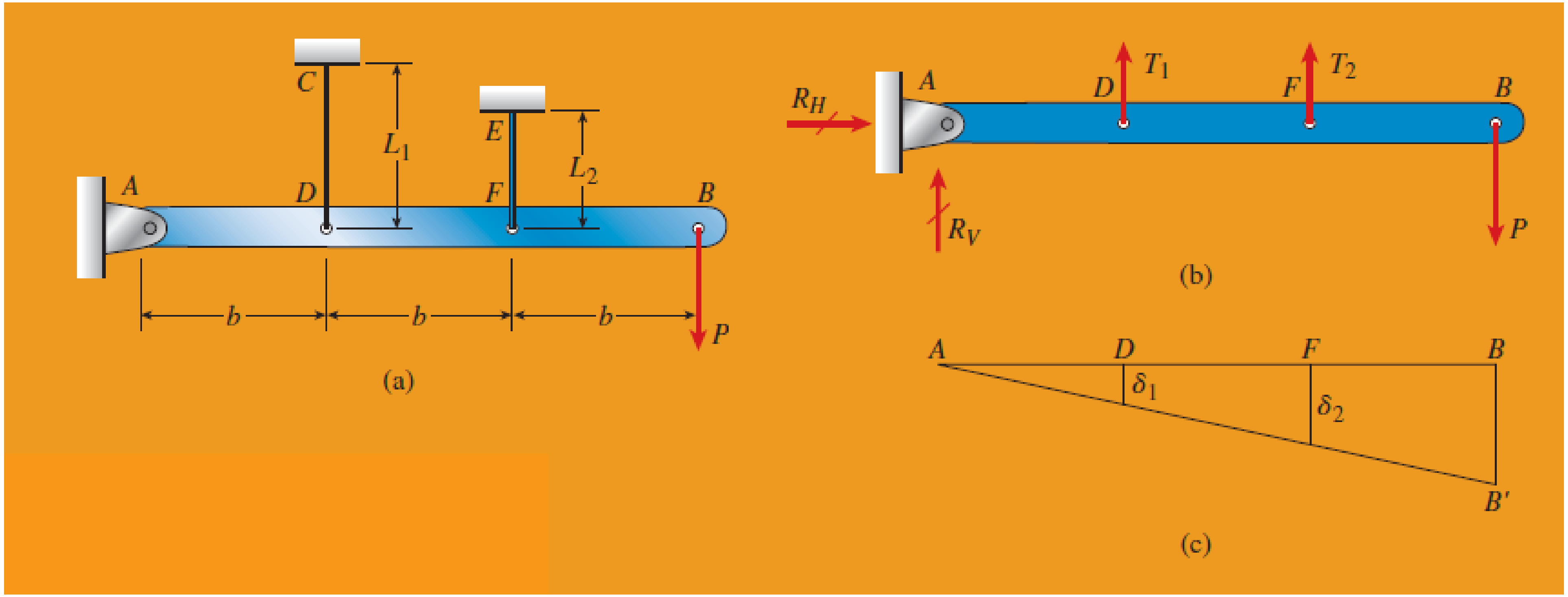
$$\sum F_{\text{vert}} = 0 \qquad R_A - P + R_B = 0$$

$$R_A = \frac{Pb}{L} \qquad R_B = \frac{Pa}{L}$$

$$\delta_C = \delta_{AC} = \frac{R_A a}{EA} = \frac{Pab}{LEA}$$

Statically indeterminate structures

- Example 2-6



Thermal effect

- Temperature change will result in thermal strain

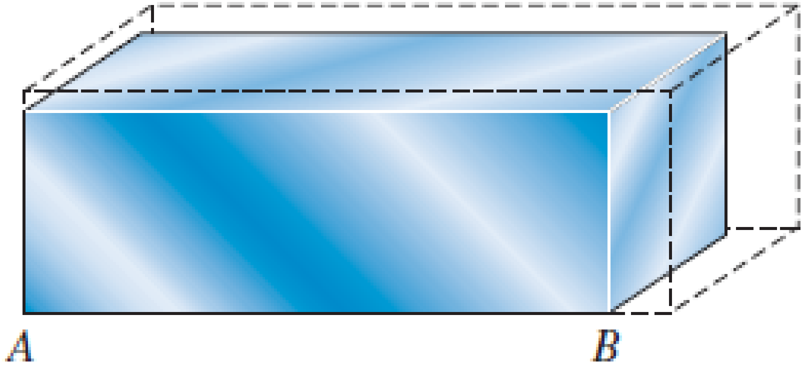


FIG. 2-19 Block of material subjected to an increase in temperature

- Thermal expansion coefficient α $\alpha_L = \frac{1}{L}\frac{dL}{dT}$ (unit: 10^{-6}/K or /K or /'C)

$$\epsilon_T = \alpha(\Delta T)$$

$$\delta_T = \epsilon_T L = \alpha(\Delta T)L$$

Temperature-displacement relation
(force-displacement relation)

- Sign convention (expansion +, contraction -)

Thermal effect

- Thermal expansion coefficient α $\alpha_L = \frac{1}{L}\frac{dL}{dT}$ (unit: 10^{-6}/K or /K)

$$\epsilon_T = \alpha(\Delta T)$$

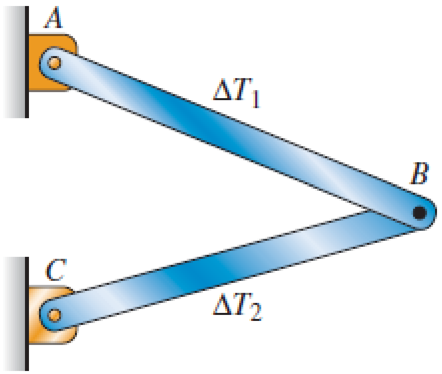
$$\delta_T = \epsilon_T L = \alpha(\Delta T)L$$

TABLE H-4 COEFFICIENTS OF THERMAL EXPANSION

Material	Coefficient of thermal expansion α		Material	Coefficient of thermal expansion α	
	10^{-6}/°F	10^{-6}/°C		10^{-6}/°F	10^{-6}/°C
Aluminum alloys	13	23	Plastics		
Brass	10.6–11.8	19.1–21.2	Nylon	40–80	70–140
Bronze	9.9–11.6	18–21	Polyethylene	80–160	140–290
Cast iron	5.5–6.6	9.9–12	Rock	3–5	5–9
Concrete	4–8	7–14	Rubber	70–110	130–200
Copper and copper alloys	9.2–9.8	16.6–17.6	Steel	5.5–9.9	10–18
Glass	3–6	5–11	High-strength	8.0	14
Magnesium alloys	14.5–16.0	26.1–28.8	Stainless	9.6	17
Monel (67% Ni, 30% Cu)	7.7	14	Structural	6.5	12
Nickel	7.2	13	Titanium alloys	4.5–6.0	8.1–11
			Tungsten	2.4	4.3

Thermal effect

$$\epsilon_T = \alpha(\Delta T)$$



- Does it always induce thermal stress?

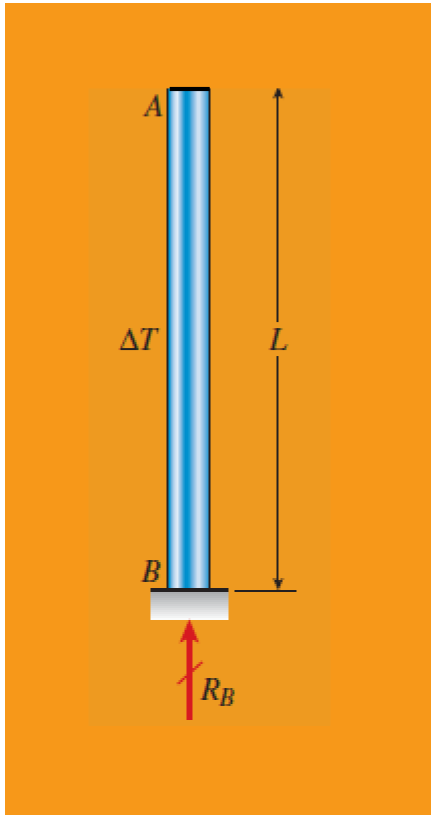
$$\sigma = E\alpha(\Delta T)$$

- If delta T1 is different from delta T2, will thermal stress occur?

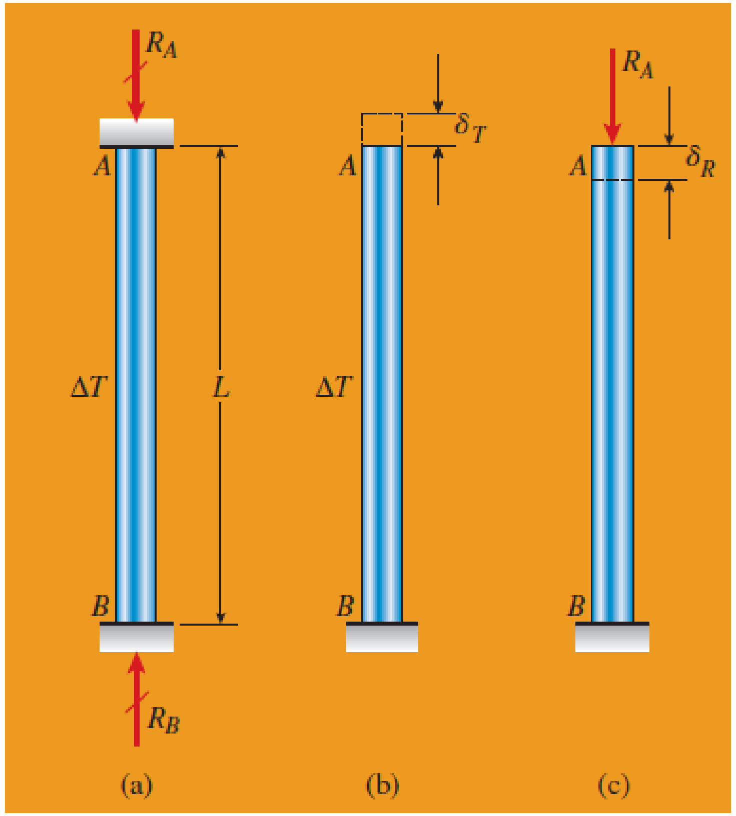
- No stresses in either bar and no reactions at supports in structurally determinate structure. (less restraint, freely movable)

Thermal effect

- Example 2-7



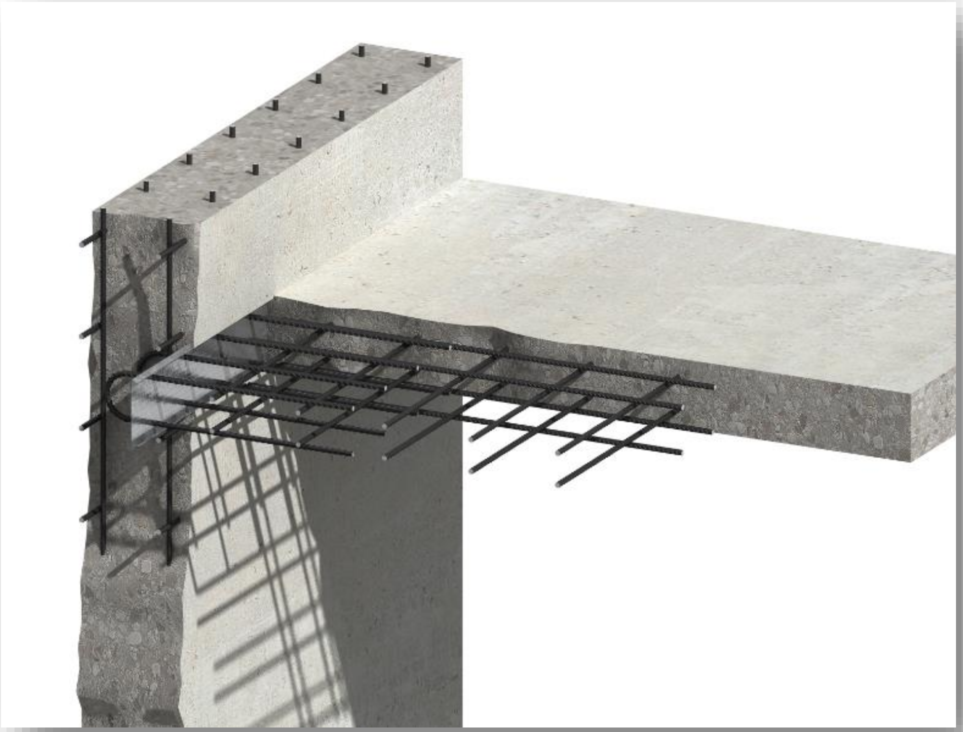
Statically Determinate Structure



Statically Indeterminate Structure

Thermal effect

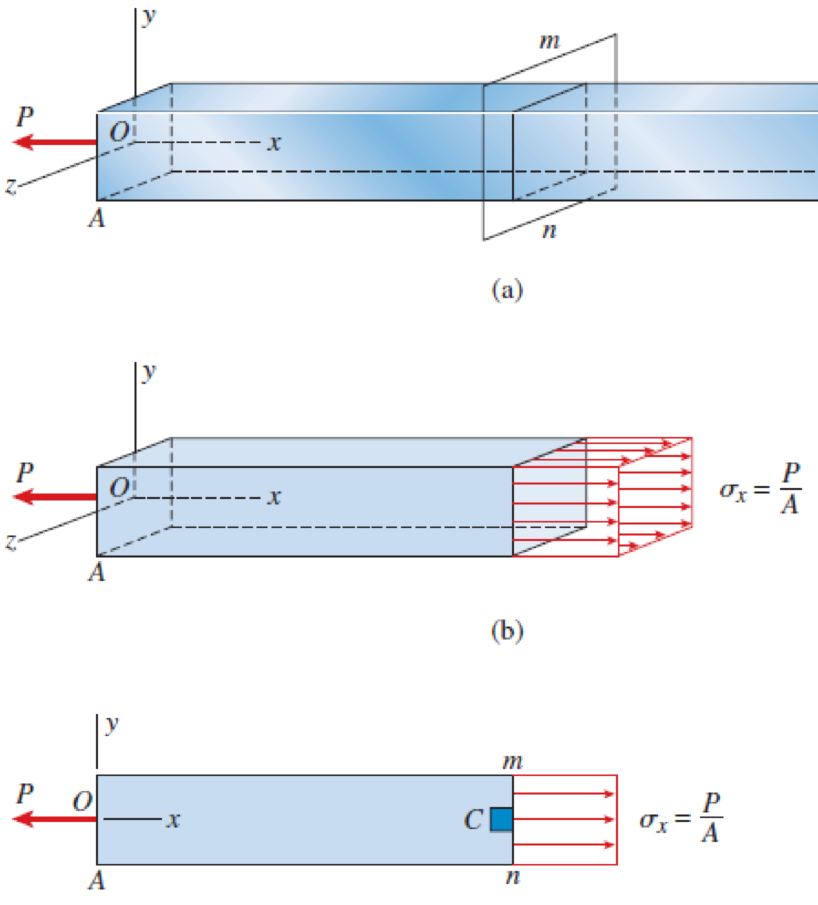
- Thermal properties of concrete and steel



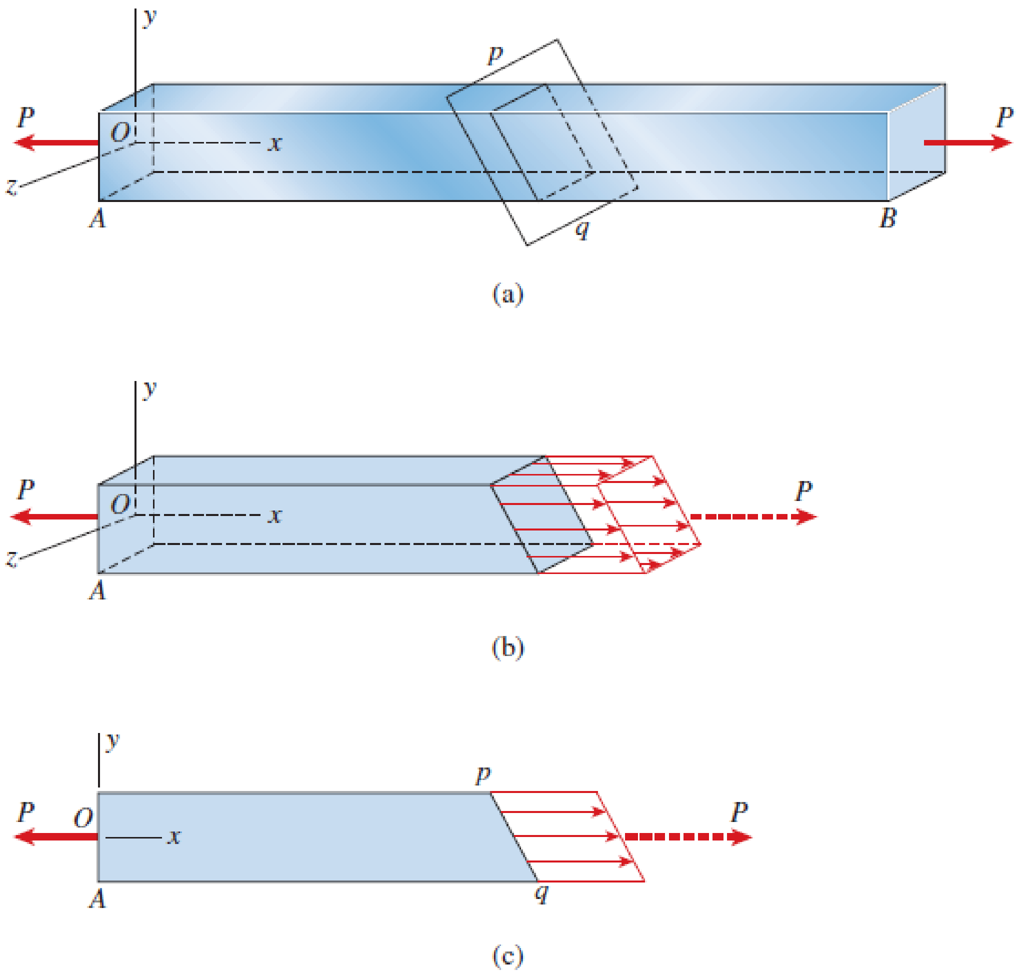
$$\alpha_L = \frac{1}{L}\frac{dL}{dT} \qquad \alpha_V = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

	Linear coefficient at 20°C (10^{-6}/K)	**Volumetric coefficient at 20°C (10^{-6}/K)**
Concrete	12	36
steel	11~13	33~39

Stresses on inclined sections

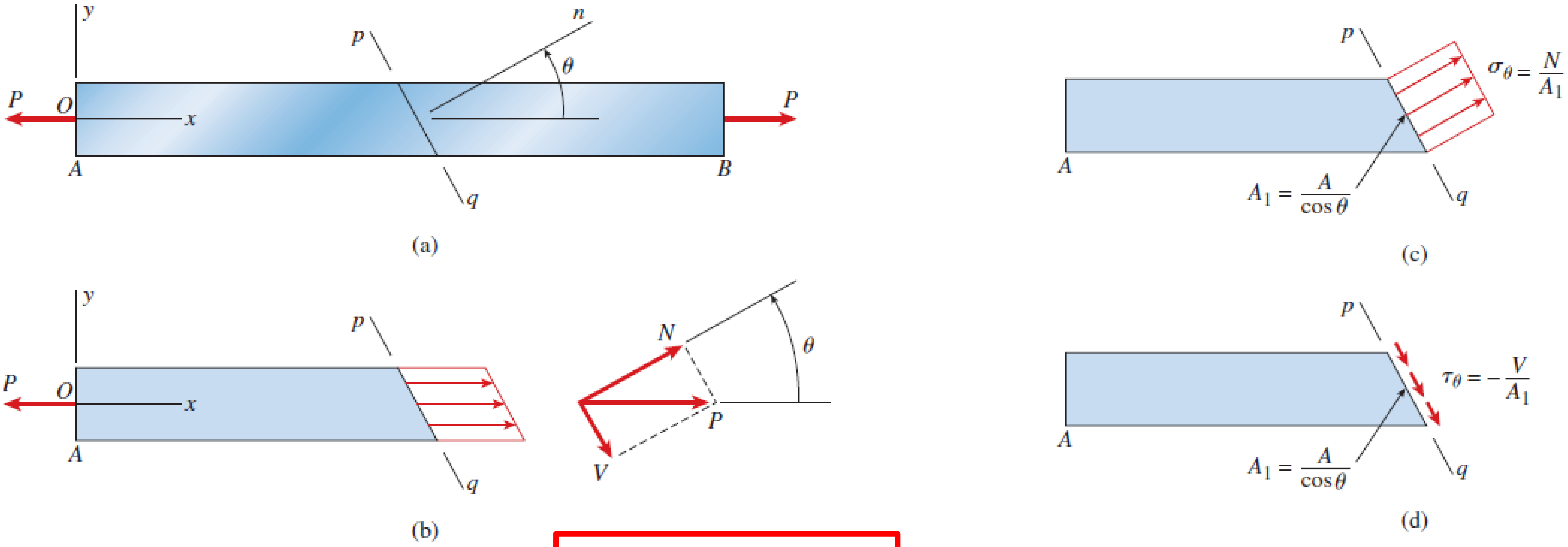


(Normal section)



(Inclined section)

Stresses on inclined sections

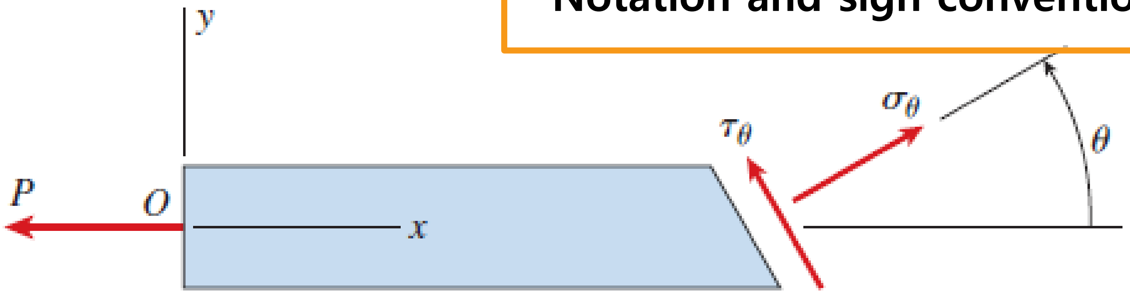


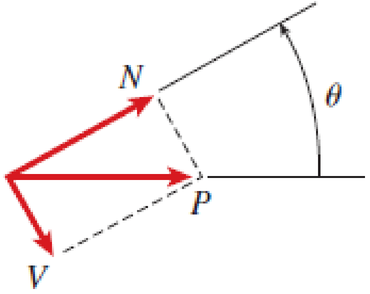
$$N = P\cos\theta \qquad V = P\sin\theta$$

Stresses on inclined sections

$$N = P\cos\theta \qquad V = P\sin\theta$$

Notation and sign convention





$$\sigma_\theta = \frac{N}{A_1} = \frac{P}{A}\cos^2\theta$$

$$\tau_\theta = -\frac{V}{A_1} = -\frac{P}{A}\sin\theta\cos\theta$$

$$\sigma_\theta = \sigma_x\cos^2\theta = \frac{\sigma_x}{2}(1 + \cos 2\theta)$$

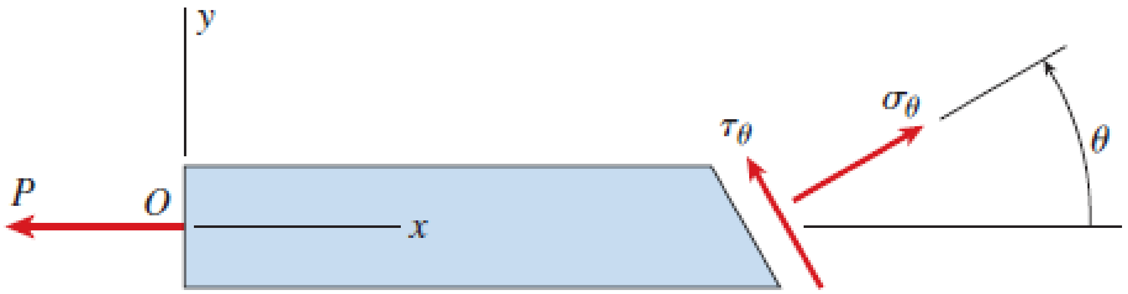
$$\tau_\theta = -\sigma_x\sin\theta\cos\theta = -\frac{\sigma_x}{2}(\sin 2\theta)$$

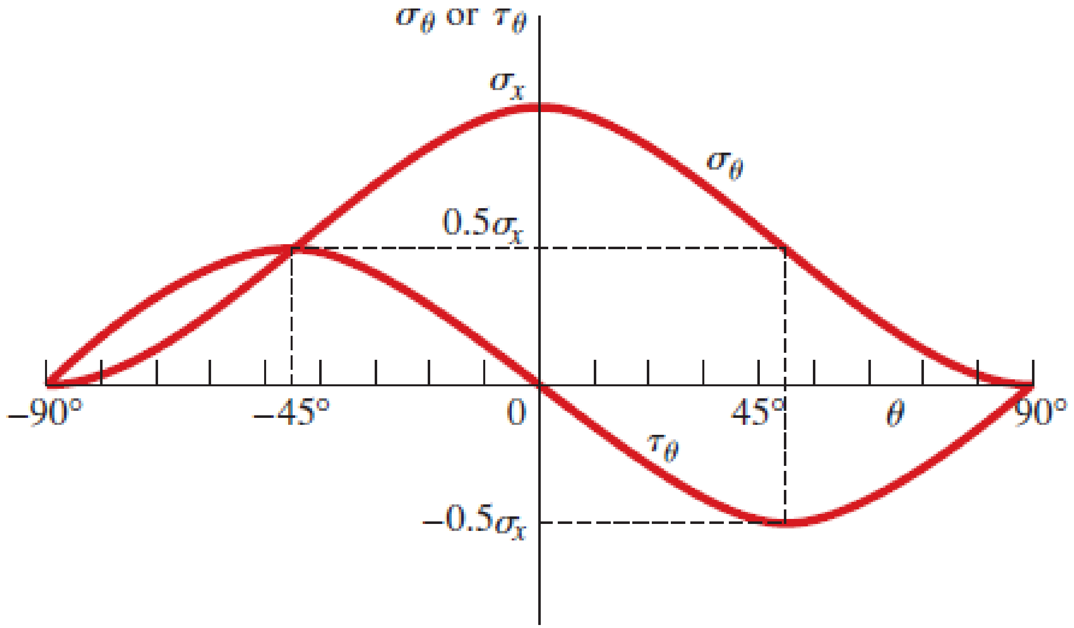
Trigonometry relation

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin\theta\cos\theta = \frac{1}{2}(\sin 2\theta)$$

Stresses on inclined sections

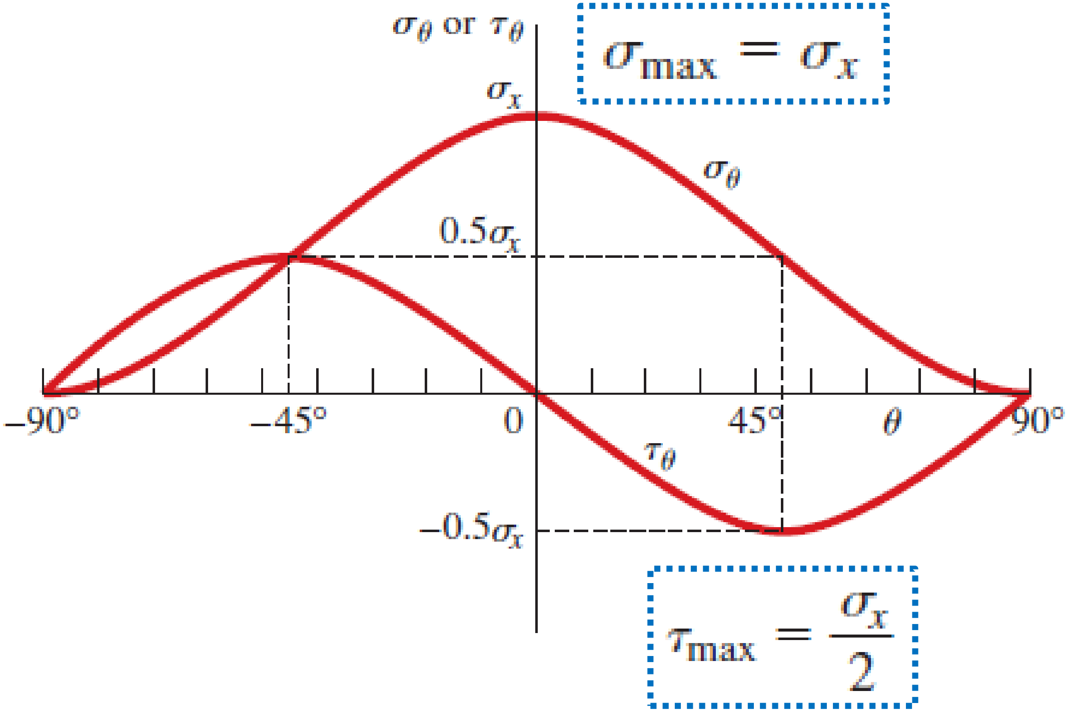


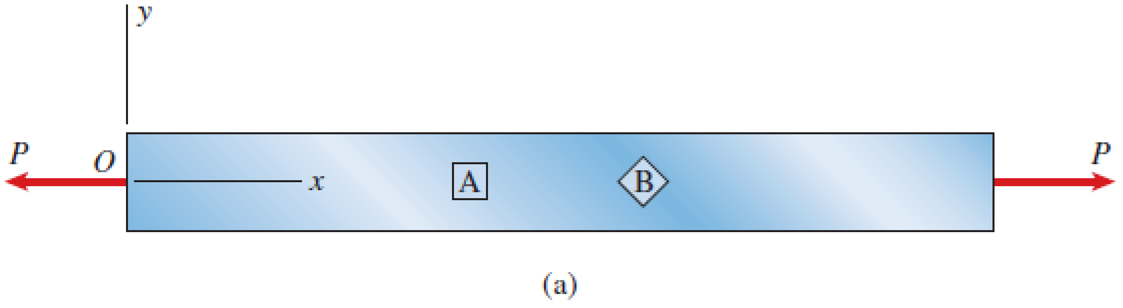


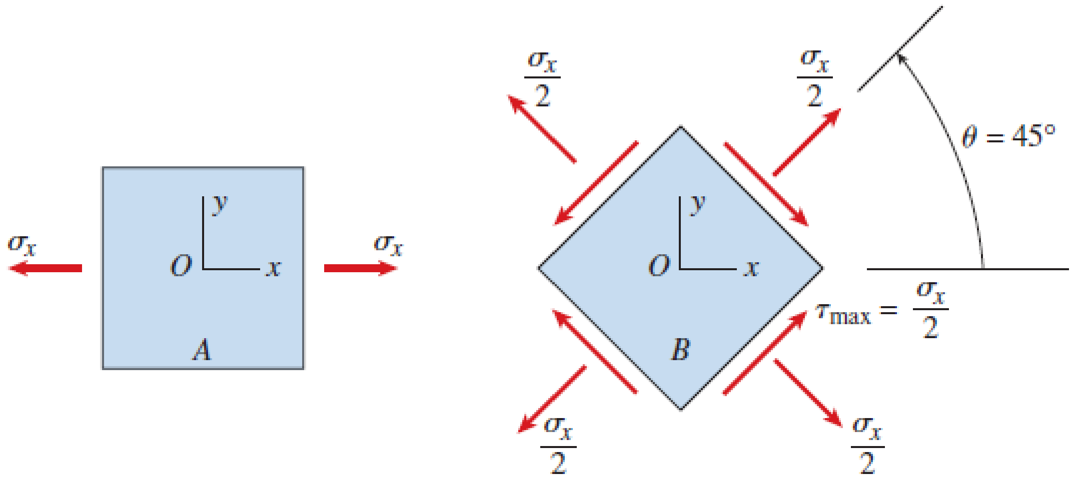
$$\sigma_\theta = \sigma_x \cos^2\theta = \frac{\sigma_x}{2}(1 + \cos 2\theta)$$

$$\tau_\theta = -\sigma_x \sin\theta \cos\theta = -\frac{\sigma_x}{2}(\sin 2\theta)$$

Stresses on inclined sections

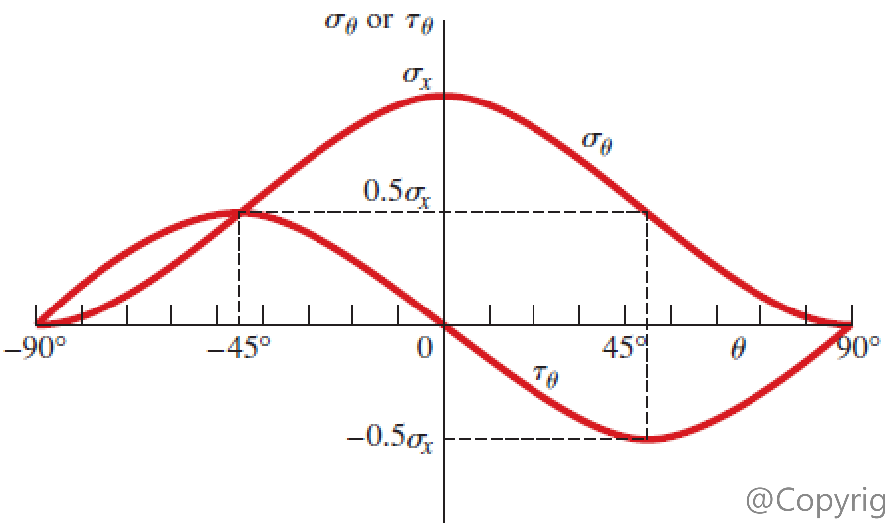






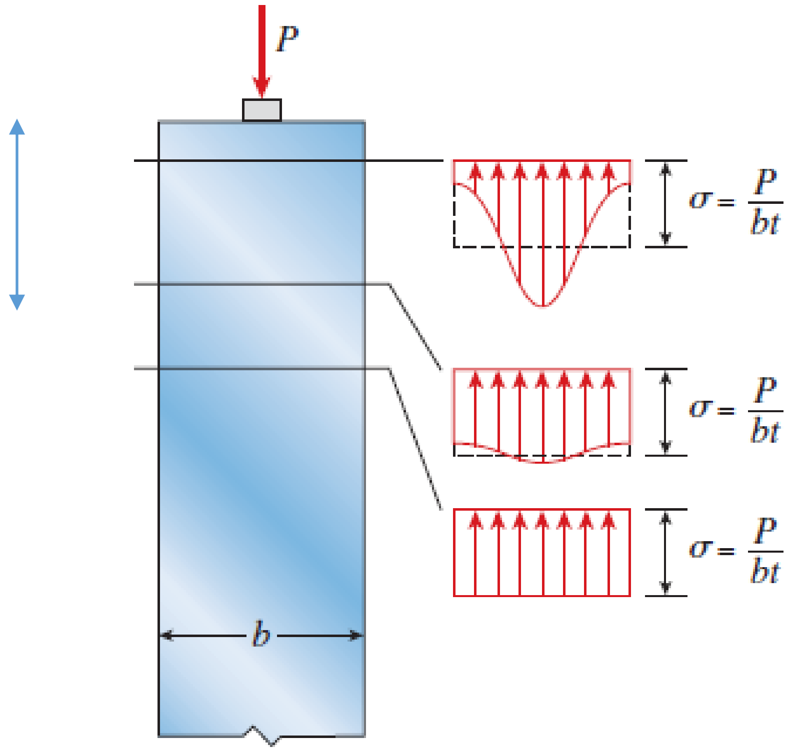
Stresses on inclined sections

- Implication:
- Although the max. shear stress in an axially loaded bar is only one-half the max. normal stress, the shear stress can cause failure if the material is much weaker in shear than in tension.
- Shear failure along a 45 degree of a wood block loaded in com
- Slip bands in a polished steel specimen loaded in tension



Other topics

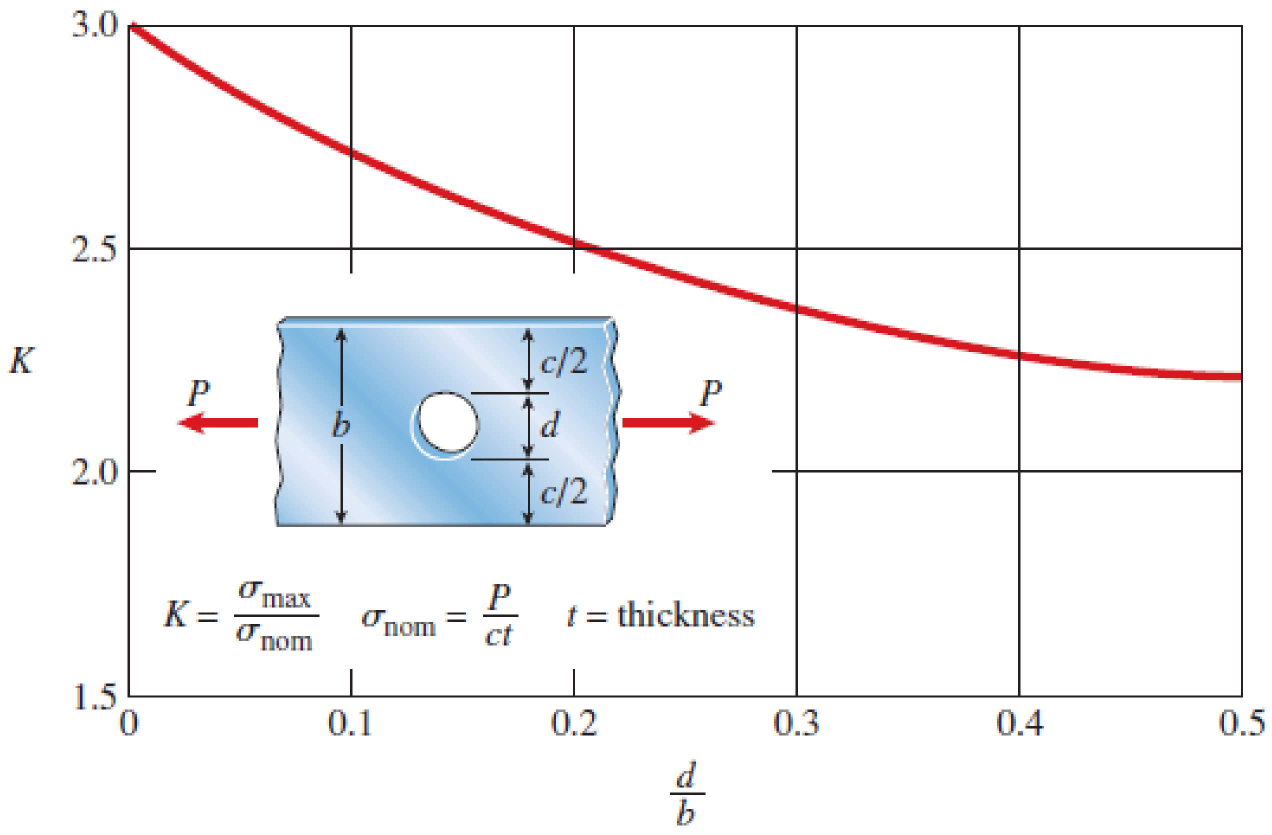
- Stress concentrations
- **Saint-Venant's principle**

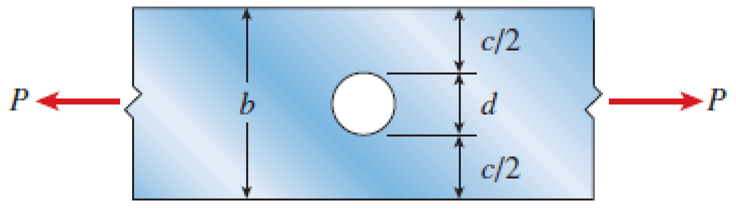


Other topics

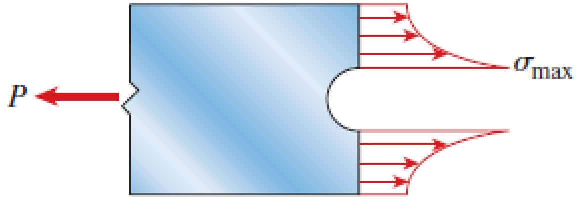
- Stress concentration factor ***K***:

$$K = \frac{\sigma_{max}}{\sigma_{nom}}$$





(a)



(b)