Rock Mechanics & Experiment
암석역학 및 실험

Lecture 4. Mechanical behavior of Intact Rock
Lecture 4. 무결암의 역학적 거동

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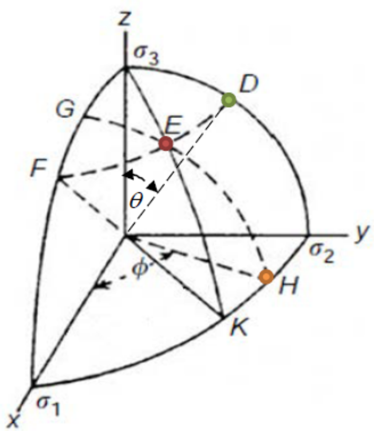
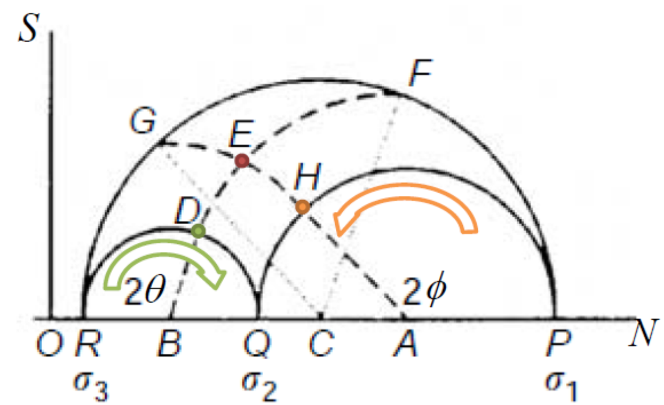
Last Lectures

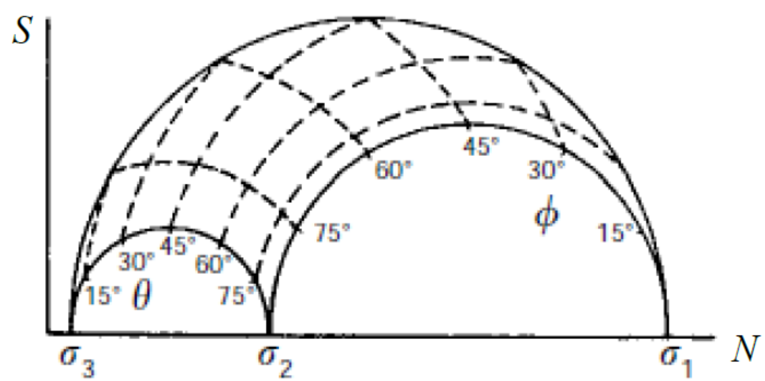
- Stress
- Plane Stress
- Principal Stresses and Maximum Shear Stresses
- Mohr's Circle
- Strain
- Hooke's Law
- Equilibrium Equation

Mohr's Circle
3D

- 3D Mohr's Circles: Particular stress value exist in the intersections of dotted lines







We can construct a diagram from which the normal and shear tractions acting on any plane can be found by locating the intersections of the circles!

Hooke's Law

- Hooke's Law in 1D

$$\sigma = E\varepsilon$$

- Shear modulus (전단계수) G

$$\tau_{xy} = G\gamma_{xy}$$

- Generalized Hooke's law (isotropy)
  - Isotropic rock has two independent parameters (E, v)
  - Shear modulus can be related to elastic modulus and Poisson's ratio

$$G = \frac{E}{2(1+\nu)}$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{pmatrix}$$

Hooke's Law
inverse form

$$
\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \nu/(1-\nu) & \nu/(1-\nu) & 0 & 0 & 0 \\ \nu/(1-\nu) & 1 & \nu/(1-\nu) & 0 & 0 & 0 \\ \nu/(1-\nu) & \nu/(1-\nu) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{(1-2\nu)}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{(1-2\nu)}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{(1-2\nu)}{2(1-\nu)} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} \quad (2.41)
$$

The inverse forms of equations 2.40, usually called Lamé's equations, are obtained from equation 2.41, i.e.

$$\sigma_{xx} = \lambda\Delta + 2G\varepsilon_{xx}, \text{ etc.}$$
$$\sigma_{xy} = G\gamma_{xy}, \text{ etc.}$$

where λ is Lamé's constant, defined by

$$\lambda = \frac{2\nu G}{(1-2\nu)} = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

and Δ is the volumetric strain.

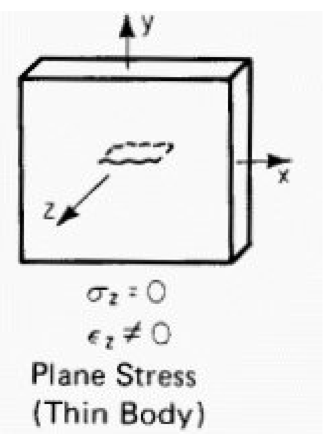
Plane Stress & Plane stress condition

Plane stress condition

- Stress and strain in different dimensions are coupled → we need a special consideration – plane strain and plane stress
- Plane stress
  - 3rd dimensional stress goes zero
  - Thin plate stressed in its own plane

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{2(1+\nu)}{E} \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix}$$



Plane Stress & Plane stress condition
Plane strain condition

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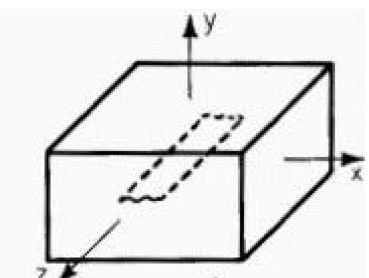
- Plane strain
  - 3rd dimensional strain goes zero
  - Stresses around drill hole or 2D tunnel

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{(1-\nu^2)}{E} & -\frac{\nu(1+\nu)}{E} & 0 \\ -\frac{\nu(1+\nu)}{E} & \frac{(1-\nu^2)}{E} & 0 \\ 0 & 0 & \frac{2(1+\nu)}{E} \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix}$$



$\epsilon_z = 0$

$\sigma_z = \nu(\sigma_x + \sigma_y)$

Plane Strain
(Thick Body)

Stress equilibrium Equation

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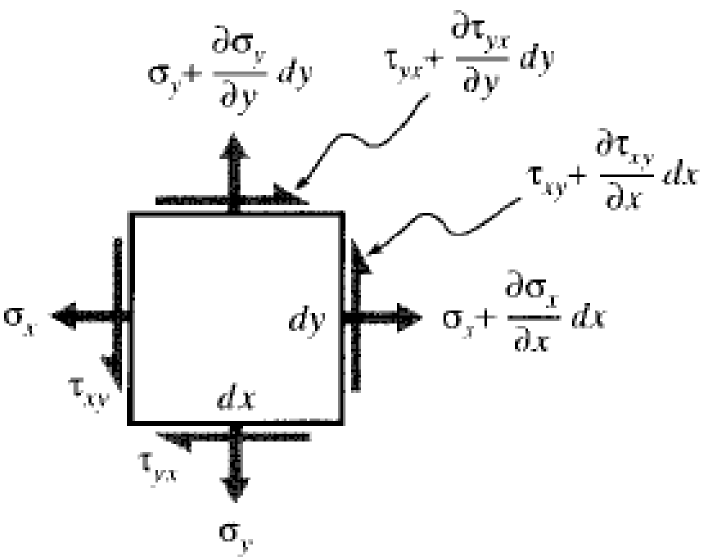
– Sum of traction, body forces (and moment) are zero (static case)



$$\sum F_i = m\frac{\partial^2 u_i}{\partial t} \xrightarrow{\text{Very slow loading}} \sum F_i = 0$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho b_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \rho b_y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho b_z = 0$$

$$\sum M_i = 0 \longrightarrow \tau_{xy} = \tau_{yx}$$

– b_x, b_y, b_z are components of acceleration due to gravity.

Stress
Definition in 2D and 3D

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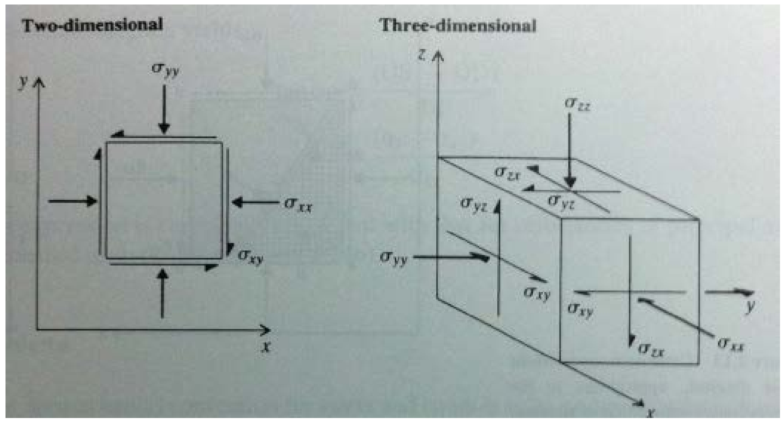
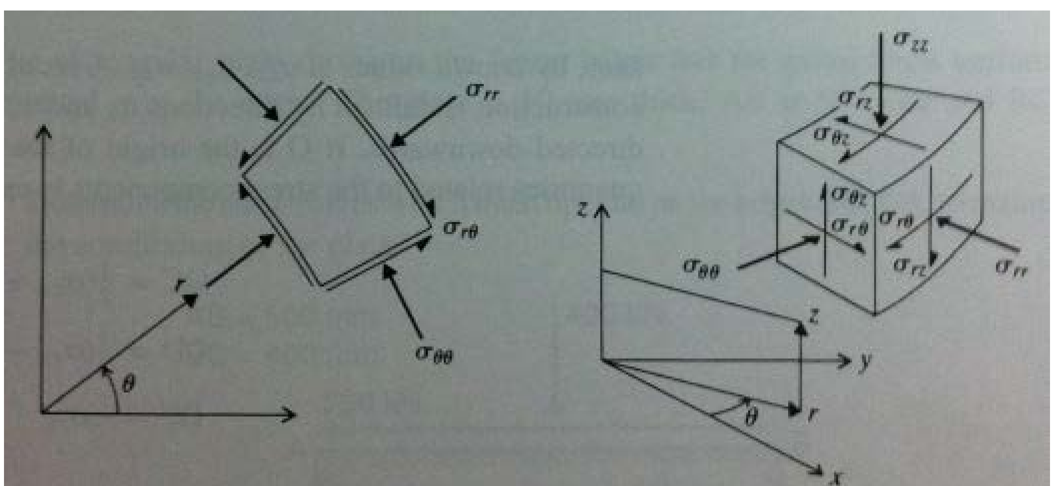
- 2D & 3D Cartesian Coordinates



- Polar & Cylindrical coordinates



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- Laboratory test
- Elastic behavior
- Uniaxial compression
  - Uniaxial Compressive Strength (UCS)
  - Full stress-strain curve
- Factors affecting laboratory test of rock
  - Size effect, Shape effect & others
- Tensile behavior
- Triaxial behavior
- Time/Temperature dependent behavior

Laboratory Test

- Various loading conditions (and specimen)
  - Test on intact rock is important
  - In real conditions, stress can be in situ stress or induced stress

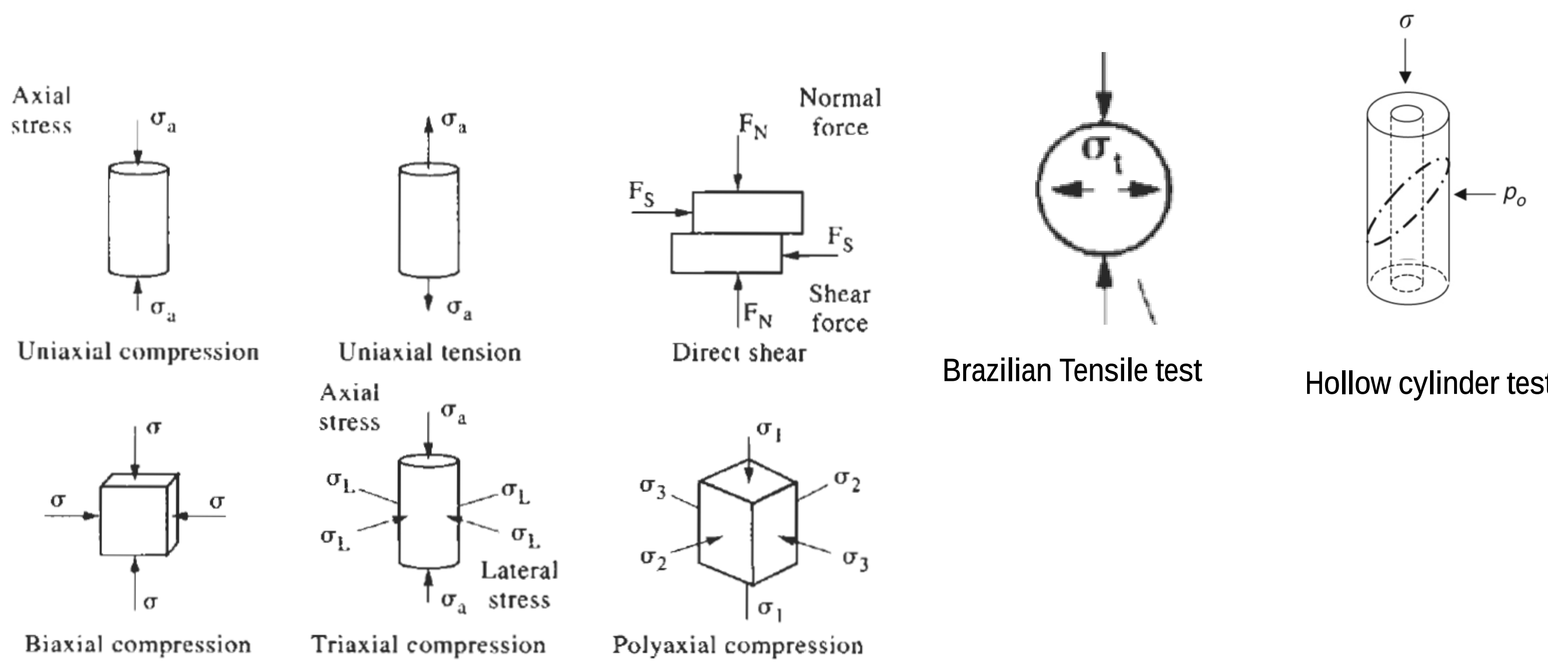

Figure 6.13 Specimen loading conditi ons in general laboratory use.

True triaxial compression

Laboratory Test
Example 1: Site Investigation in Forsmark, Sweden

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- Geological Repository for Nuclear Waste

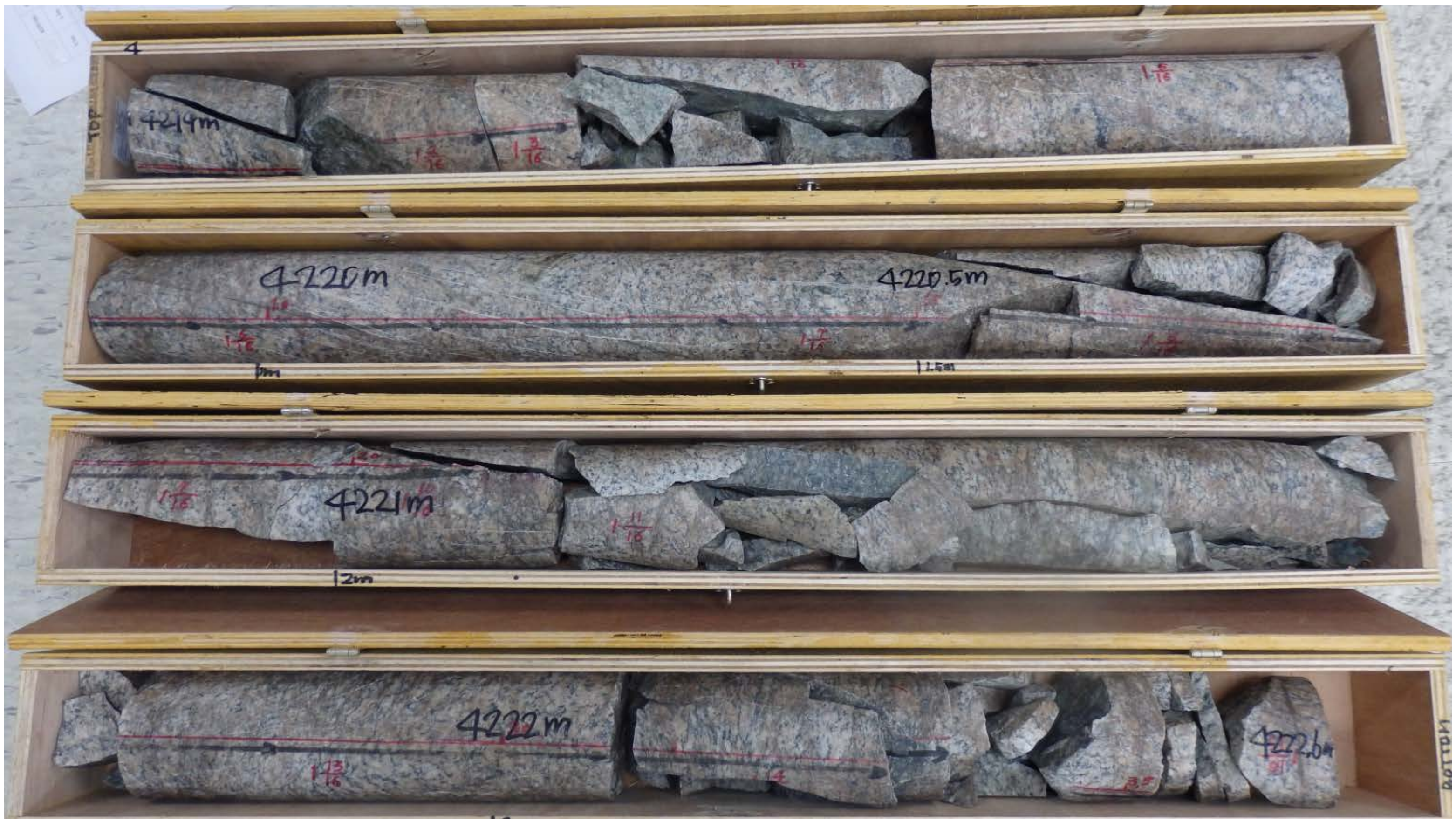
Rock core collection (Forsmark, Oct 2004)

- 25 core-drilled boreholes up to 1,000 m depth.
- 17.8 km core length in total

Core Drilling site (Forsmark, June 2003)

Laboratory Test
Example 2: Pohang EGS project site

- 3.6 m long 4 inch (~10 cm) core at 4.2 km depth



Laboratory Test
Example 2: Pohang EGS project site.

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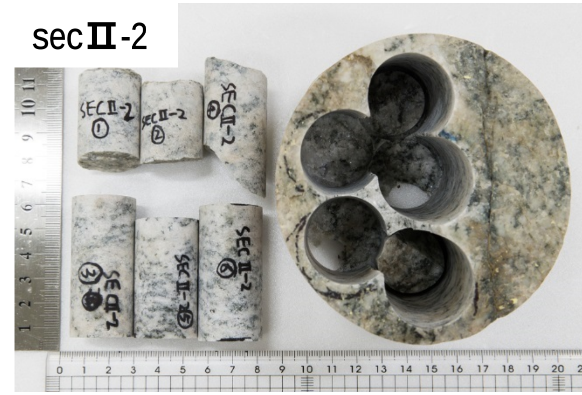
sec Ⅰ -1

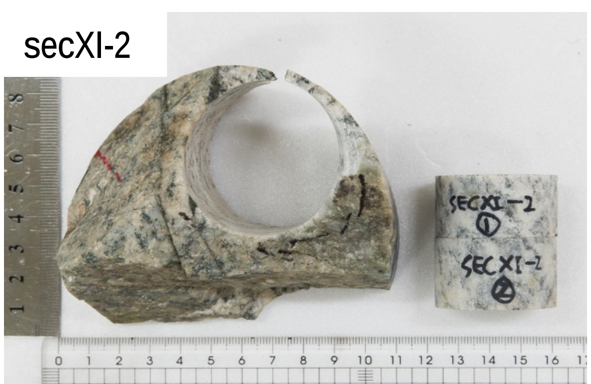

secⅡ-1


secⅡ-2

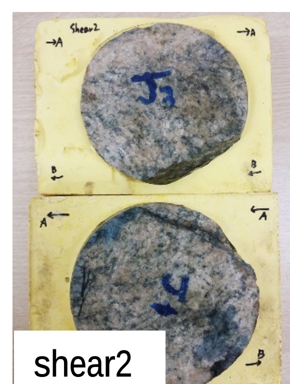

secⅦ-4


secⅦ-5


secXI-1


secXI-2


shear1


shear2


shear3

Laboratory Test
Example 2: Pohang EGS project site.

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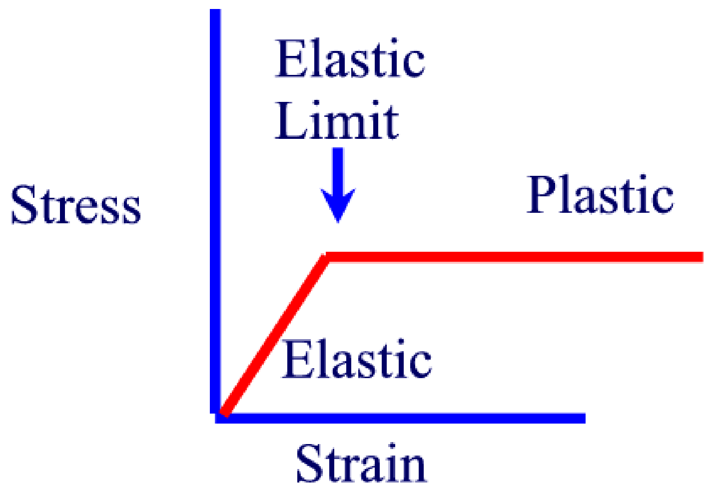
UCS & Triaxial: 1 inch cores
BTS: 1.5 inch cores
Divided-bar (thermal conductivity measurement): NX (54mm) disks

Elastic Modulus & Poisson’s ratio

- Elastic (탄성)
  - Recovers to the initial state when unloaded (하중에 따른 변형이 하중에 제하(없애는 것)되었을 때 원상태로 복귀하는 성질)
- Plastic (소성)
  - Not recoverable (복귀가 안됨)



Elastic behavior
Elastic Modulus & Poisson's ratio

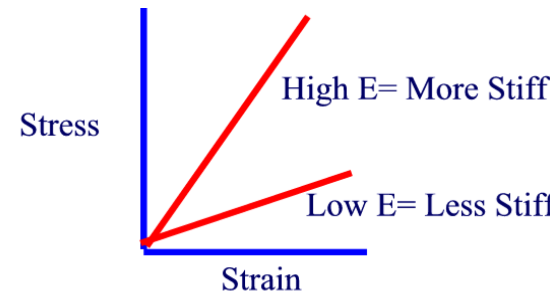
$$q''_x = -k\frac{dT}{dx}$$

$$q_x = -K_x\frac{\partial h}{\partial x}$$

- Hooke's law

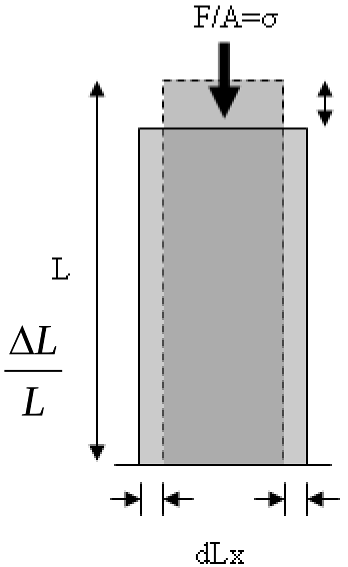
$$\sigma = E\varepsilon = E\frac{du}{dx}$$

- Elastic (Young's) modulus(탄성계수), E (N/m^2=Pa)
  - Tangent modulus
  - Secant modulus



- Poisson's ratio (포아송 비), v (dimensionless)

$$\nu = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_x}{\varepsilon_y}$$



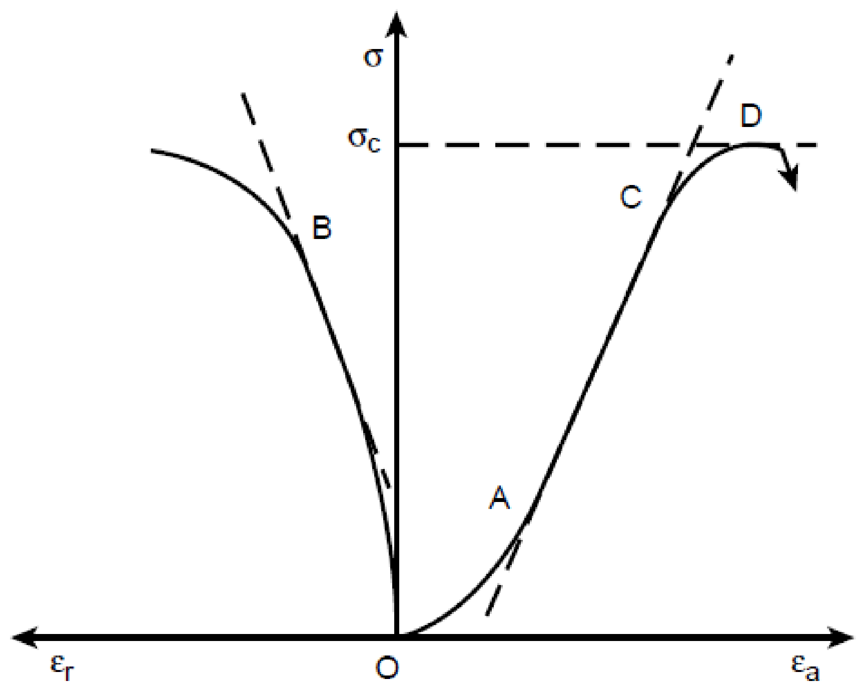
$$\varepsilon = \frac{\Delta L}{L}$$

- Typical values
  - ???

Uniaxial compression test
Uniaxial Compressive Strength

- Uniaxial Compressive Strength (UCS, 단축압축강도): maximum sustainable stress under uniaxial stress condition
  - Unit: same as stress (MPa or psi)

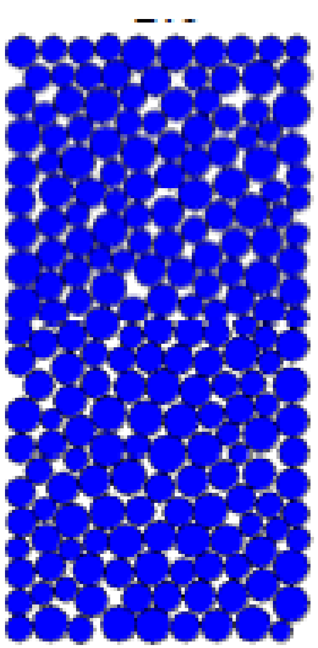
$$\sigma_c = \frac{F_{\max}}{A}$$



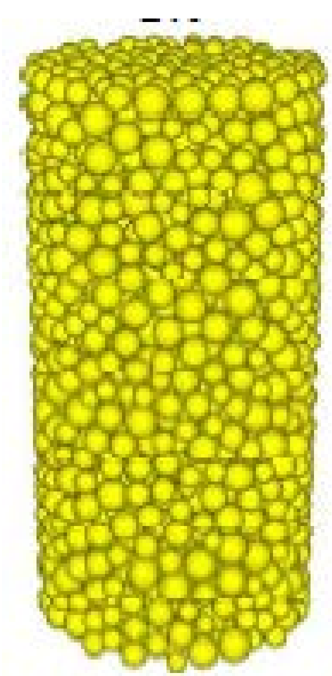
Roegiers JC, 2000, Formation Characterization: Rock Mechanics, Eds: Economides & Nolte, Reservoir Stimulation, 3rd Ed., Wiley

Uniaxial compression test
Uniaxial Compressive Strength

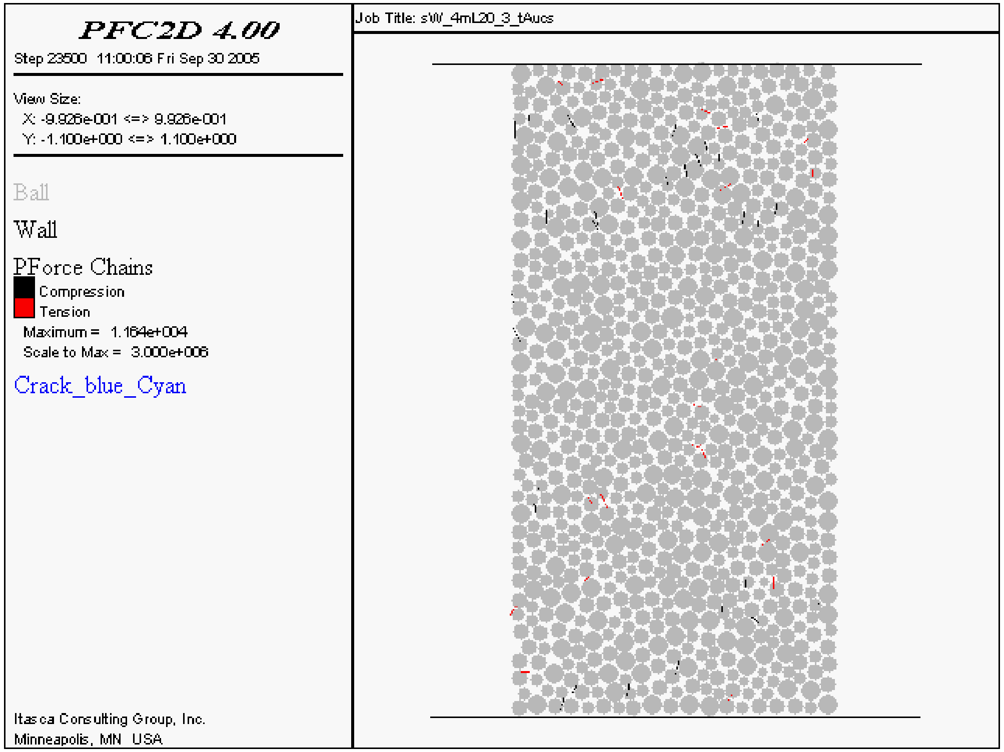
– Example of Uniaxial Compressive Strength test by a numerical simulation using Discrete Element Method



2D



3D



Uniaxial compression test
Uniaxial Compressive Strength

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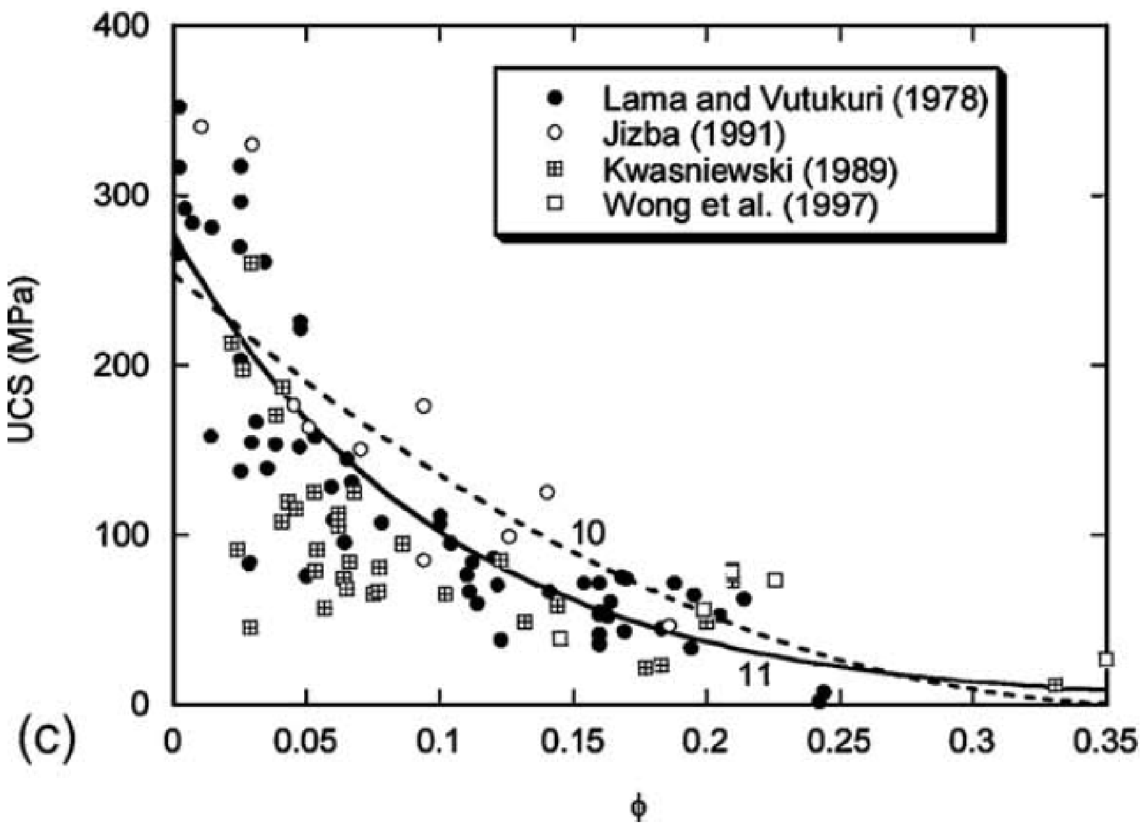
- Relationship with other parameters
  - UCS tends to decrease with porosity



Chang, Chandong, Mark D. Zoback, and Abbas Khaksar. "Empirical Relations between Rock Strength and Physical Properties in Sedimentary Rocks." *Journal of Petroleum Science and Engineering* 51, no. 3–4 (2006): 223-37.

Uniaxial compression test
Full stress-strain curve

- Initial behavior: concave upward
- Acoustic emission at 50% level
- Post peak behavior may not be important in civil engineering but it is important (encouraged) for some applications. e.g., block caving

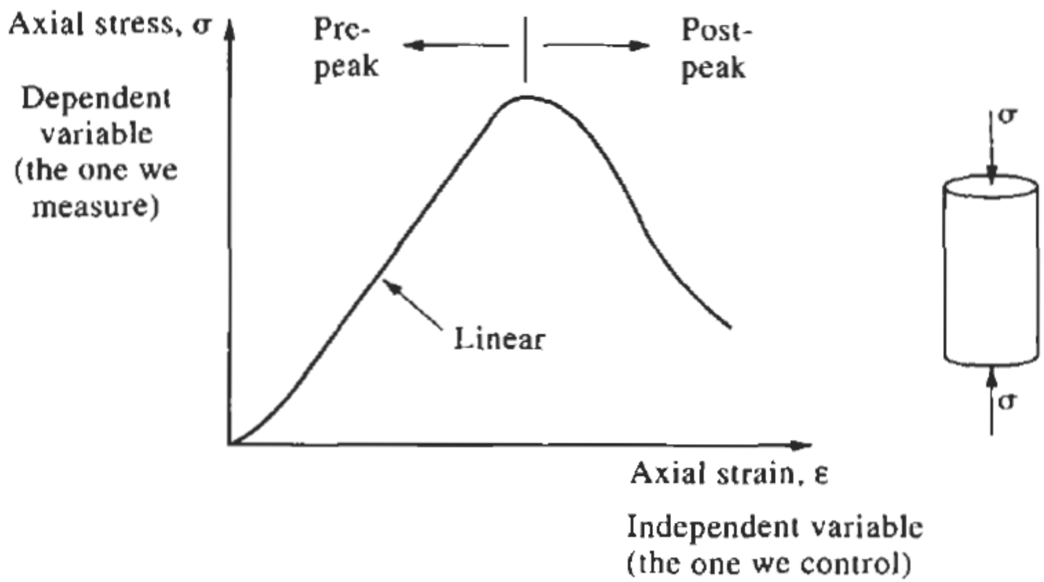


Figure 6.1 The complete stress–strain curve.

Uniaxial compression test
Full stress-strain curve

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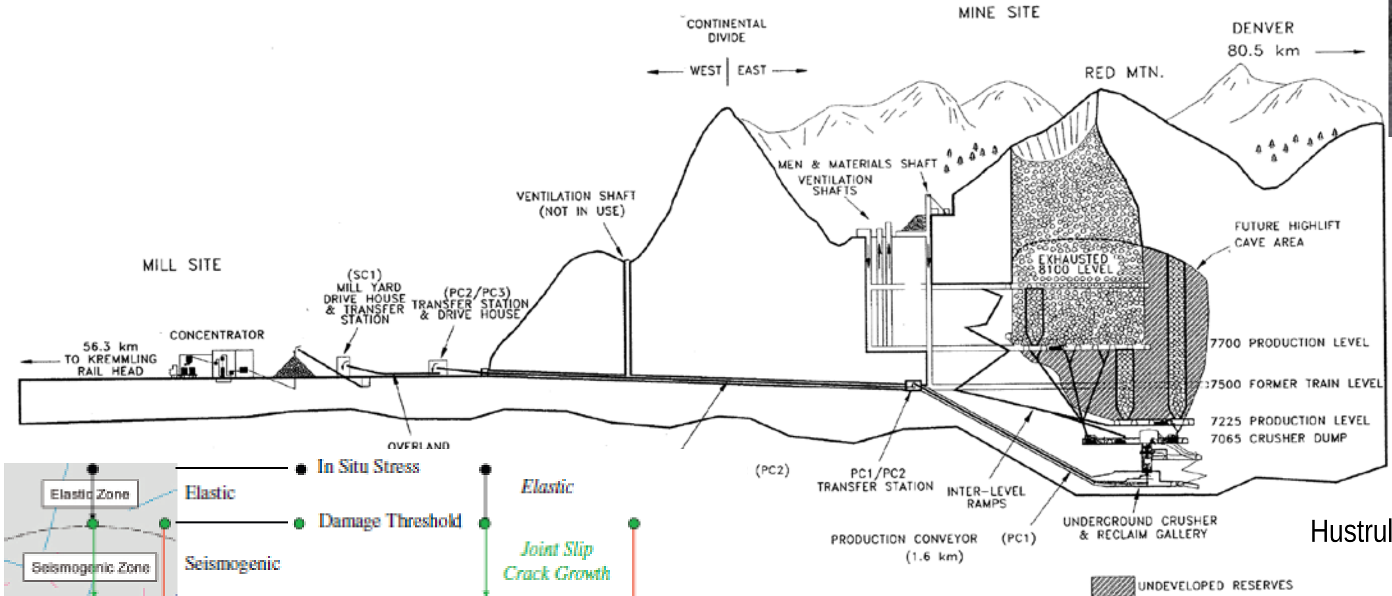
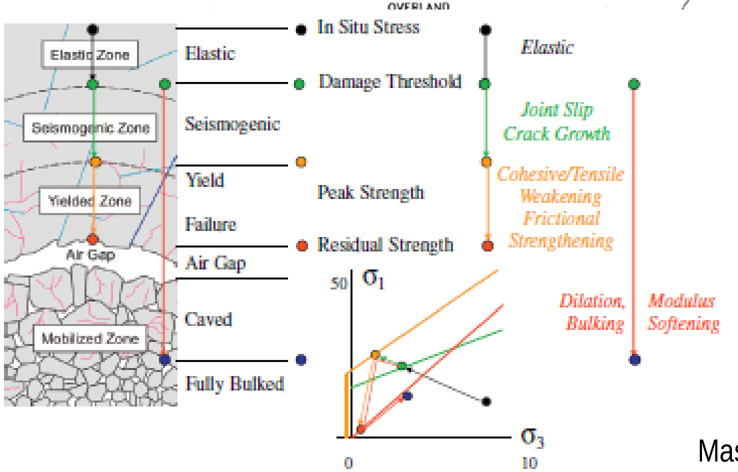
- 헨데센 광산 (Hendersen Mine), 콜로라도, 미국
  - 1976 년 운영시작 (세계최대의 몰리브덴 광산)
  - 1000 미터 하부에 광체, 최대심도 1,600 미터

Min, 2006



Hustrulid & Bullock, 2001



Caving mechanics

Mas Ivars et al., 2011

Uniaxial compression test
Full stress-strain curve

- Full stress-strain curve by servo-controlled testing
  - Stress controlled test will generate uncontrolled failure

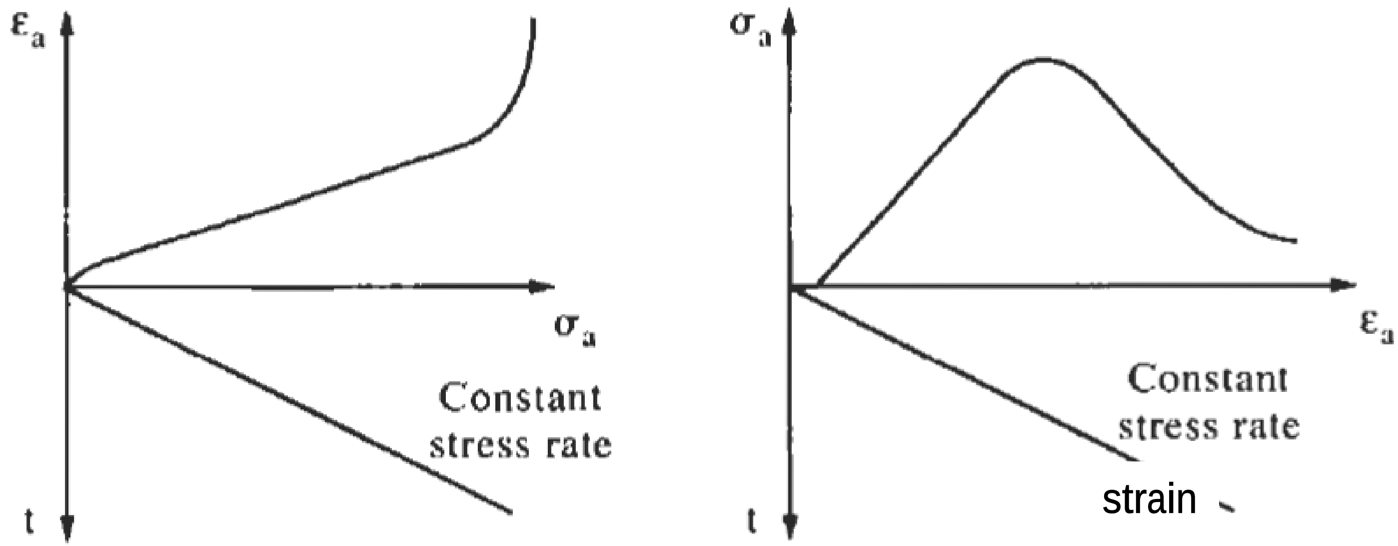


Figure 6.5 Stress- and strain-controlled stress–strain curves.

Uniaxial compression test
Full stress-strain curve

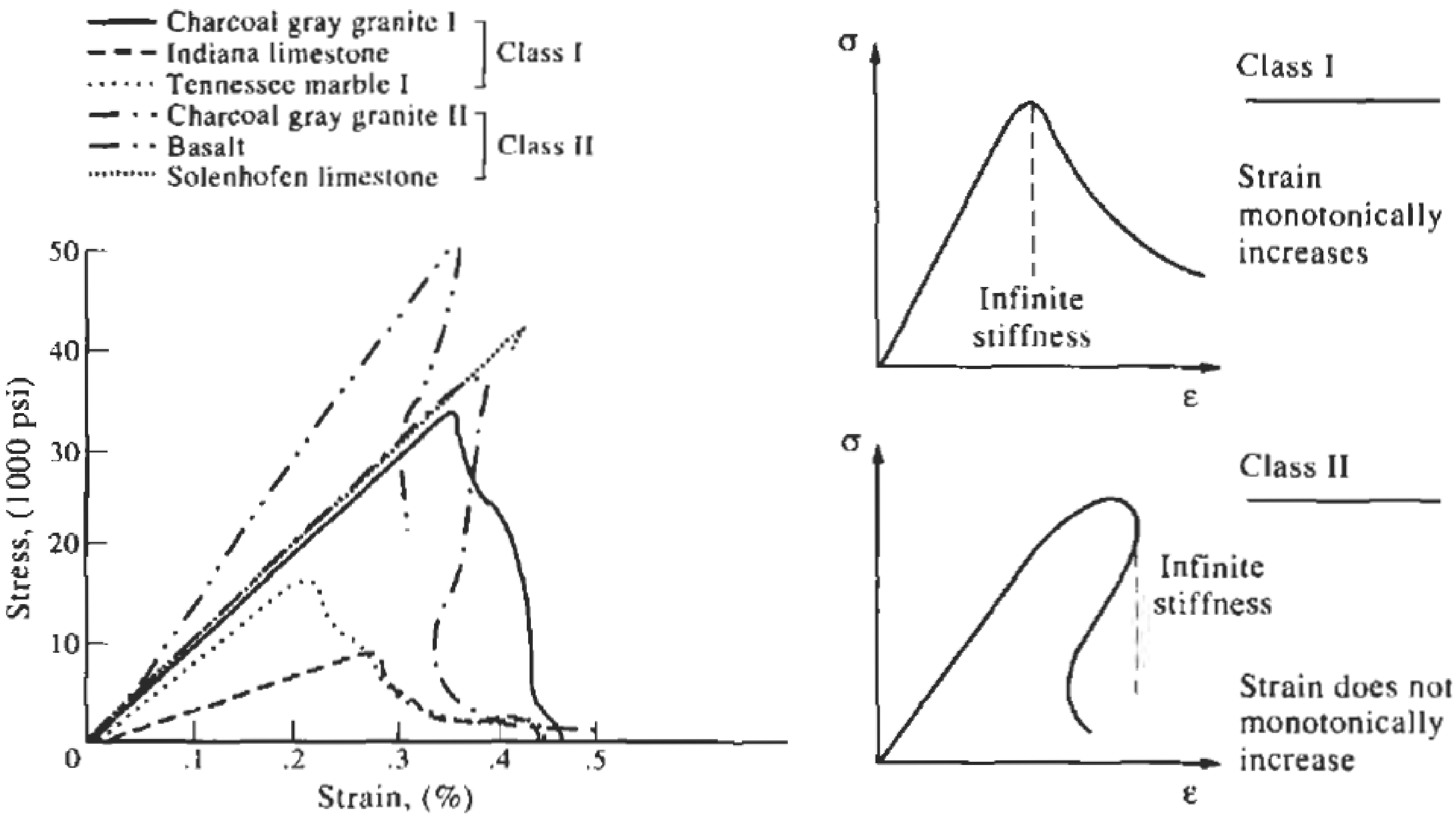


Figure 6.8 Examples of complete stress–strain curves for different rocks (from Wawersik and Fairhurst, 1970).

Factors affecting laboratory test
Size effect

- Size effect: properties varies with size
  - Elastic modulus: relatively less affected
  - Strength: tends to decreases with increase of size. Why?
  - One could choose “representative elementary volume (REV)” to overcome this.

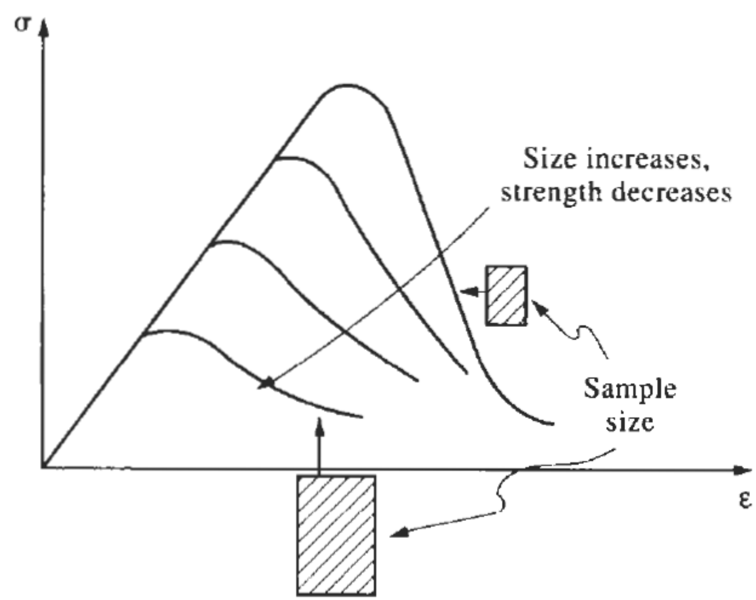


Figure 6.11 The size effect in the uniaxial complete stress–strain curve.

Factors affecting laboratory test
Size effect

- Representative Elementary Volume: Volume after which a property does not vary

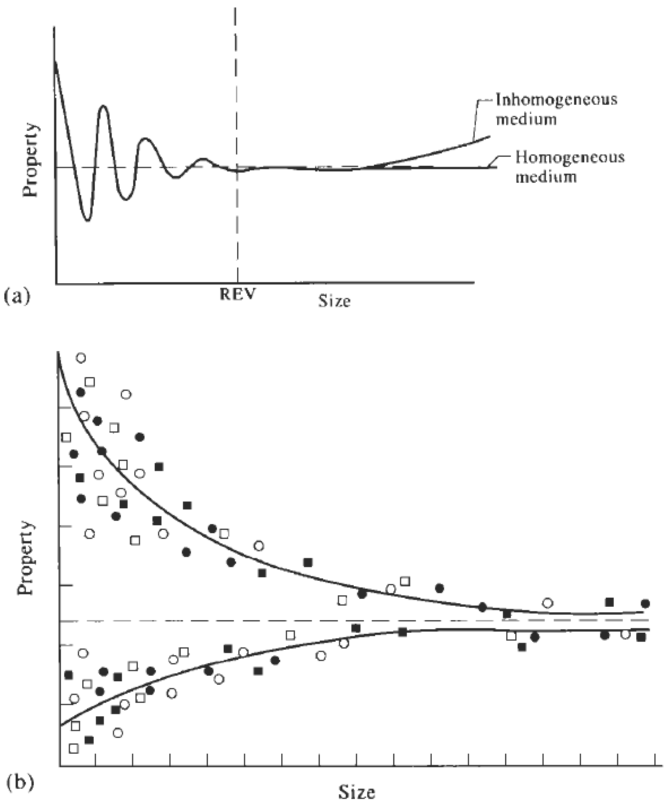
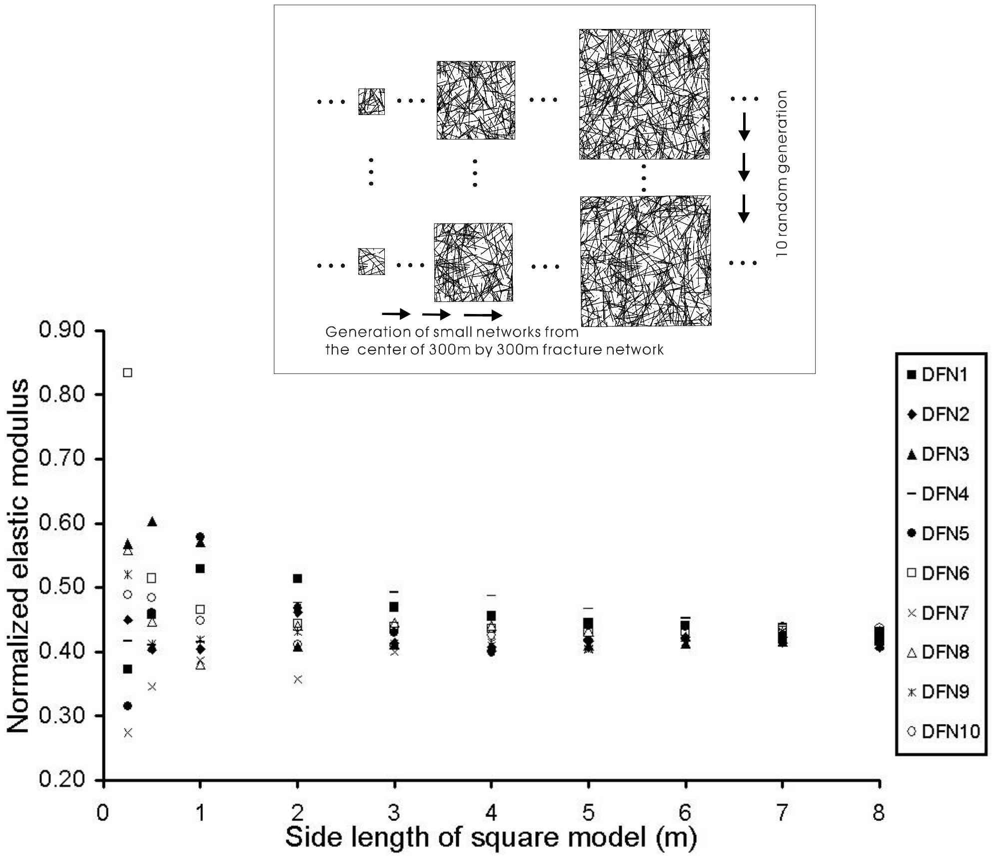


Figure 4.12 Variability in measured values with respect to sample volume, illustrating the REV. (a) General concept. (b) Example data scatter.



Normalized elastic moduli of fractured rock in various scales

Min KB, Jing L, 2003, Numerical determination of the equivalent elastic compliance tensor for fractured rock masses using the distinct element method, *International Journal of Rock Mechanics & Mining Sciences* 2003;40(6):795-816.

Factors affecting laboratory test
Shape effect

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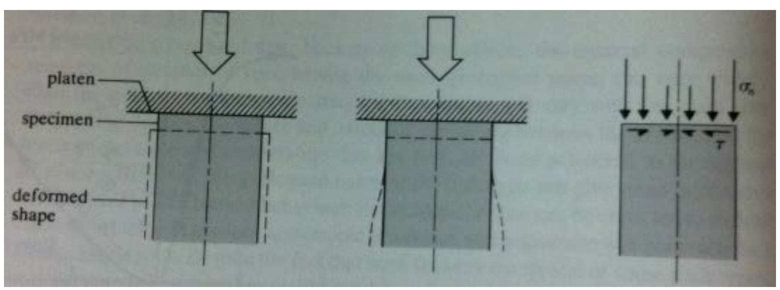
- Aspect ratio matters
  - Low ratio tends to have larger strength. Why?
  - Solution?
    - use large enough ratio >2.0-2.5
    - Improve testing procedure



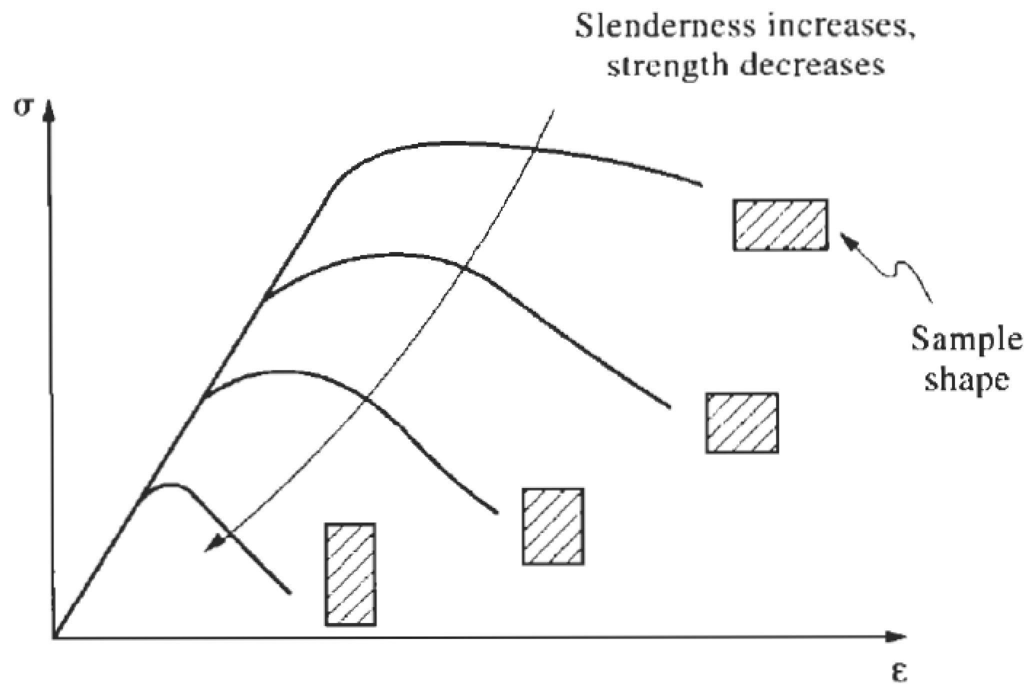


Figure 6.12 The shape effect in uniaxial compression.

Factors affecting laboratory test

Other factors for lab test and in situ behavior

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- Moisture content
- Desiccation – especially for clay
- Slaking
- Swelling - bentonite
- Pore pressure – via effective stress
- Groundwater chemistry – dissolution (chalk, limestone)
- Free-thaw mechanism – cold region

Tensile behavior
Tensile strength

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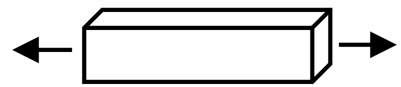
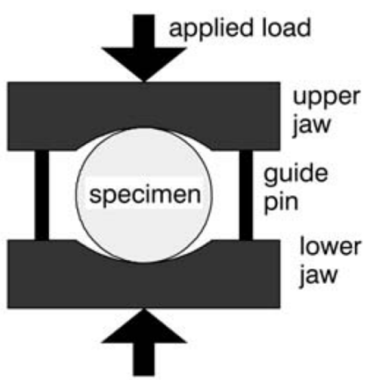
- Tensile strength : Maximum sustainable stress under tensile condition

$$\sigma_t = \frac{T_{\max}}{A}$$



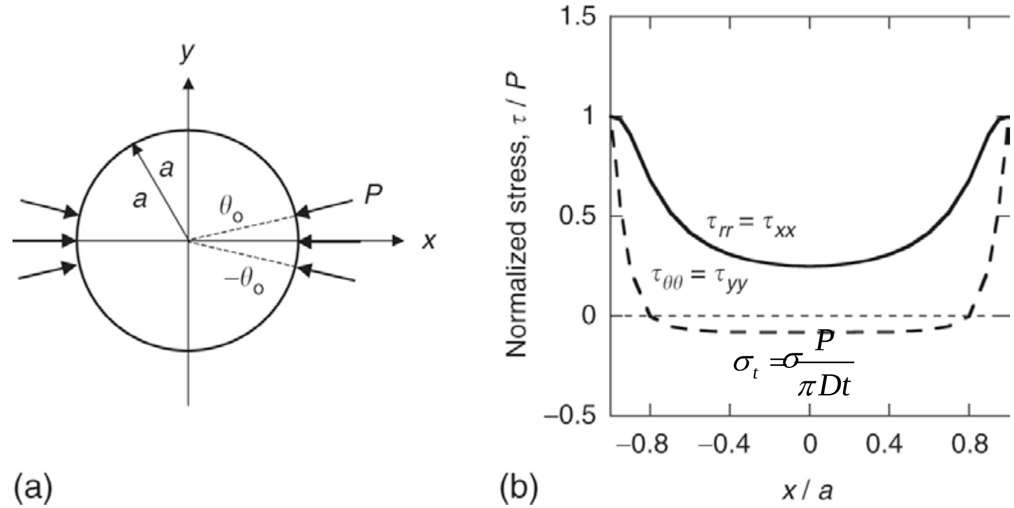
Tensile loading

- <u>Tensile strength is 1/10 ~ 1/20 of UCS</u>
- Tensile strength is measured by Brazilian Test = $2P/(\pi dt)$



Tensile behavior
Brazilian Tensile test

- The reason why Brazilian Test Works…



When $2\theta_0$=15°

  - Stress distribution along the x-axis $\rho = r / a$

$$\tau_{rr}(\theta = 0) = \frac{2P}{\pi}\left\{\frac{(1-\rho^2)\sin 2\theta_o}{(1-2\rho^2\cos 2\theta_o+\rho^4)} + \arctan\left[\frac{(1+\rho^2)}{(1-\rho^2)}\tan\theta_o\right]\right\}, \tag{8.161}$$

$$\tau_{\theta\theta}(\theta = 0) = -\frac{2P}{\pi}\left\{\frac{(1-\rho^2)\sin 2\theta_o}{(1-2\rho^2\cos 2\theta_o+\rho^4)} - \arctan\left[\frac{(1+\rho^2)}{(1-\rho^2)}\tan\theta_o\right]\right\}, \tag{8.162}$$

$$\tau_{rr}(\theta = \pi/2) = -\frac{2P}{\pi}\left\{\frac{(1-\rho^2)\sin 2\theta_o}{(1+2\rho^2\cos 2\theta_o+\rho^4)} - \arctan\left[\frac{(1-\rho^2)}{(1+\rho^2)}\tan\theta_o\right]\right\}, \tag{8.163}$$

$$\tau_{\theta\theta}(\theta = \pi/2) = \frac{2P}{\pi}\left\{\frac{(1-\rho^2)\sin 2\theta_o}{(1+2\rho^2\cos 2\theta_o+\rho^4)} + \arctan\left[\frac{(1-\rho^2)}{(1+\rho^2)}\tan\theta_o\right]\right\}. \tag{8.164}$$

$$\tau_{rr}(\theta = 0) = \frac{W(3+\rho^2)}{\pi a(1-\rho^2)}, \quad \tau_{\theta\theta}(\theta = 0) = -\frac{W}{\pi a},$$

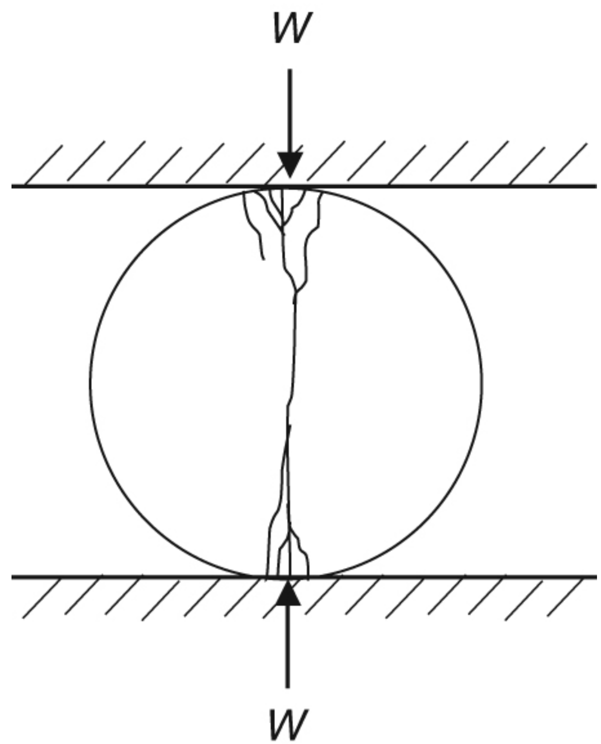
$$\sigma_t = \frac{W}{\pi a} = \frac{2P}{\pi D t}$$

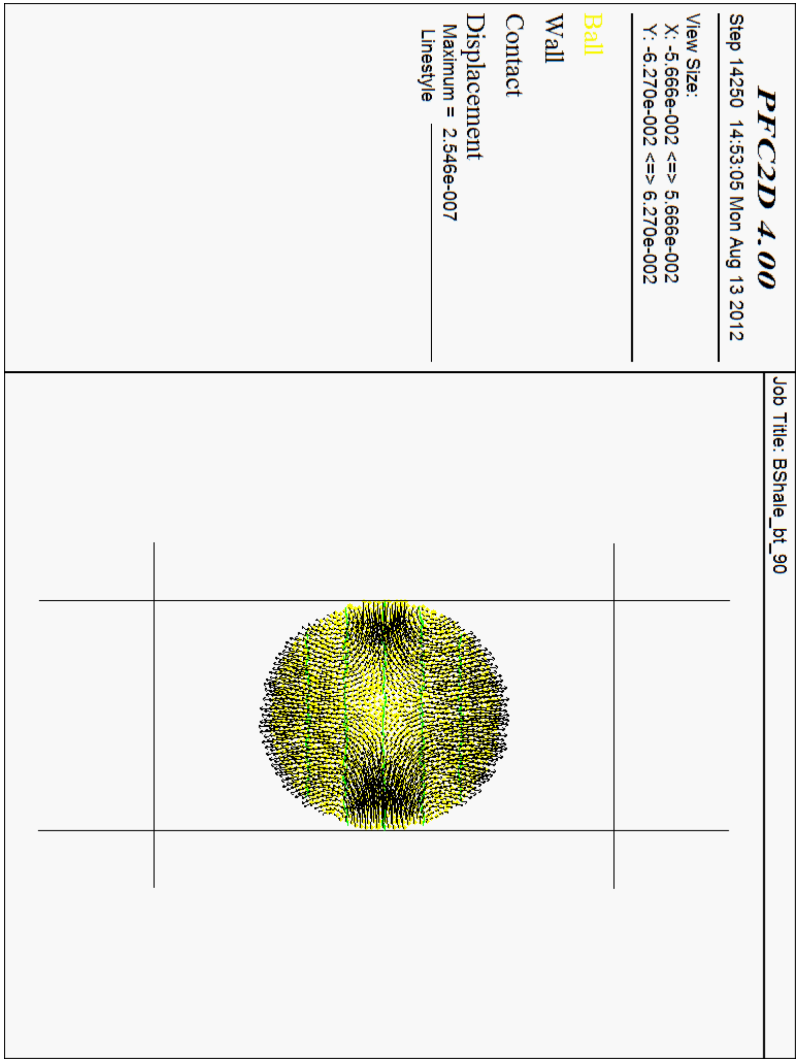
W: line load per unit length

Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Tensile behavior
Brazilian Tensile test

Example of Brazilian Tensile Strength Test by a numerical simulation using Discrete Element Method





Tensile behavior
Brazilian Tensile test

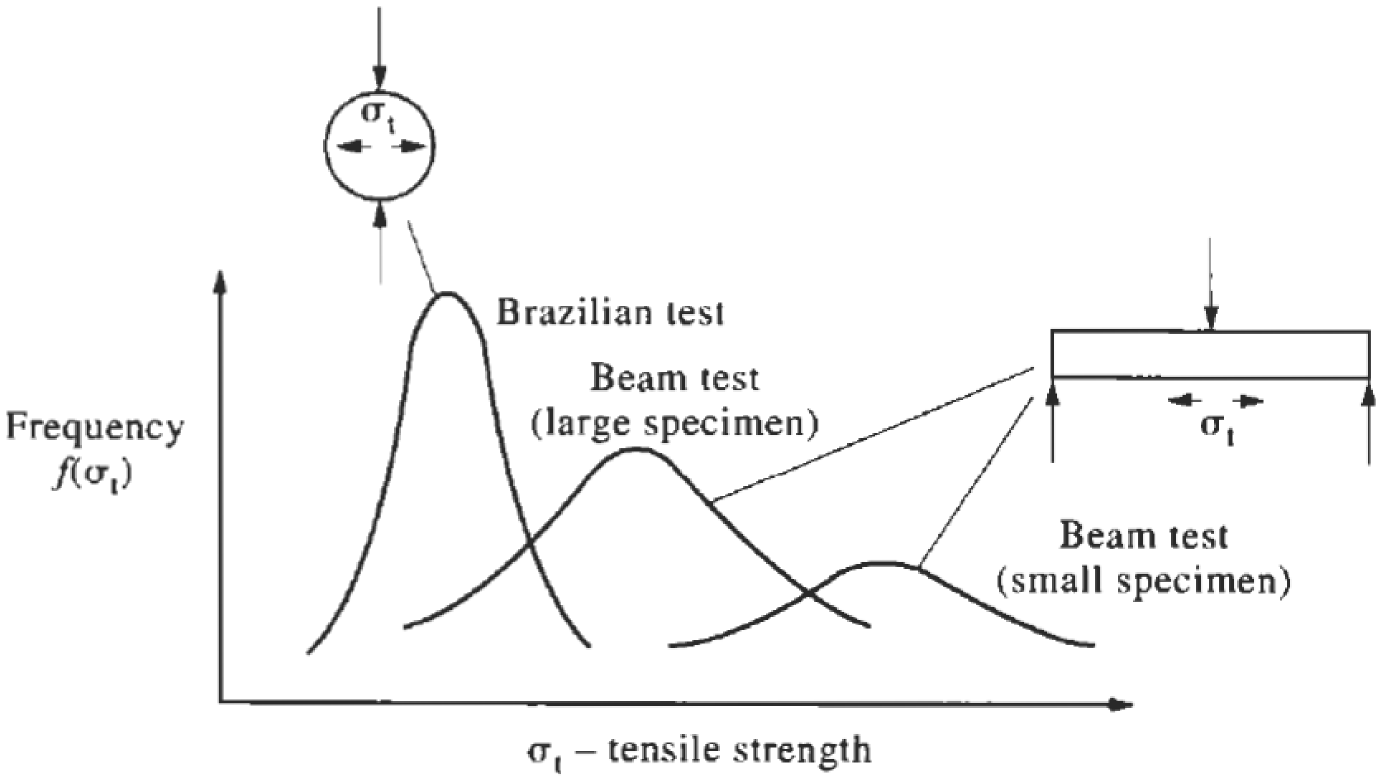


Figure 6.14 Tensile strength variation as a function of specimen volume and type of test.

- How about direct tensile test?

Triaxial behavior
Brittle/ductile behavior

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- Brittle vs ductile
  - Ductile: rock support an increasing load as it deforms
  - Brittle: load decreases as the strain increases
- Brittle-ductile transition
  - Rock becomes more ductile with increasing confining pressure

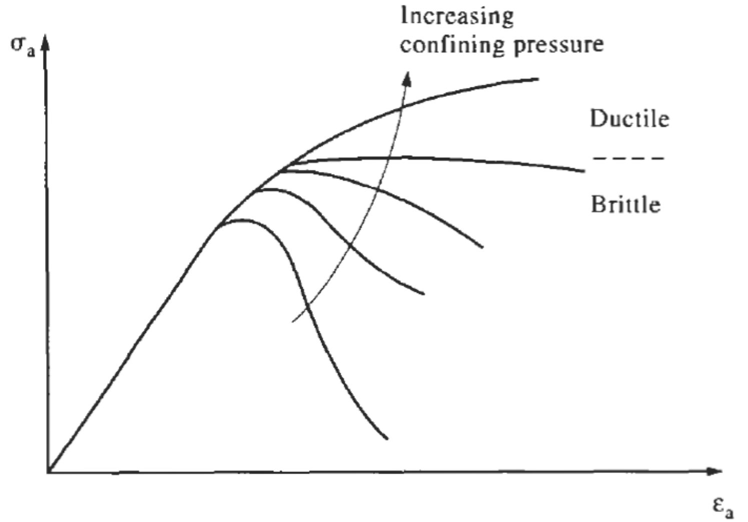


Figure 6.15 The effect of confining pressure in the triaxial test and the brittle–ductile transition.

Triaxial behavior
Brittle/ductile behavior

- Quantitative description of brittleness is still an open question
  - There are many definitions for brittleness index

$$B_1 = \frac{\varepsilon_{el}}{\varepsilon_{tot}} = \frac{\varepsilon_{el}}{\varepsilon_{el} + \varepsilon_{pl}}$$

$$B_2 = \frac{W_{el}}{W_{tot}}$$

$$B_3 = \frac{C_0 - T_0}{C_0 + T_0}$$

$$B_4 = \sin\varphi$$

$$B_5 = \frac{\tau_{max} - \tau_{res}}{\tau_{max}}$$

$$B_6 = \left|\frac{\varepsilon_f^p - \varepsilon_c^p}{\varepsilon_c^p}\right|$$

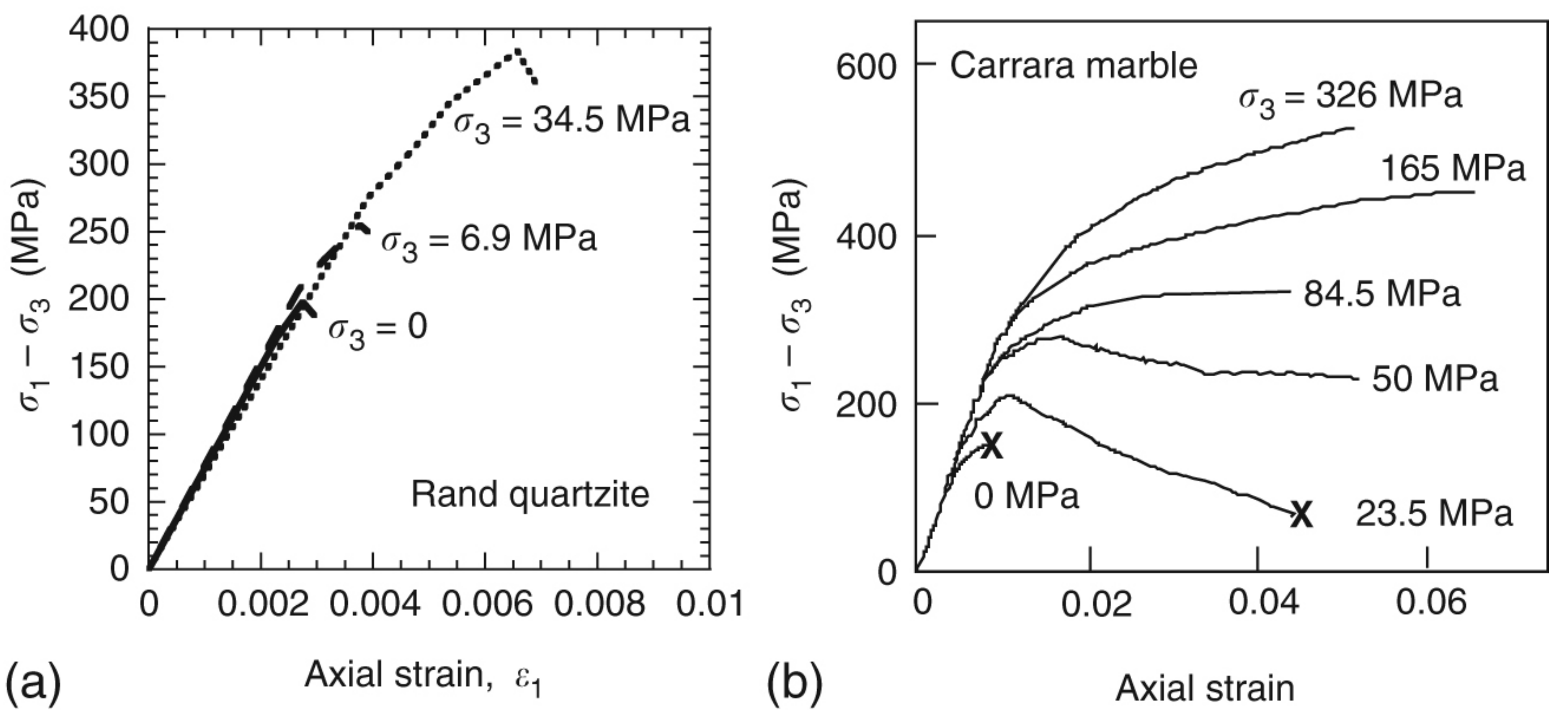
$$B_7 = OCR^b$$

$$B_8 = \frac{1}{2}\cdot\left(\frac{E_{dyn}[Mpsi](0.8 - \phi) - 1}{8 - 1} + \frac{\upsilon_{dyn} - 0.4}{0.15 - 0.4}\right)\cdot 100$$

Holt R. M. et al (2011)

Triaxial behavior
Effect of confining pressure

- With increasing confining pressure
  - Strength increases
  - Becomes more ductile

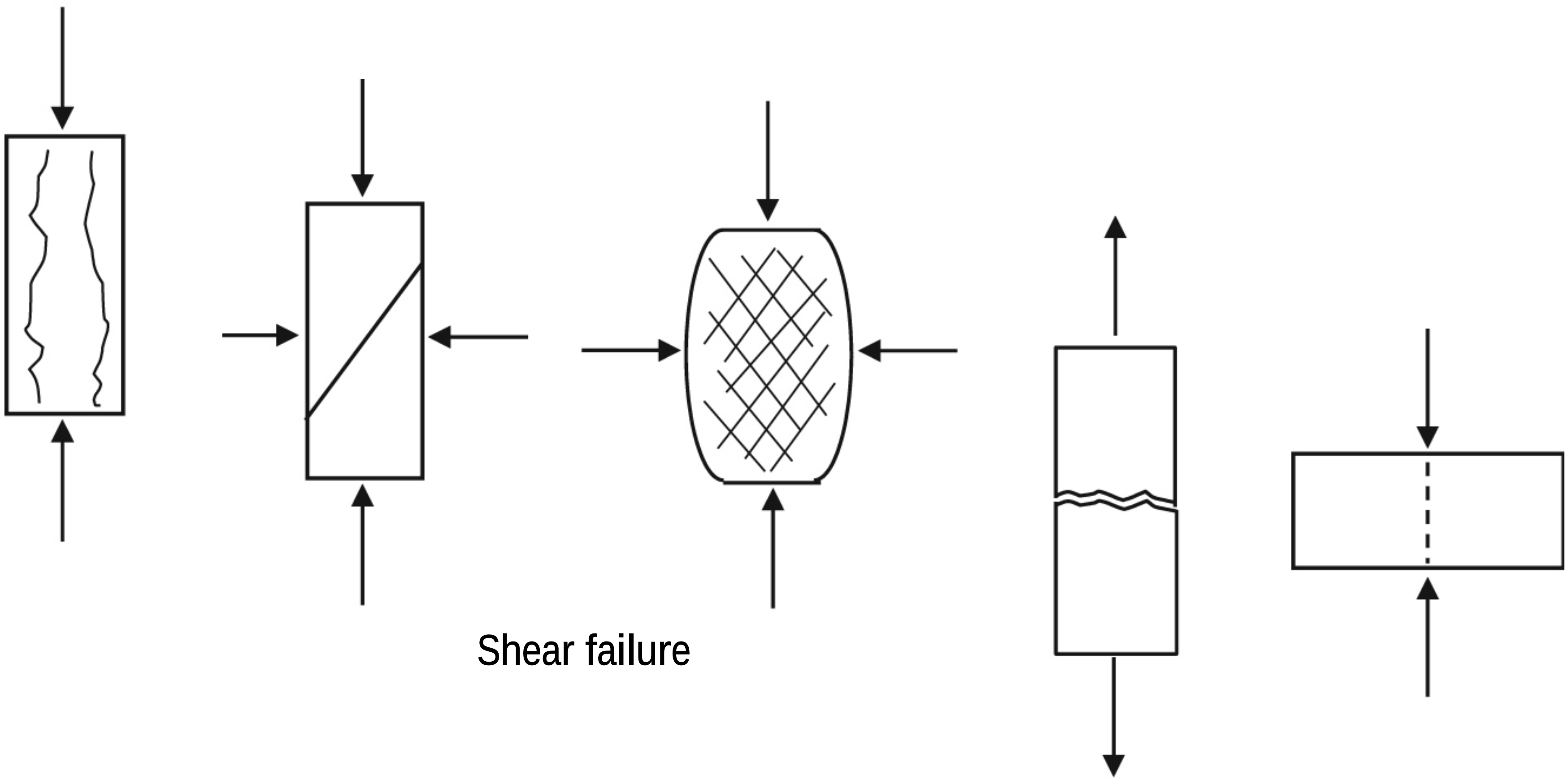


Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Laboratory Test

- Failure patterns vs. loading conditions

Longitudinal splitting



Shear failure

extension

Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Time dependent behavior
Creep/relaxation

- Creep:
  - Continued deformation when the applied stress is held constant
- Relaxation:
  - Decrease in stress when applied strain is held constant
- Fatigue
  - Increase in strain due to cyclic loading

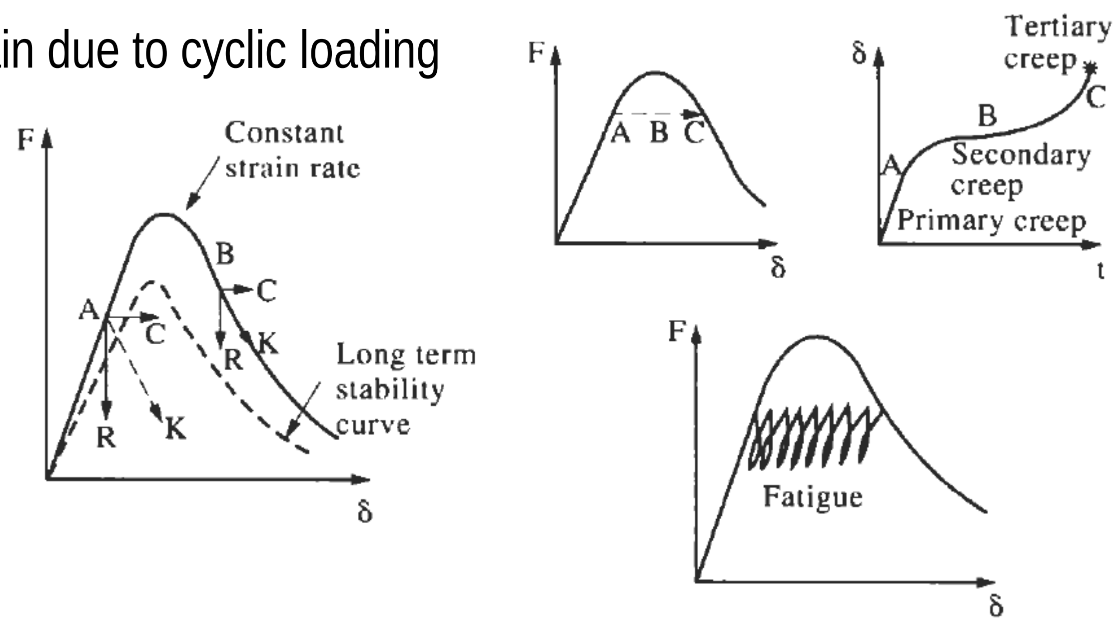


Figure 6.16 Time-dependent effects and the complete stress–strain curve.

Temperature dependent behavior

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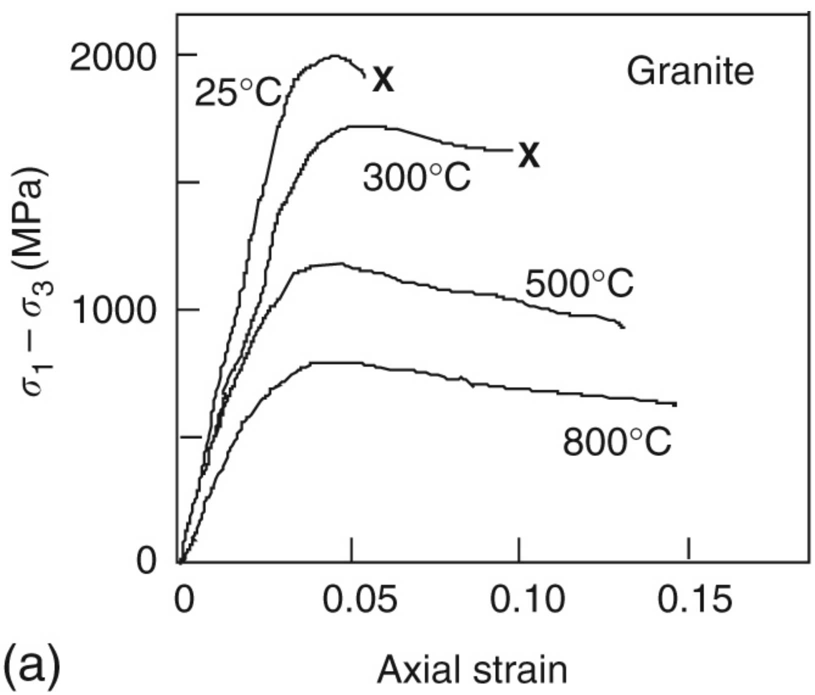
- Increase in temperature tends to:
  - Reduces elastic modulus & compressive strength
  - Increases the ductility



(a)

At confining pressure of 500 MPa

Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing