
Combinational Building Blocks – Decoder

One-hot representation

- Represent a set of N elements with N bits
- Exactly one bit is set
- Example – encode numbers 0-7

Binary	One-hot
000	00000001
001	00000010
010	00000100
...	...
110	01000000
111	10000000

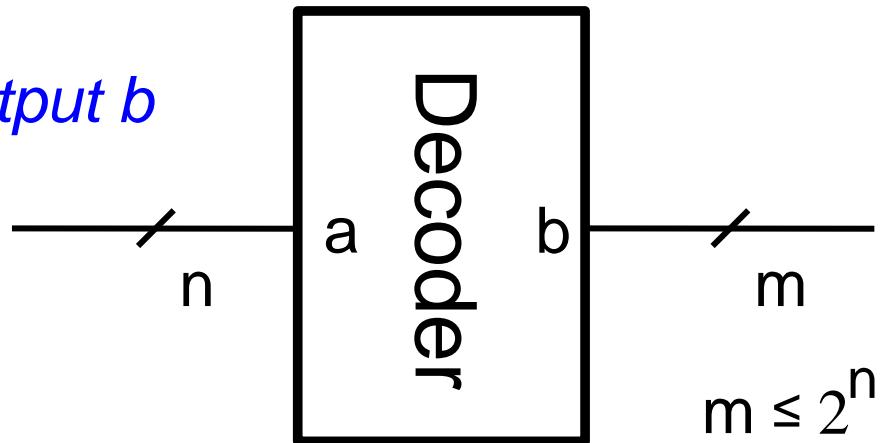
Decoder

- A decoder converts symbols from one code to another.
- A binary to one-hot decoder converts a symbol from binary code to a one-hot code.
 - One-hot code: exactly one bit is high at any given time and each bit represents a symbol.

Binary input a to one-hot output b

$$b[i] = 1 \text{ if } a = i$$

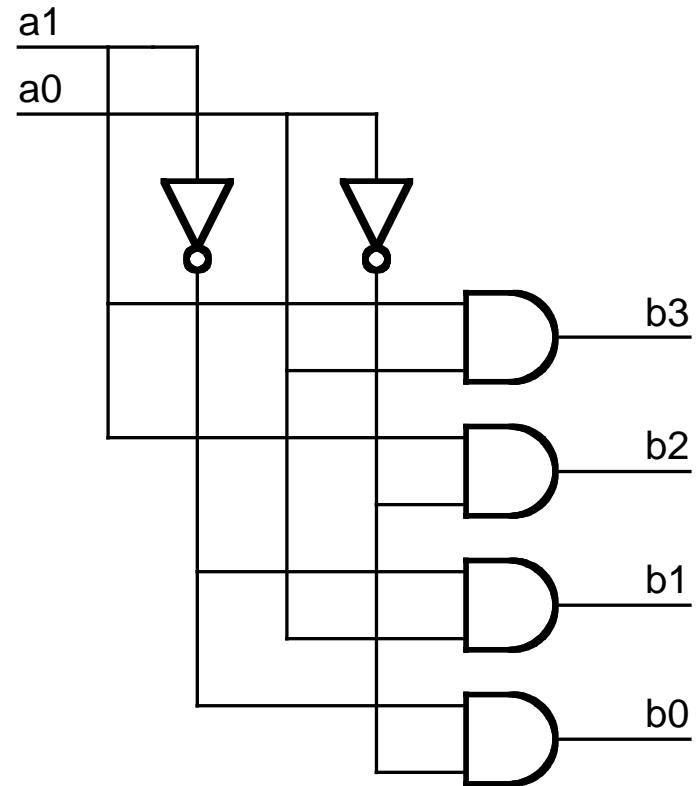
$$b = 1 << a$$



Example

2 → 4 Decoder

a1	a0	b3	b2	b1	b0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0



Can build a large decoder from several small decoders

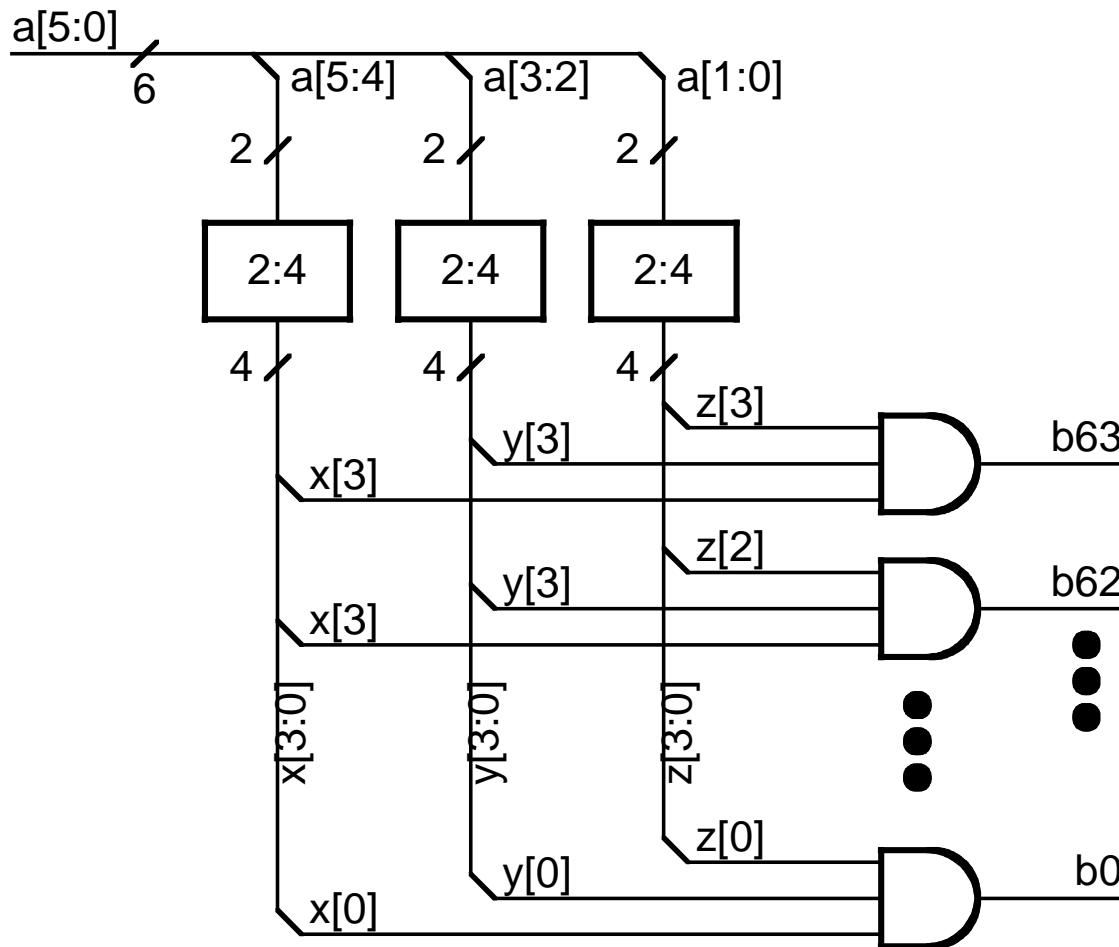
- Example – build a 6 : 64 decoder from three 2 : 4 decoders
- Each 2 : 4 decodes 2 bits
 - $a[5:4] \rightarrow x[3:0]$, $a[3:2] \rightarrow y[3:0]$, $a[1:0] \rightarrow z[3:0]$
- AND one bit each of x, y, and z to generate an output
 - $b[a] = x[a[5:4]] \& y[a[3:2]] \& z[a[1:0]]$

a5	a4	x3	x2	x1	x0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

a3	a2	y3	y2	y1	y0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

a1	a0	z3	z2	z1	z0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

2-Stage decoders

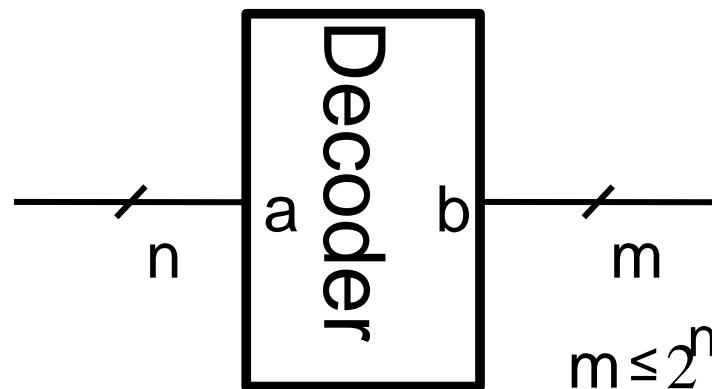


Verilog implementation of a decoder

```
// a - binary input      (n bits wide)
// b - one hot output   (m bits wide)
module Dec(a, b) ;
  parameter n=2 ;
  parameter m=4 ;

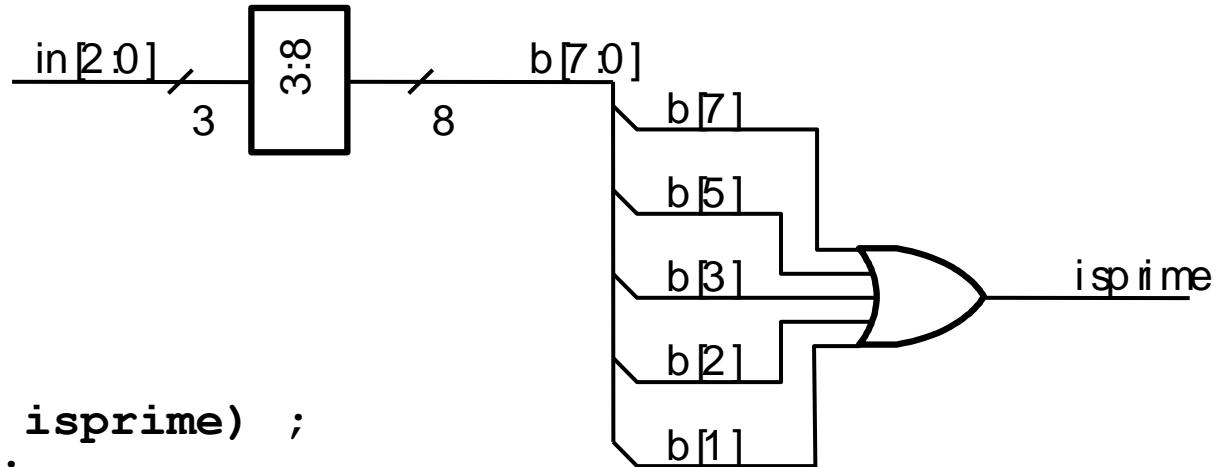
  input  [n-1:0] a ;
  output [m-1:0] b ;

  wire [m-1:0] b = 1<<a ;
endmodule
```



Logic implementation using decoder

Example – 3-bit prime number function



```
module Primed(in, isprime) ;
  input [2:0] in ;
  output      isprime ;
  wire [7:0] b ;

  // compute the output as the OR of the required minterms
  wire          isprime = b[1] | b[2] | b[3] | b[5] | b[7] ;

  // instantiate a 3->8 decoder
  Dec #(3,8) d(in,b) ;
endmodule
```

Combinational Building Blocks – Encoder

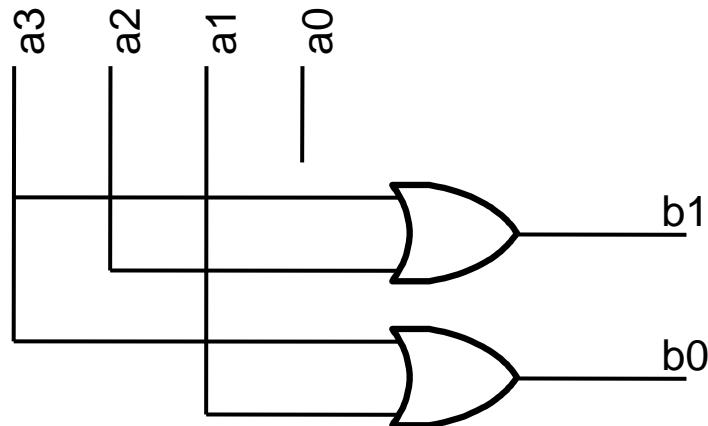
Encoder

- An encoder is an inverse of a decoder.
- Encoder is a logic module that converts a one-hot input signal to a binary-encoded output signal.
- Example: a 4 : 2 encoder.

a3	a2	a1	a0	b1	b0
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

$$b_0 = a_3 \vee a_1$$

$$b_1 = a_3 \vee a_2$$

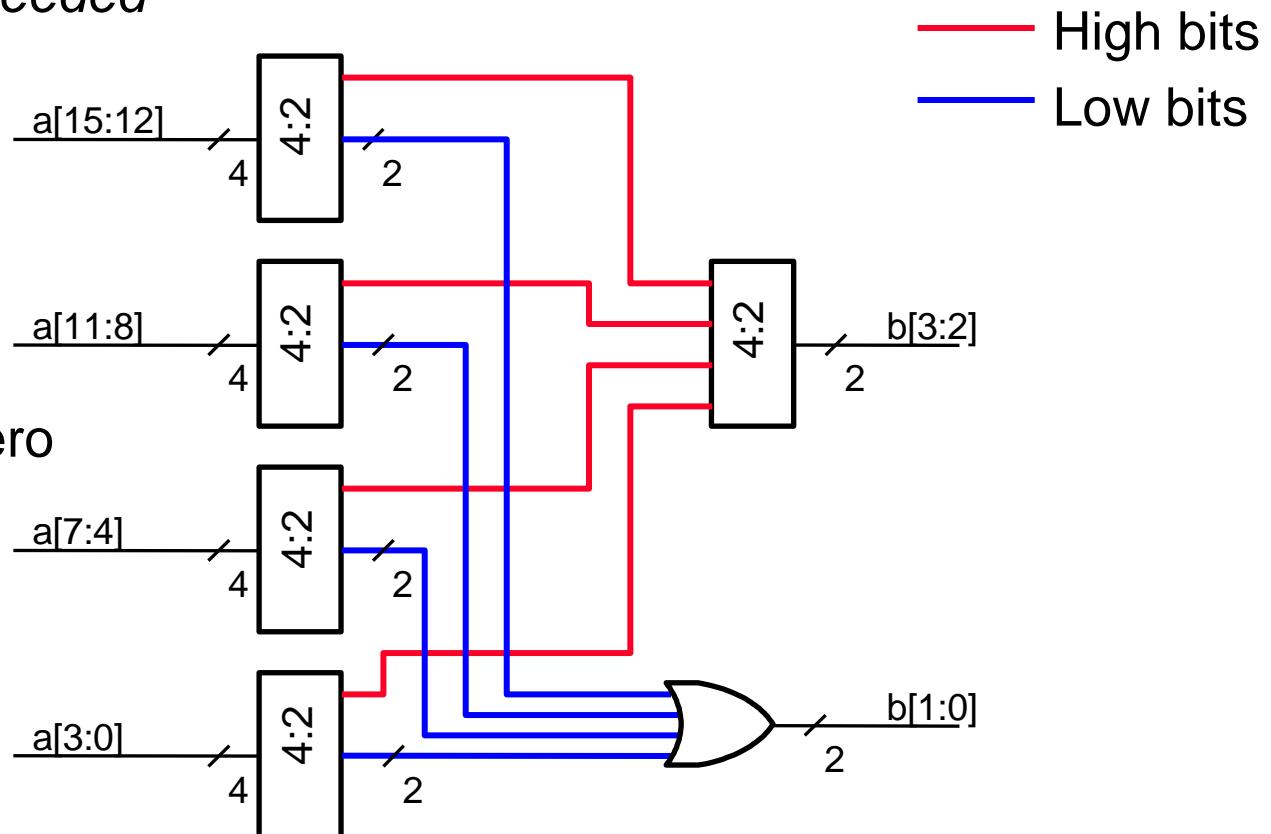


Designing a large encoder – 16 : 4 from five 4 : 2 encoders

Need an ‘any input true’ output on first rank of encoders

First rank encodes low bits, second rank encodes high bits

Repeat as needed



Summary

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-

Combinational Building Blocks – MUX

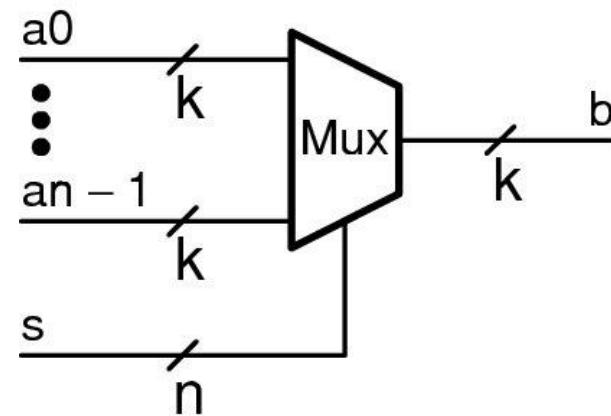
Multiplexer

- Multiplexer:
 - n k -bit inputs
 - n -bit one-hot select signal s
 - Multiplexers are commonly used as *data selectors*

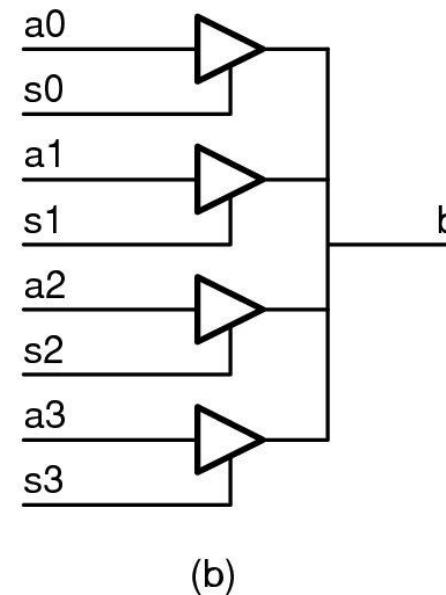
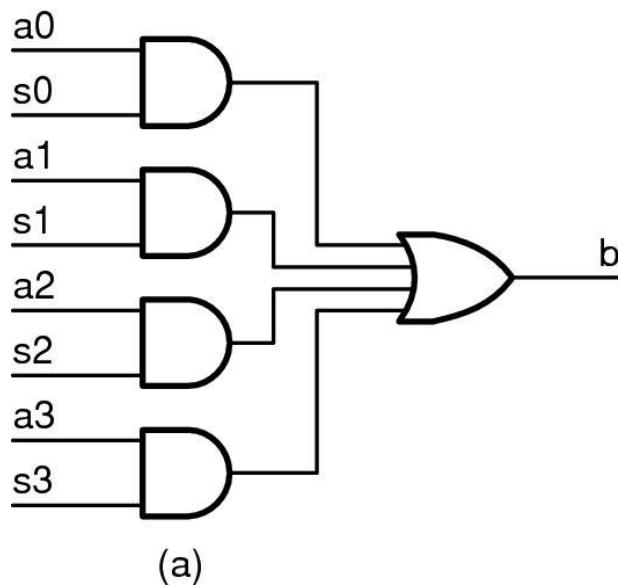
Selects one of n k -bit inputs

s must be one-hot

$b = a_i$ if $s[i] = 1$



Multiplexer Implementation



```
// four input k-wide mux with one-hot select
module Mux4(a3, a2, a1, a0, s, b) ;
    parameter k = 1 ;
    input [k-1:0] a0, a1, a2, a3 ; // inputs
    input [3:0]   s ; // one-hot select
    output[k-1:0] b ;
    wire [k-1:0] b = ({k{s[0]}} & a0) |
                      ({k{s[1]}} & a1) |
                      ({k{s[2]}} & a2) |
                      ({k{s[3]}} & a3) ;
endmodule
```

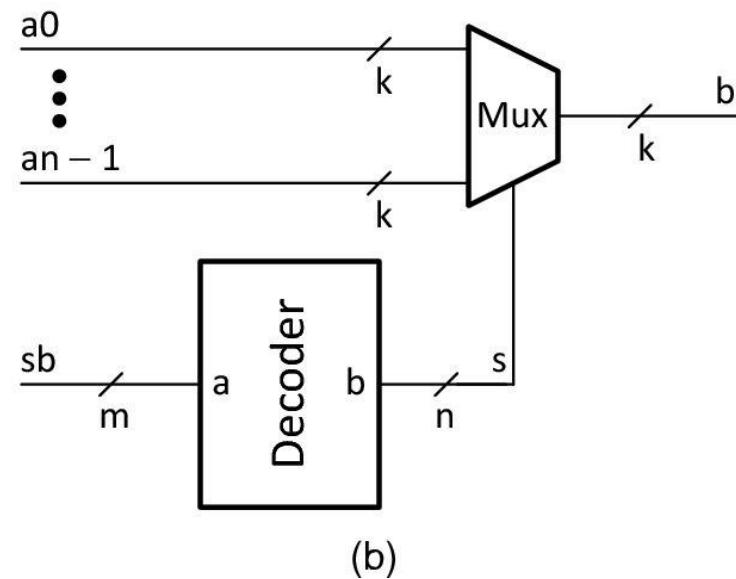
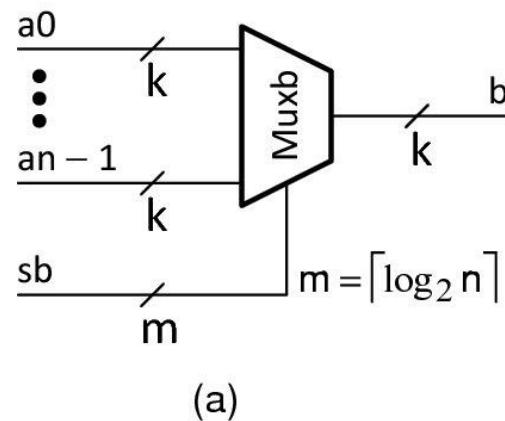
```
Mux4 #(2) mx(2'd3, 2'd2, 2'd1, 2'd0, f, h) ;
```

	f	h
#	0001	00
#	0010	01
#	0100	10
#	1000	11

```
module Mux3a(a2, a1, a0, s, b) ;
parameter k = 1 ;
input [k-1:0] a0, a1, a2 ; // inputs
input [2:0] s ; // one-hot select
output[k-1:0] b ;
reg [k-1:0] b ;

always @(*) begin
  case(s)
    3'b001: b = a0 ;
    3'b010: b = a1 ;
    3'b100: b = a2 ;
    default: b = {k{1'bx}} ;
  endcase
end
endmodule
```

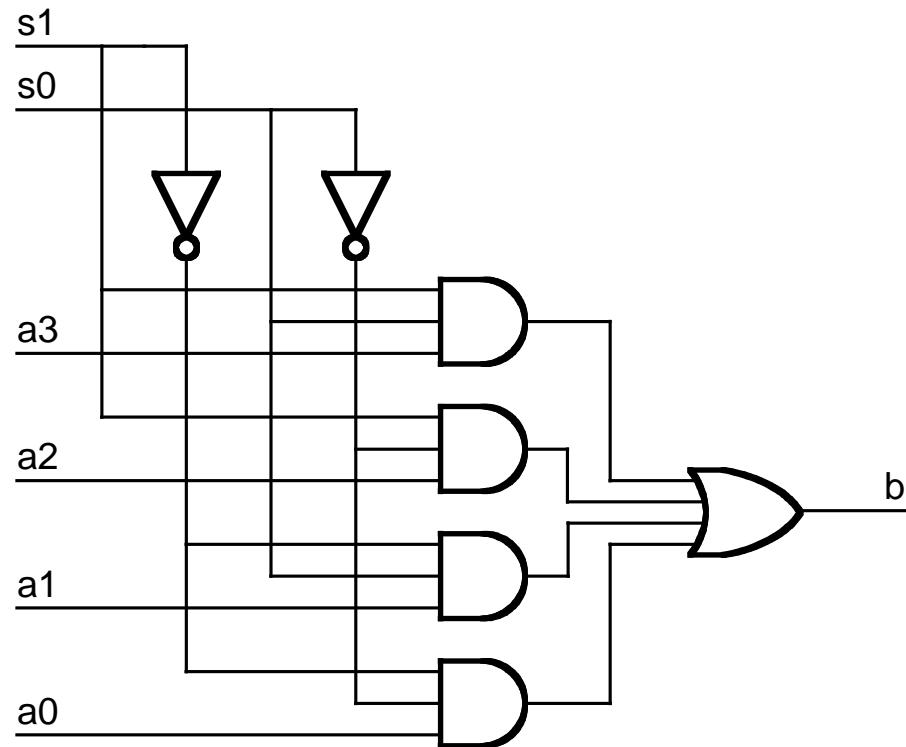
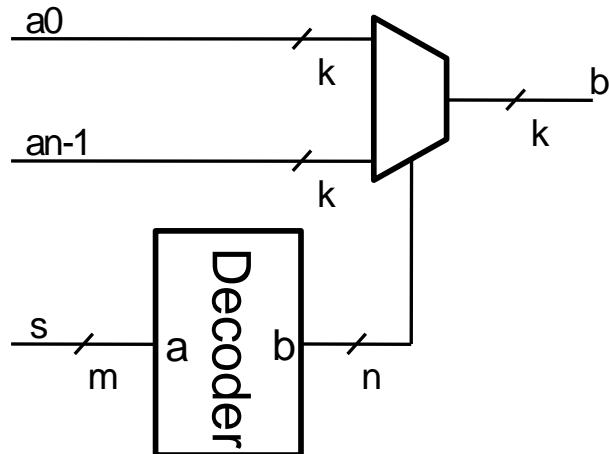
k-bit Binary-Select Multiplexer



Selects one of n k-bit inputs

$$b = a_i \text{ if } sb = i$$

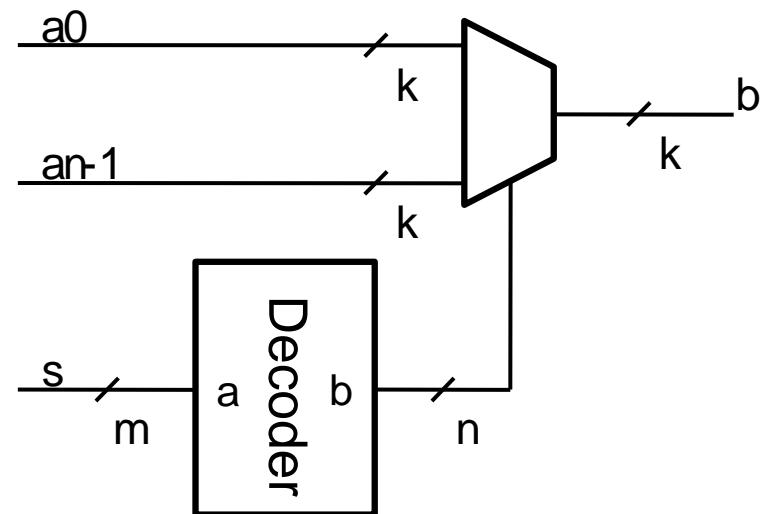
k-bit Binary-Select Multiplexer



Implementing k-bit Binary-Select Multiplexer using Verilog

```
// 3:1 multiplexer with binary select (arbitrary width)
module Muxb3(a2, a1, a0, sb, b) ;
  parameter k = 1 ;
  input [k-1:0] a0, a1, a2 ; // inputs
  input [1:0]   sb ;        // binary select
  output[k-1:0] b ;         // output
  wire [2:0]    s ;         // one-hot select

  Dec #(2,3) d(sb,s) ; // Decoder converts binary to one-hot
  Mux3 #(k) m(a2, a1, a0, s, b) ; // multiplexer selects input
endmodule
```



```
// 3:1 multiplexer with binary select (arbitrary width)
module Muxb3a(a2, a1, a0, sb, b) ;
  parameter k = 1 ;
  input [k-1:0] a0, a1, a2 ; // inputs
  input [1:0]   sb ;        // binary select
  output[k-1:0] b ;
  reg   [k-1:0] b ;

  always @(*) begin
    case (sb)
      0 : b = a0 ;
      1 : b = a1 ;
      2 : b = a2 ;
      default: b = {k{1'bX}} ;
    endcase
  end

endmodule
```

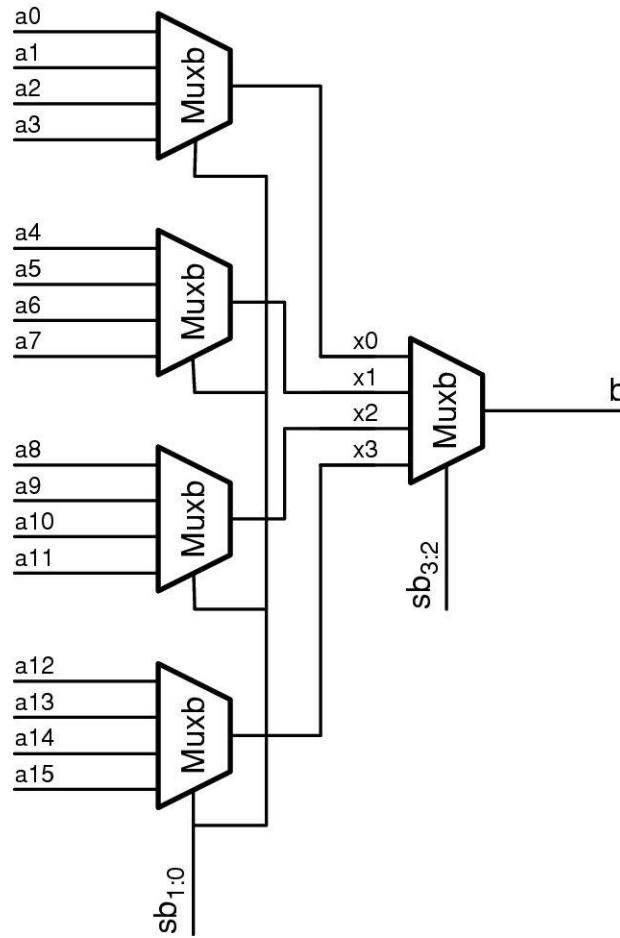
Implementing a large one-hot select mux using small ones

```
// 6:1 multiplexer with one-hot select (arbitrary width)
module Mux6a(a5, a4, a3, a2, a1, a0, s, b) ;
parameter k = 1 ;
input [k-1:0] a0, a1, a2, a3, a4, a5 ; // inputs
input [5:0]   s ;                      // one-hot select
output[k-1:0] b ; 
wire   [k-1:0] ba, bb ;

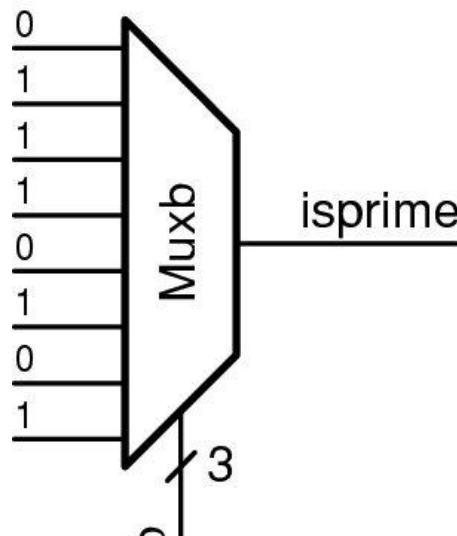
assign b = ba | bb ;

Mux3 #(k) ma(a2, a1, a0, s[2:0], ba) ;
Mux3 #(k) mb(a5, a4, a3, s[5:3], bb) ;
endmodule
```

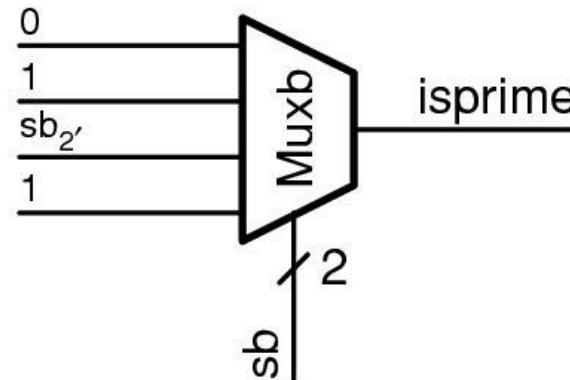
Implementing a large binary-select mux using small ones



Logic with Muxes



(a)



(b)

```
module Primem(in, isprime) ;
  input [2:0] in ;
  output      isprime ;

  Muxb8 #(1) m(1, 0, 1, 0, 1, 1, 1, 0, in, isprime) ;
endmodule
```

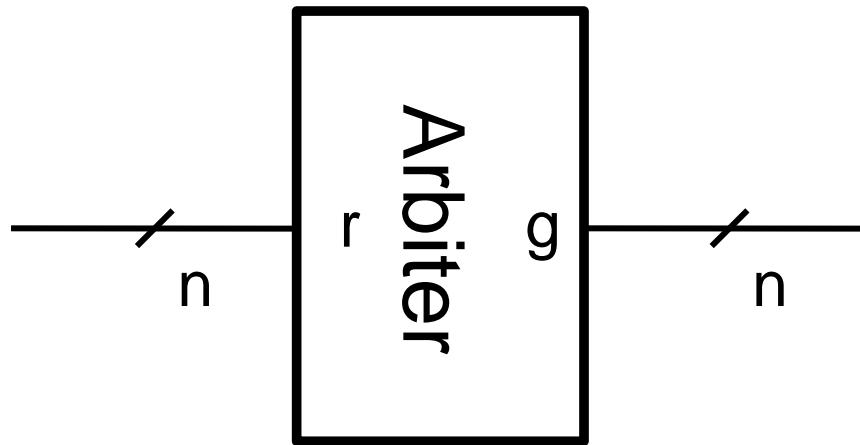
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Combinational Building Blocks - Arbiter

Arbiter

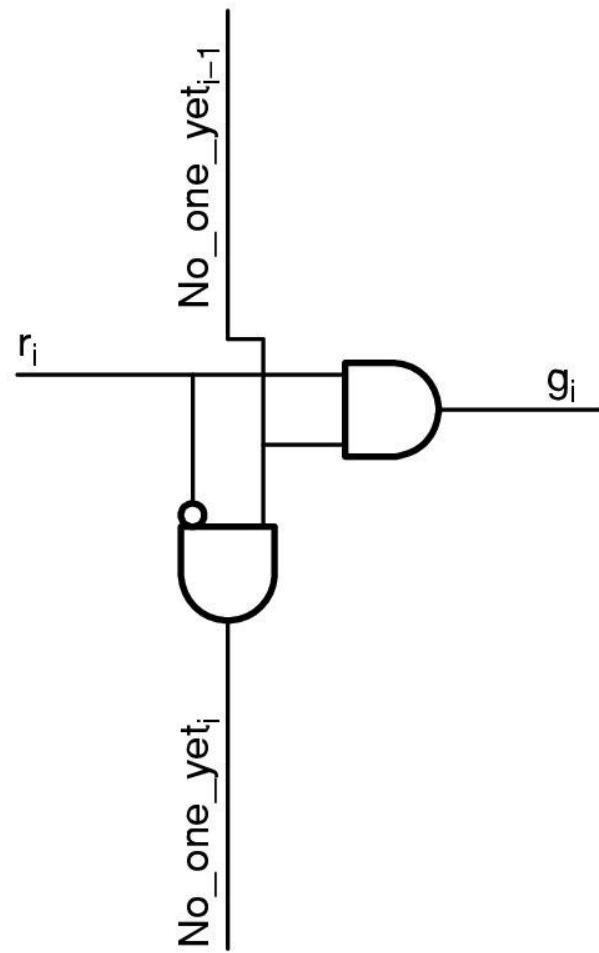
Arbiter handles requests from multiple devices to use a single resource



Finds first “1” bit in r

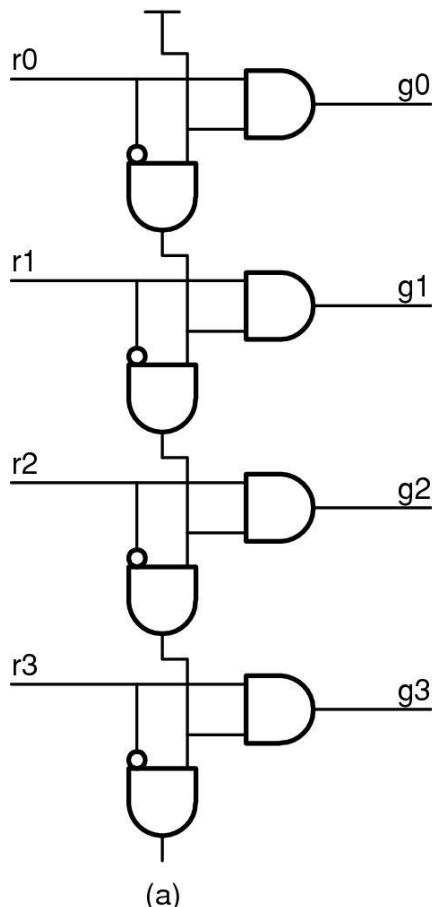
$g[i]=1$ if $r[i]=1$ and $r[j]=0$ for $j>i$

Logic Diagram of one Bit of an Arbiter

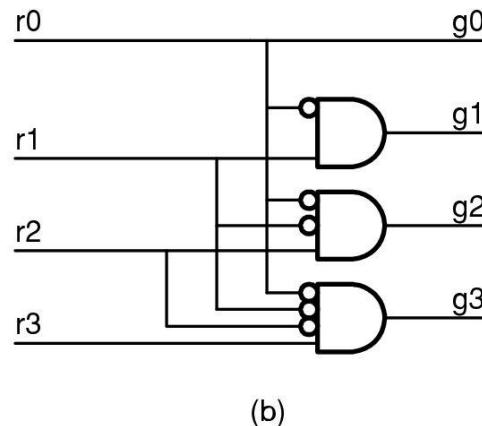


Two Implementations of a 4-bit Arbiter

Using bit-cell



Using look-ahead



(b)

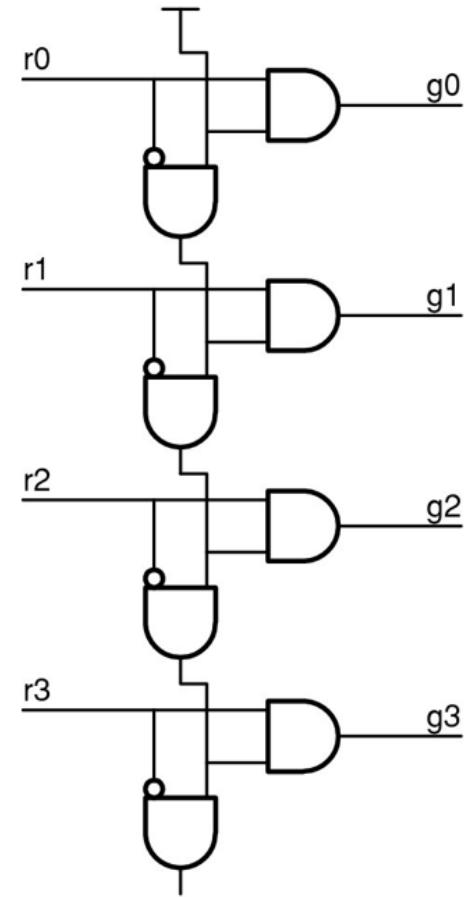
Arbiter using Verilog with LSB as the highest priority

```
// arbiter (arbitrary width)
module Arb(r, g) ;
parameter n=8 ;
input [n-1:0] r ;
output [n-1:0] g ;
wire [n-1:0] c ;

assign c = { (~r[n-2:0] & c[n-2:0]),1'b1} ;

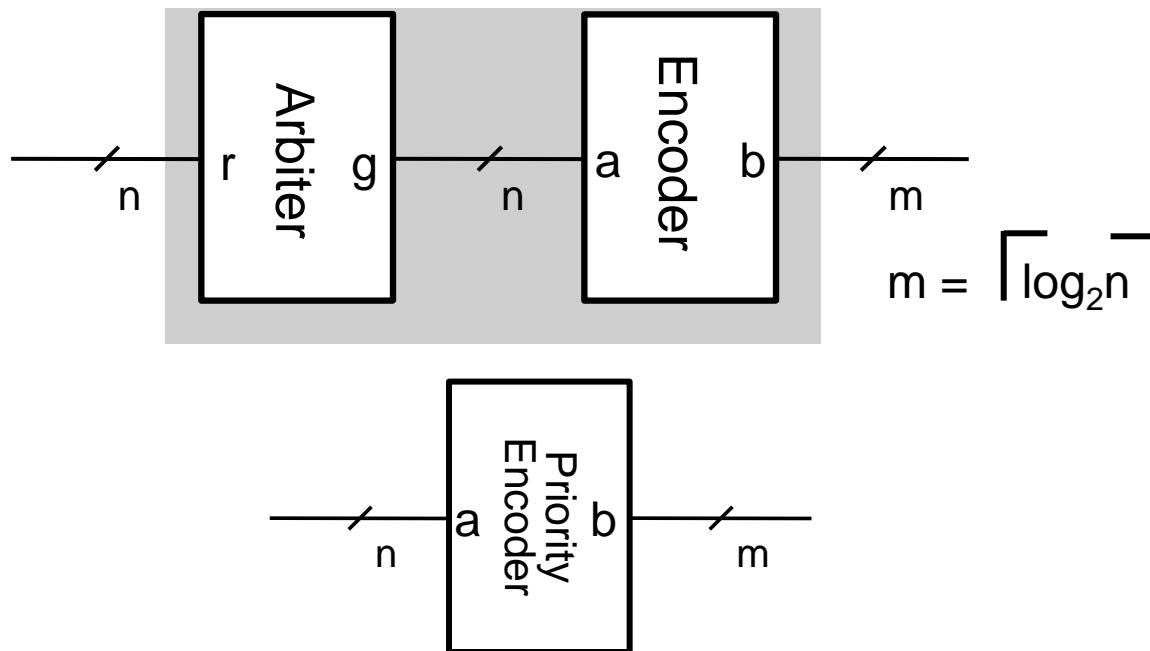
assign g = r & c ;

endmodule
```



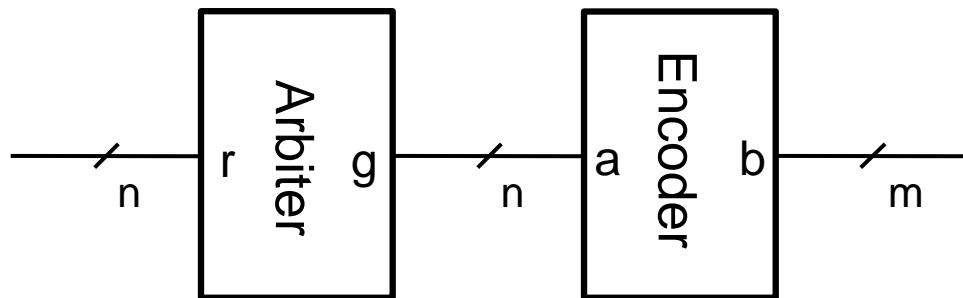
Priority Encoder

- Priority encoder:
 - n-bit input signal **a**
 - m-bit output signal **b**
 - **b** indicates the position of the first 1 bit in **a**



Verilog for Priority Encoder

```
// priority encoder (arbitrary width)
module PriorityEncoder83(r, b) ;
    input [7:0] r ;
    output [2:0] b ;
    wire [7:0] g ;
    Arb #(8) a(r, g) ;
    Enc83 e(g, b) ;
endmodule
```



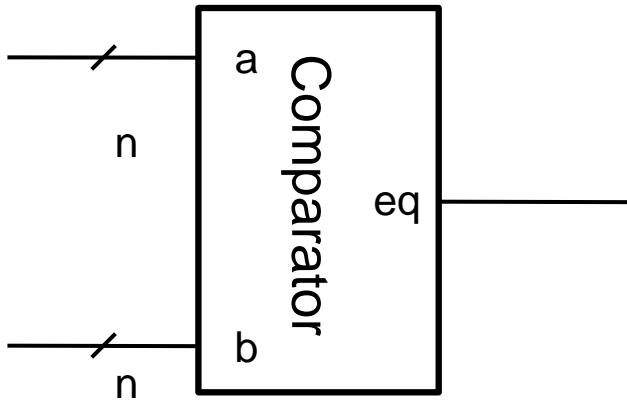
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Combinational Building Blocks

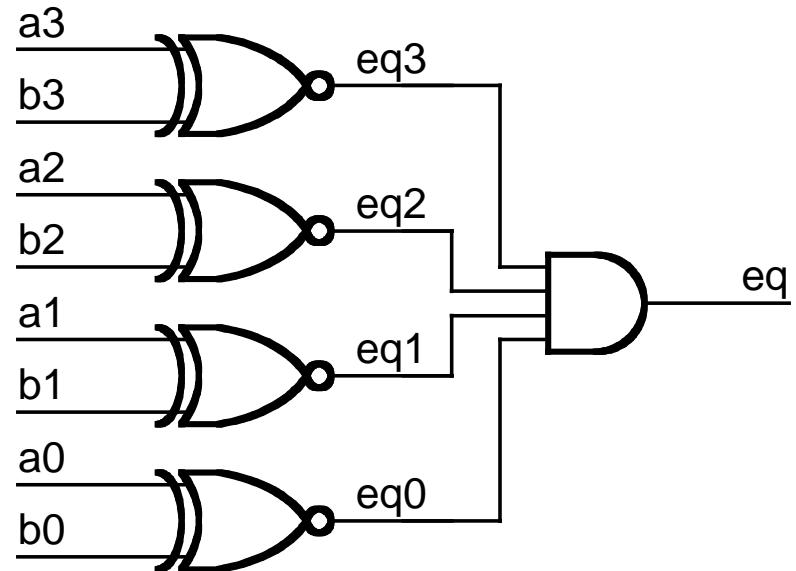
- Comparator and shifter

Equality Comparator

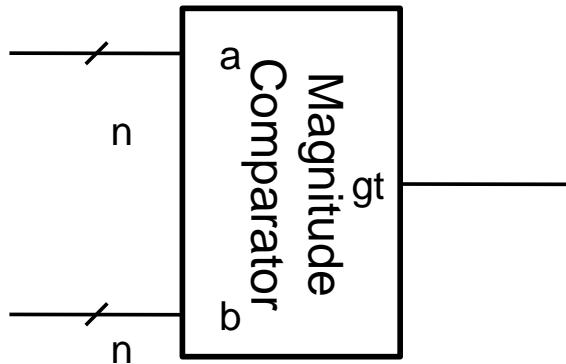


```
// equality comparator
module EqComp(a, b, eq) ;
parameter k=8;
input [k-1:0] a, b;
output eq ;

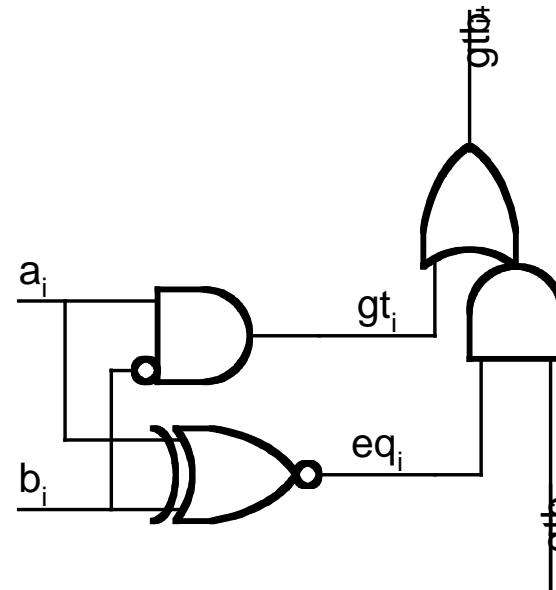
assign eq = (a==b) ;
endmodule
```



Magnitude Comparator



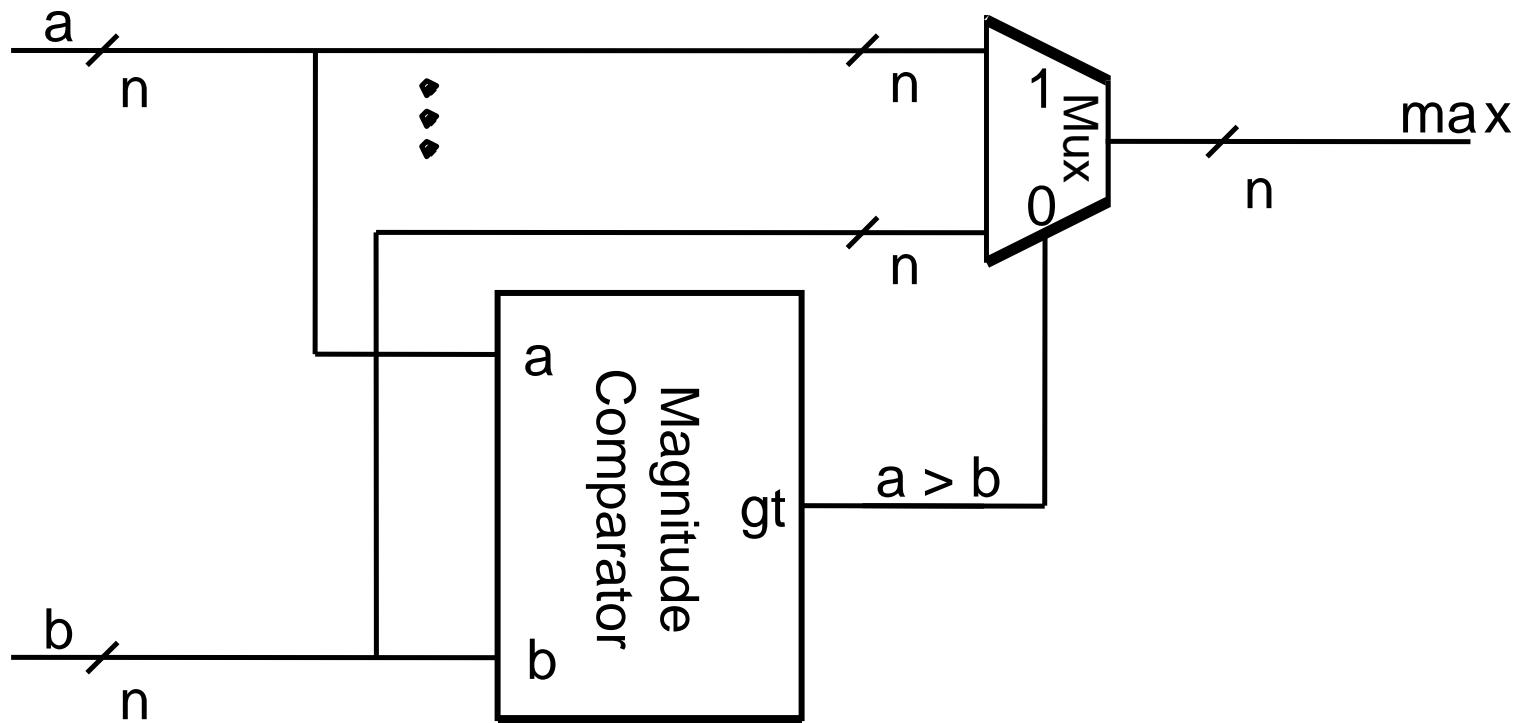
```
// magnitude comparator
module MagComp(a, b, gt) ;
  parameter k=8 ;
  input [k-1:0] a, b ;
  output gt ;
  wire [k-1:0] eqi = a ~^ b ;
  wire [k-1:0] gti = a & ~b ;
  wire [k:0] gtb {((eqi[k-1:0] & gtb[k-1:0]) | gti[k-1:0]), 1'b0} ;
  wire gt = gtb[k] ;
endmodule
```



```
// Behavioral Magnitude comparator
module MagComp_b(a, b, gt) ;
  parameter k=8 ;
  input [k-1:0] a, b ;
  output gt ;

  assign gt = (a > b) ;
endmodule
```

Putting things together – Maximum unit



Shifters

```
module ShiftLeft(n, a, b) ;
  parameter k=8 ;
  parameter lk=3 ;
  input [lk-1:0] n ;
  input [k-1:0] a ;
  output [2*k-2:0] b ;
  assign b = a<<n ;
endmodule
```

```
module BarrelLeftShift(n, a, b) ;
parameter k=8 ;
parameter lk=3 ;
input [lk-1:0] n ;
input [k-1:0] a ;
output [k-1:0] b ;
wire [2*k-2:0] x = a<<n ;
assign b = x[k-1:0] | {1'b0, x[2*k-2:k]} ;
endmodule
```

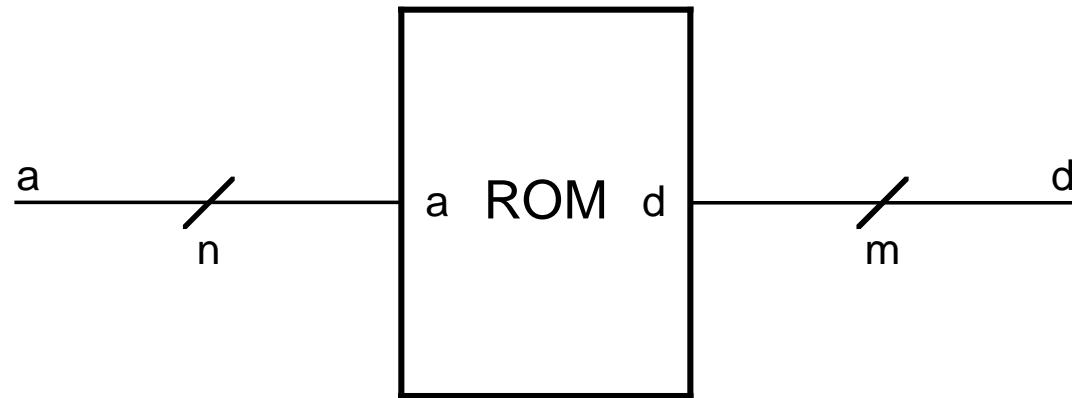
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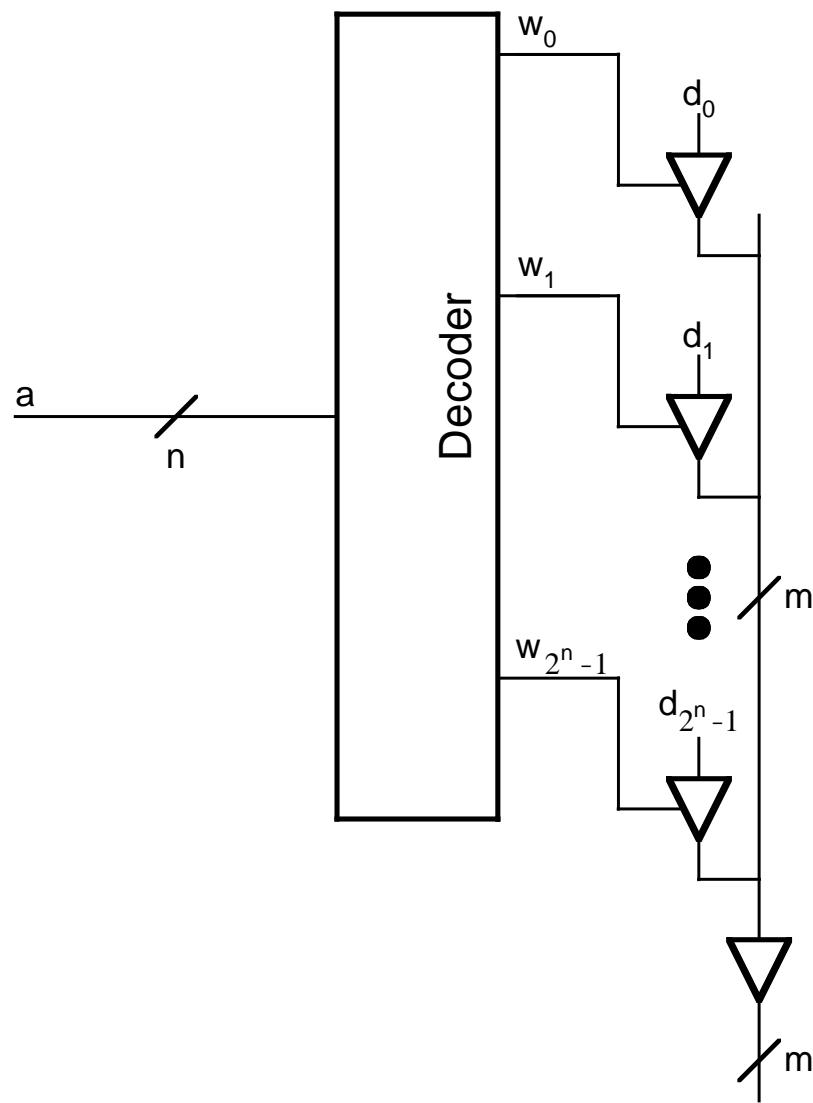
Combinational Building Blocks

- ROM, RAM, PLA

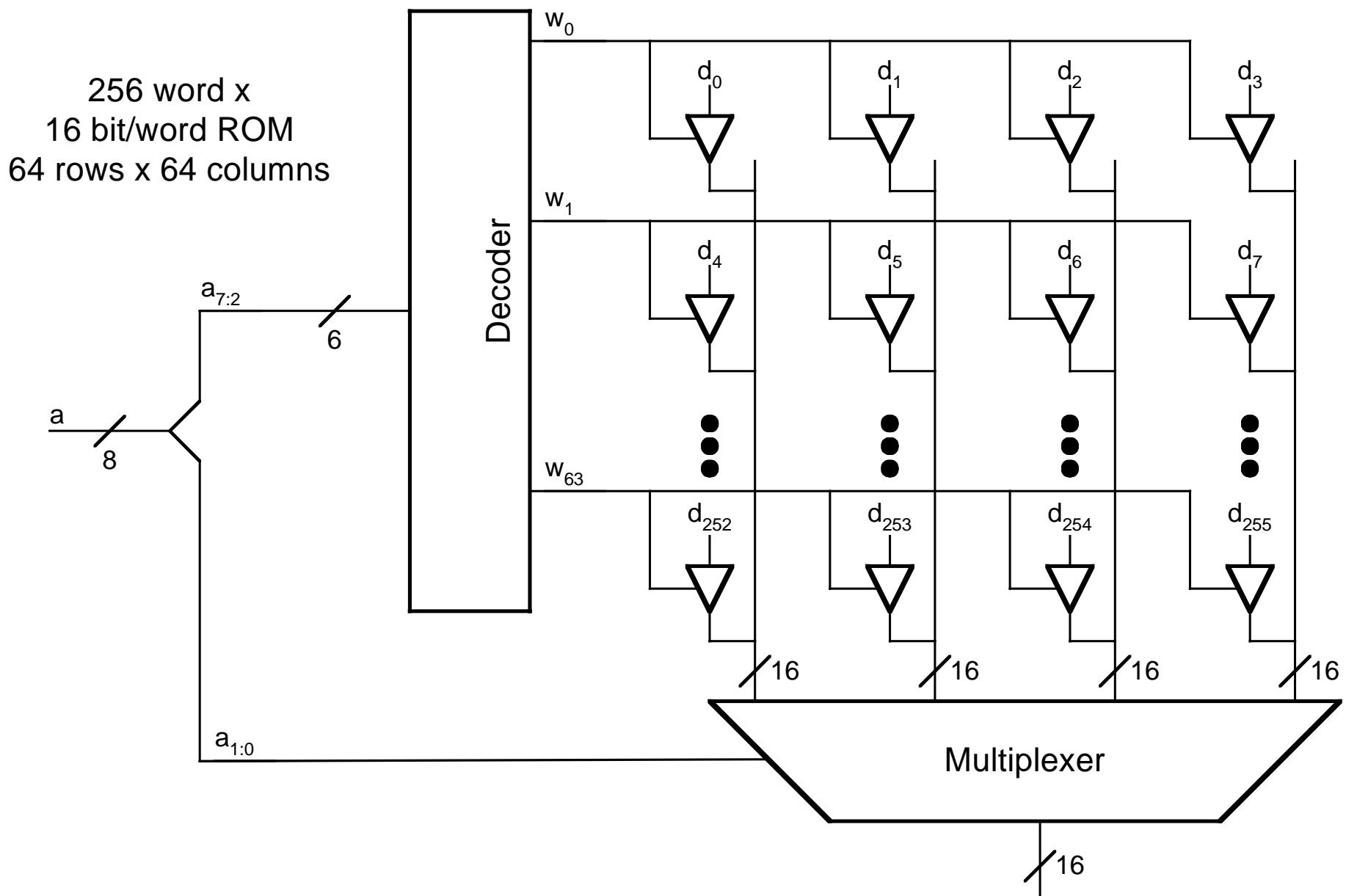
Read-only memory (ROM)



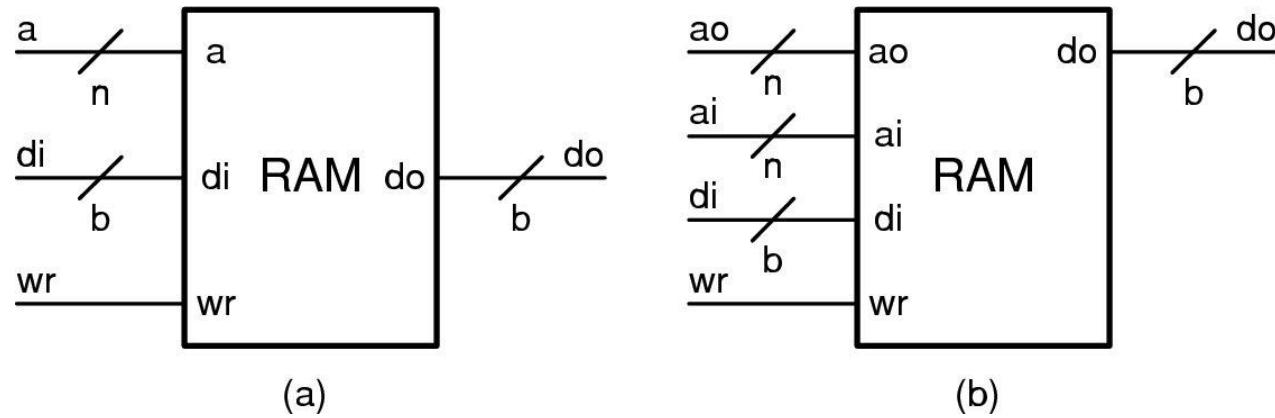
Conceptual block diagram



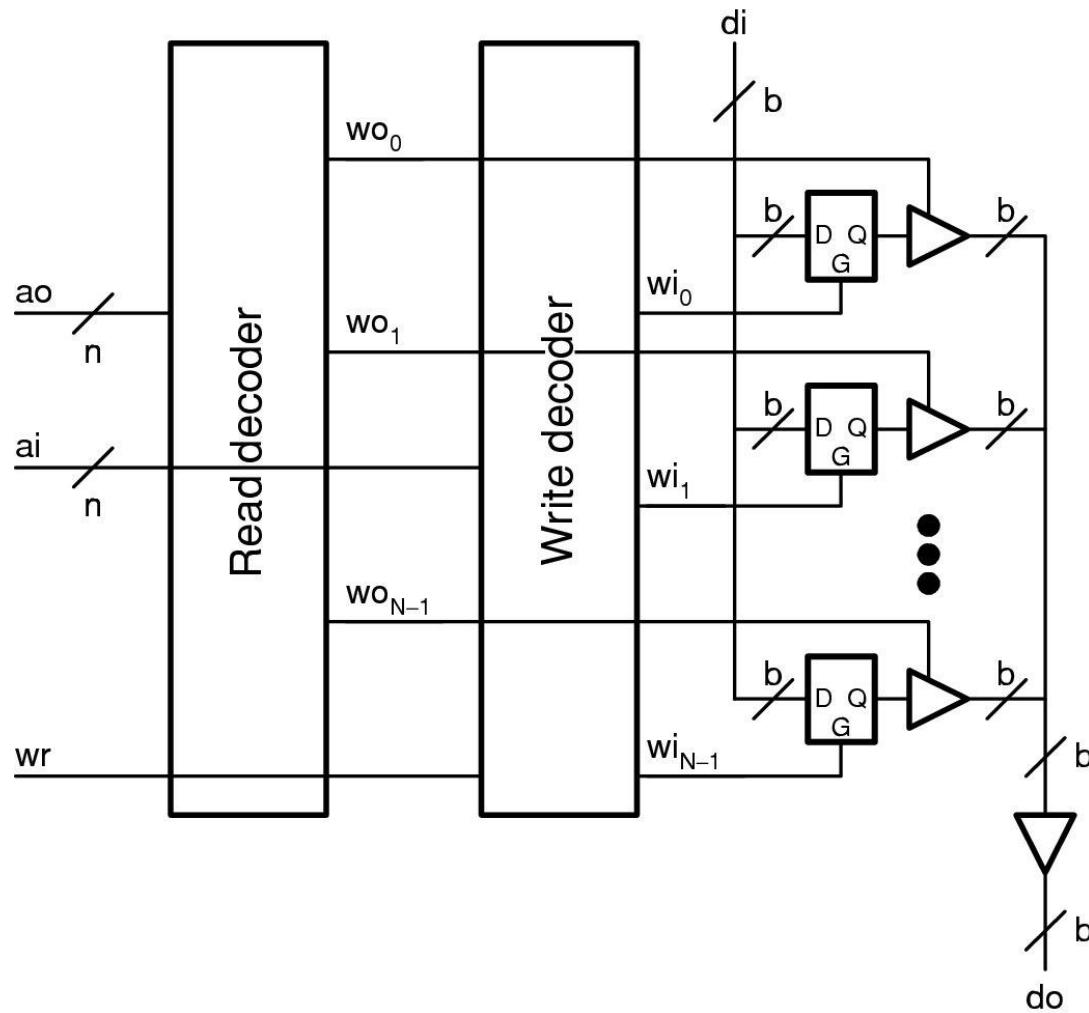
2-D array implementation



Read-write memory (RAM)



Dual-port RAM implementation



Programmable logic array

