Announcement

- Homework due April 10th
- Lab session on April 5th **(Lab report due April 12th)**
- Full Attendance **10**

  (If you missed 2 classes of 20, mark will be **9**)

  (Minimum absence is 25%)
- Participation = max **5**

  (If you are 10th out of 50 students, you will get additional **4**)

  (If you are 40th out of 50 students, you will get additional **1**)

-> No one will take disadvantage!

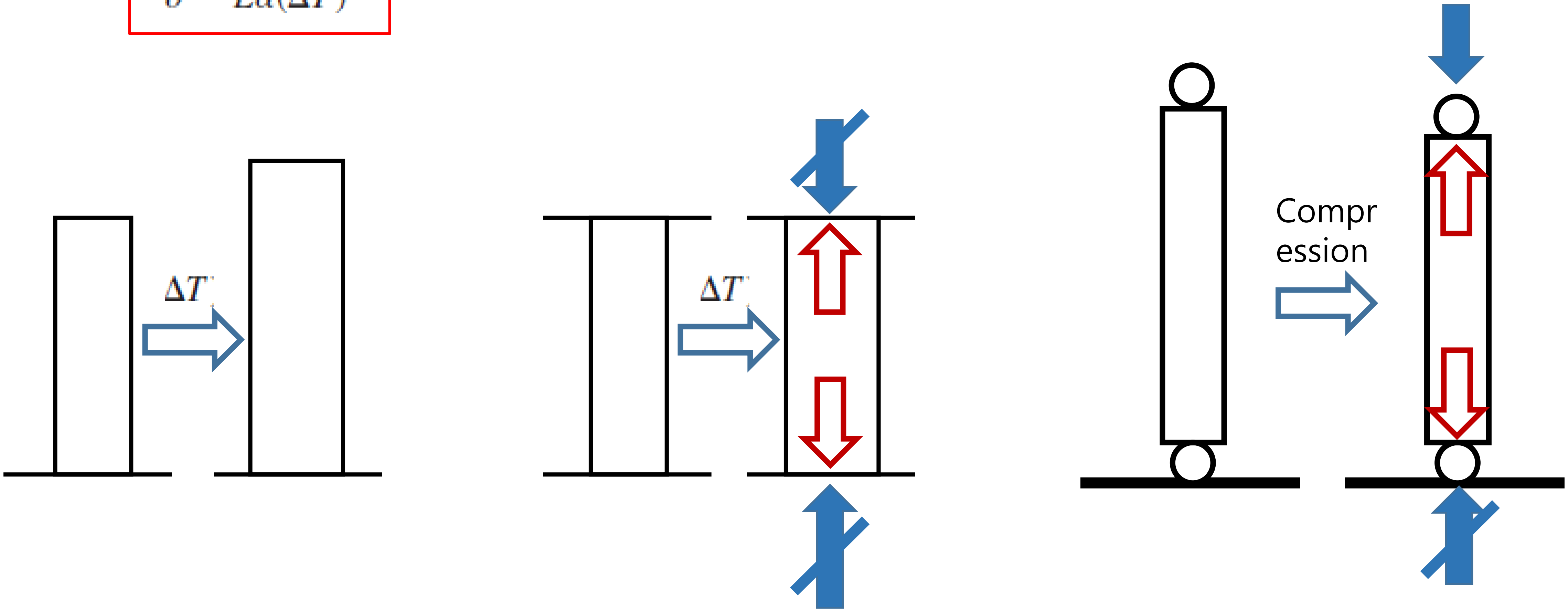
Thermal stress-strain

$$\sigma = E\alpha(\Delta T)$$

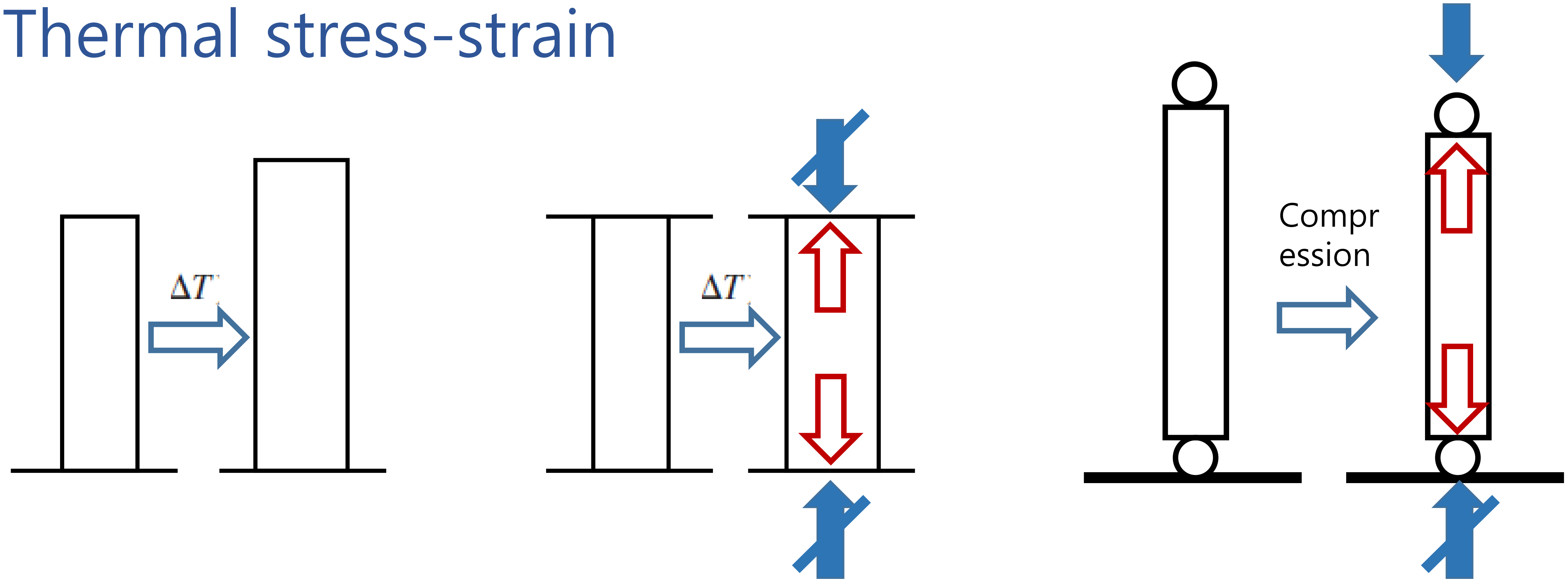
Node equilibrium

Internal equilibrium

Global equilibrium



Thermal stress-strain



$$\begin{array}{c} \delta \neq 0 \\ \varepsilon \neq 0 \\ \sigma_{\text{int}} = 0 \\ R_{rec} = 0 \end{array} = \alpha(\Delta T)$$

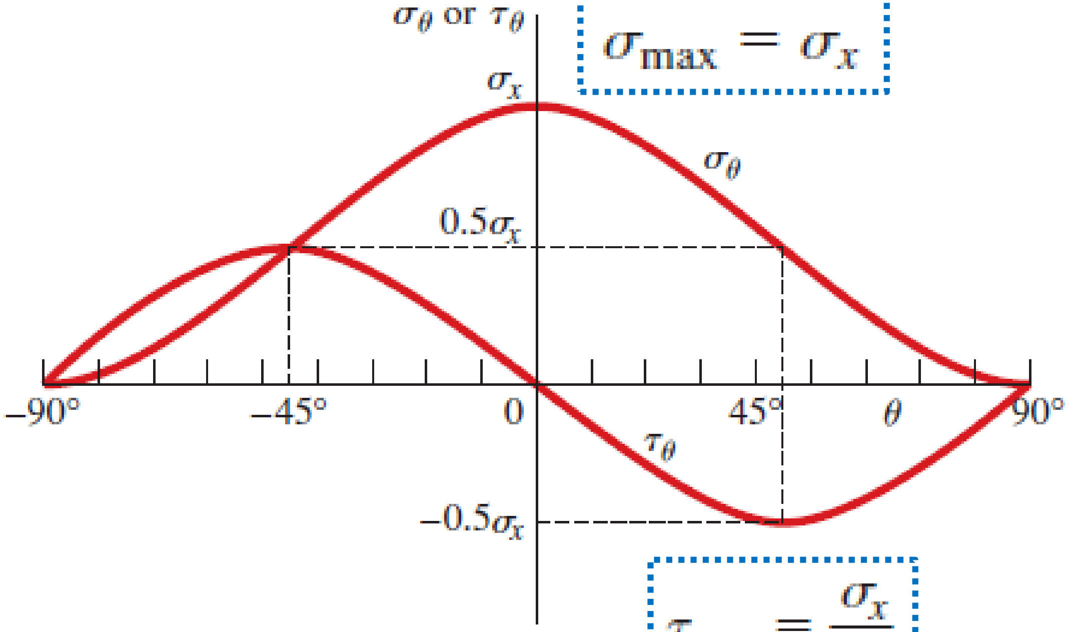
$$\begin{array}{c} \delta = 0 \\ \varepsilon = 0 \\ \sigma_{\text{int}} \neq 0 \\ R_{rec} \neq 0 \end{array} = E\alpha(\Delta T)$$

$$\begin{array}{c} \delta \neq 0 \\ \varepsilon \neq 0 \\ \sigma_{\text{int}} \neq 0 \\ R_{rec} \neq 0 \end{array}$$

Stresses on inclined sections

$$\sigma_x = \frac{P}{A}$$

$$\sigma_{max} = \sigma_x$$

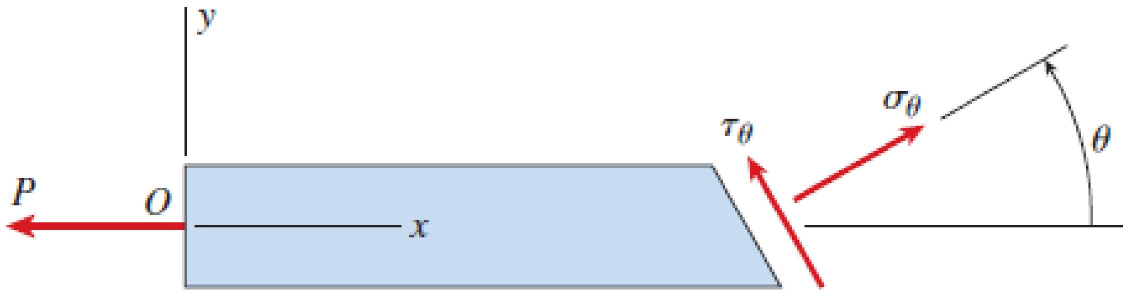


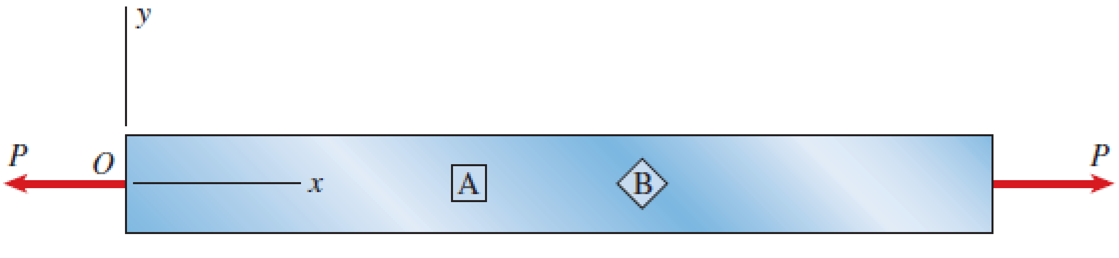
$$\tau_{max} = \frac{\sigma_x}{2}$$

$$\sigma_\theta = \sigma_x \cos^2\theta = \frac{\sigma_x}{2}(1 + \cos 2\theta)$$

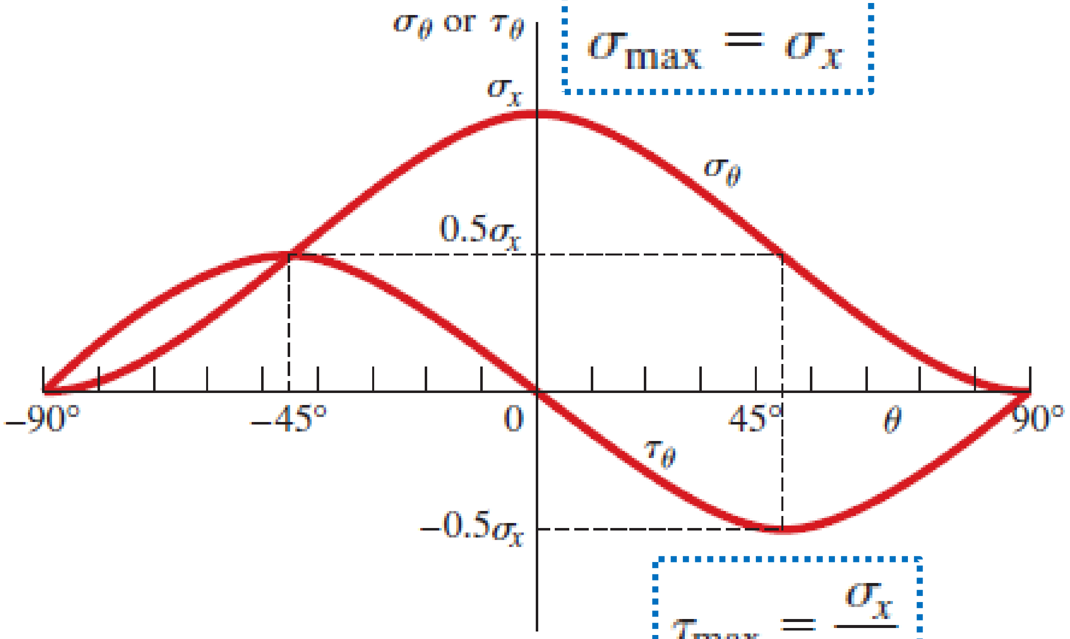
$$\tau_\theta = -\sigma_x \sin\theta \cos\theta = -\frac{\sigma_x}{2}(\sin 2\theta)$$

Stresses on inclined sections



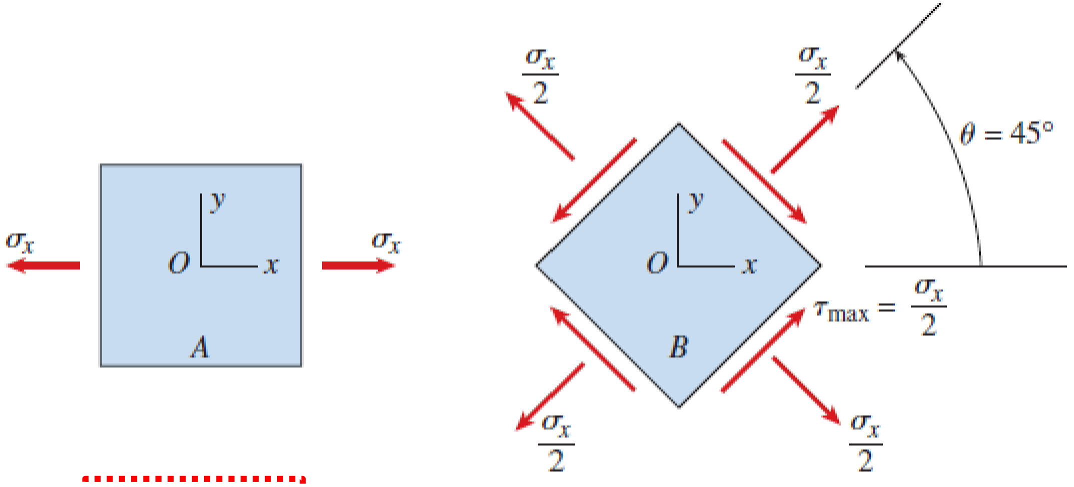


(a)



$$\sigma_{max} = \sigma_x$$

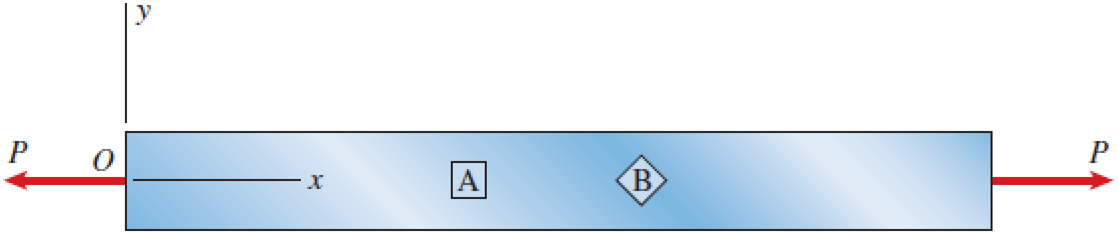
$$\tau_{max} = \frac{\sigma_x}{2}$$



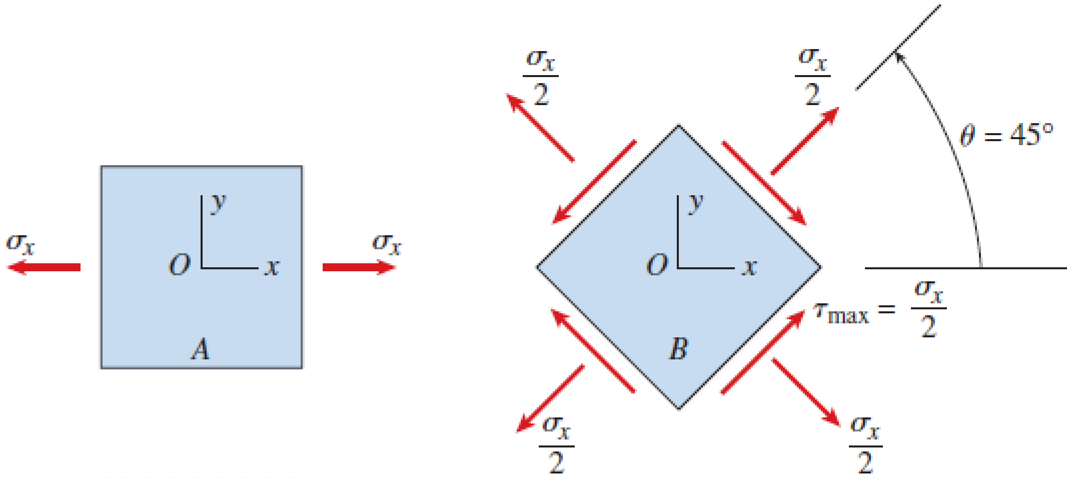
$$\sigma_x = \frac{P}{A}$$

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{P}{2A}$$

Stresses on inclined sections



(a)



$$\sigma_x = \frac{P}{A}$$

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{P}{2A}$$

E.g.,

If one material has:

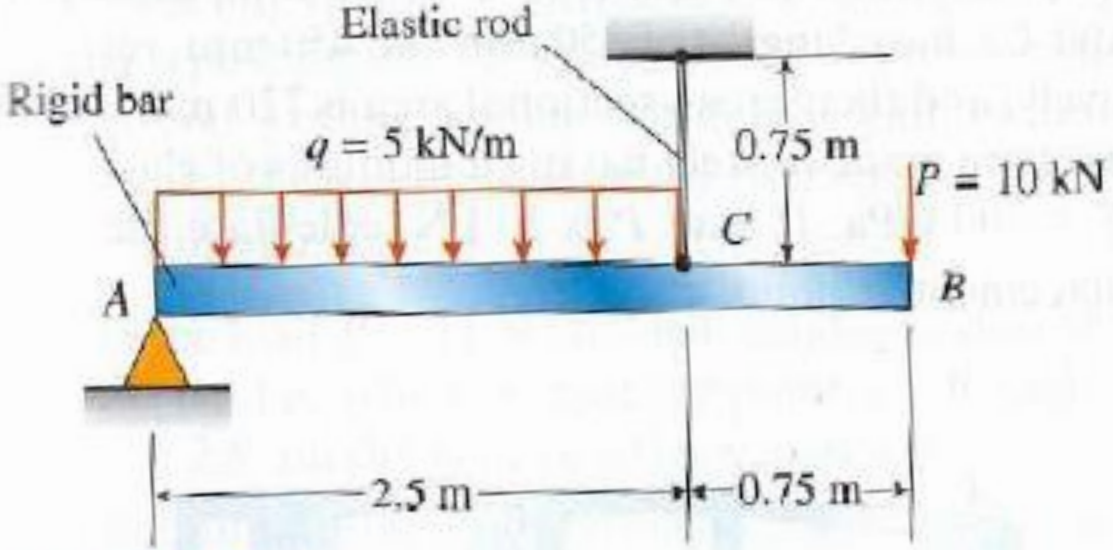
Allowable shear stress = 40 MPa

Allowable normal stress = 100 MPa

Then, the material under **axially loaded** will fail at **45 degree** with axial stress of 80 MPa.

Homework

2.2-2 Rigid bar *ABC* is supported with a pin at *A* and an elastic steel rod at *C*. The elastic rod has a diameter of 25 mm and modulus of elasticity $E = 200$ GPa. The bar is subjected to a uniform load *q* on span *AC* and a point load at *B*. Calculate the change in length of the elastic rod. What is the vertical displacement at point *B*?

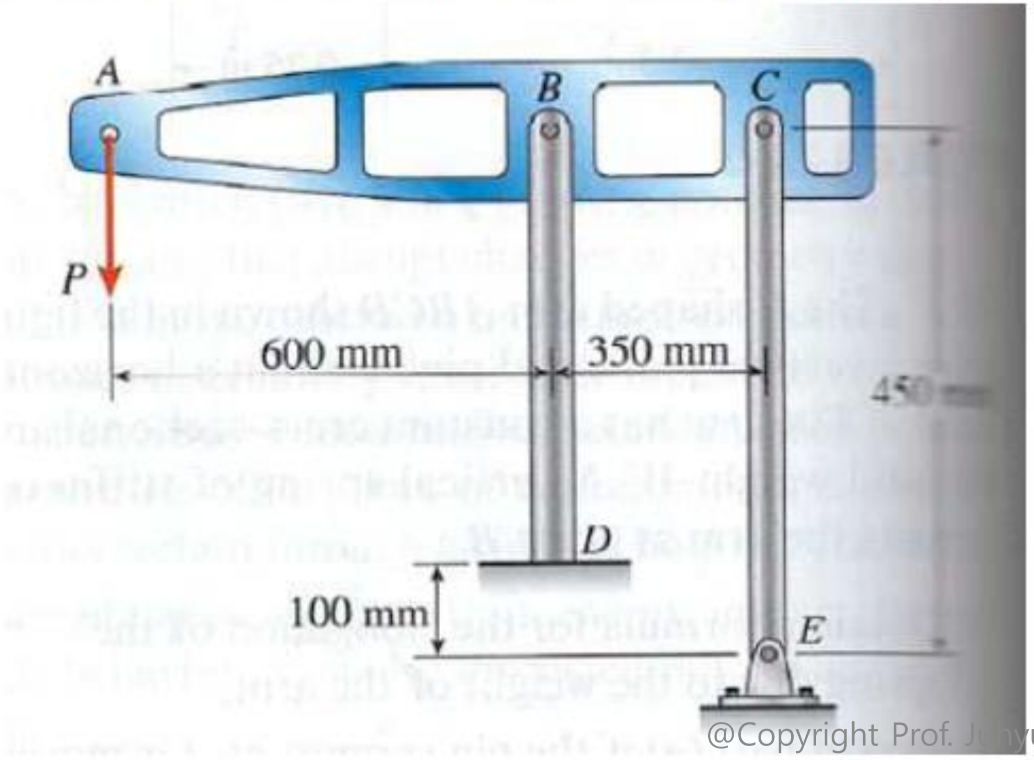


PROBLEM 2.2-2

Homework

2.

A device consists of a horizontal beam A, B, C Supported by two vertical bars BD and CE. Bar CE is pinned at both ends but bars BD is fixed to the foundation at its lower end. The distance from A to B is 600mm and from B to C is 350mm. Bars BD and CE have lengths of 350mm and 450mm, respectively, and their cross-sectional area is 720 mm^2. The bars are made of steel having a modulus of elasticity E=200GPa. If load P is 20kN, calculate the displacement at point A.



Homework

2.3-1 The length of the end segments of the bar (see figure) is 500 mm and the length of the prismatic middle segment is 1250 mm. Also, the diameters at cross sections *A*, *B*, *C*, and *D* are 12, 24, 24, and 12 mm, respectively, and the modulus of elasticity is 120 GPa.

(a) Calculate the elongation of a copper bar of solid circular cross section with tapered ends when it is stretched by axial loads of magnitude 14 kN (see figure).

(b) If the total elongation of the bar cannot exceed 0.635 mm, what are the required diameters at *B* and *C*? Assume that diameters at *A* and *D* remain at 12 mm.

PROBLEM 2.3-1

Chapter 3 Torsion

- Torsional deformation
- Circular bars of linearly elastic materials
- Non-uniform torsion
- Stress-strain in pure torsion
- Relationship between E and G
- ~~Statically indeterminate torsional member~~
- ~~Strain energy in torsion and pure shear~~
- ~~And others~~

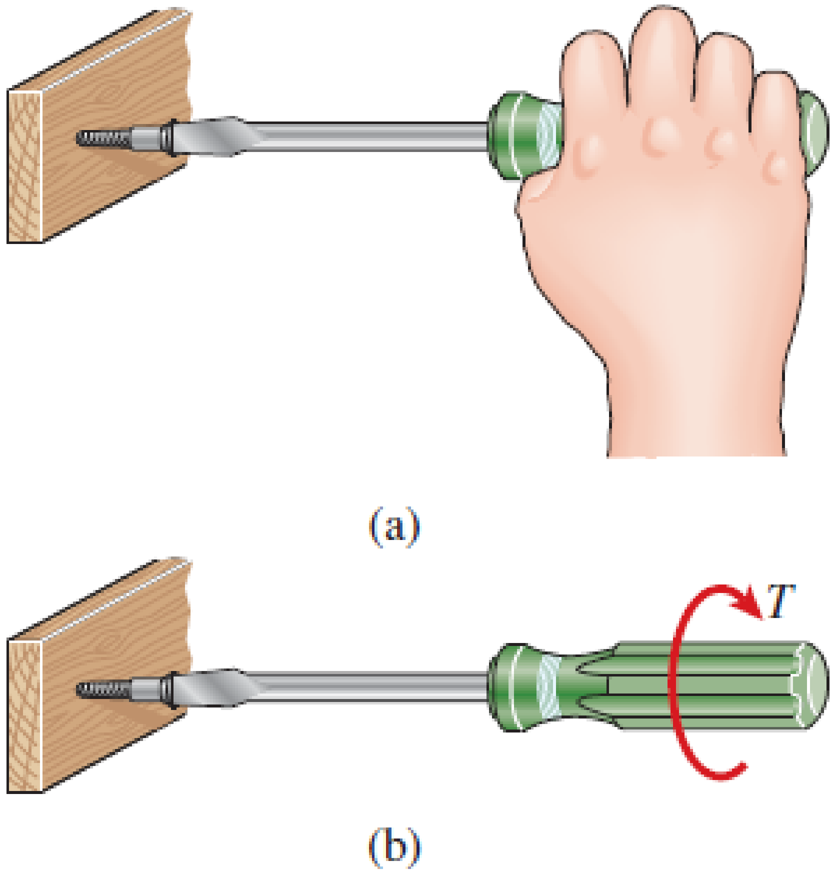


FIG. 3-1 Torsion of a screwdriver due to a torque T applied to the handle

Notation

- Torque (twisting moment)
- A moment of couple
- Notation

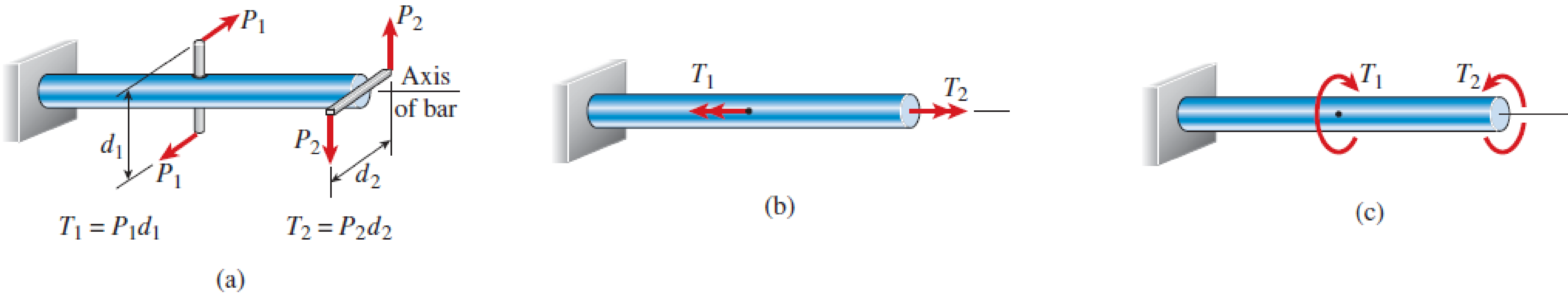
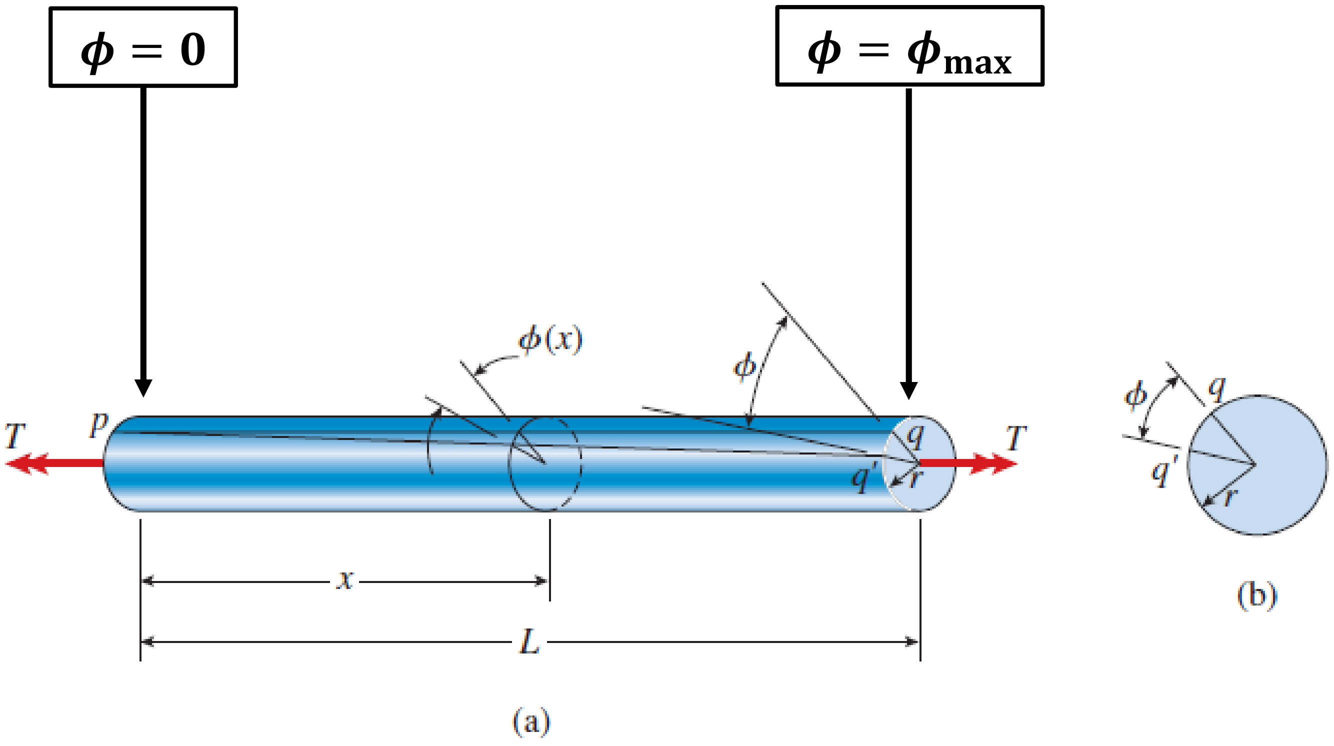
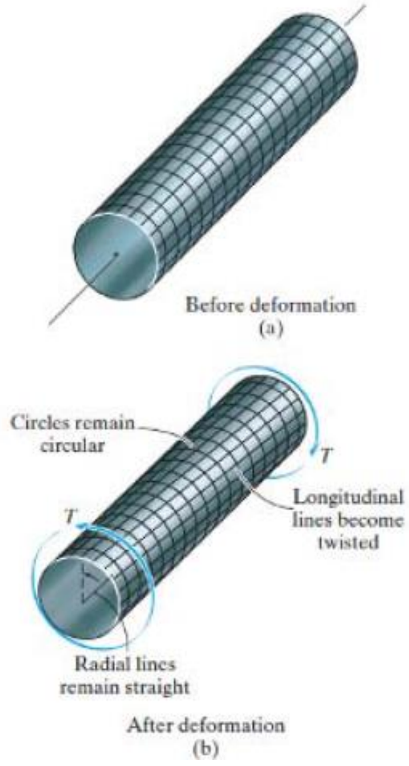


FIG. 3-2 Circular bar subjected to torsion by torques T_1 and T_2

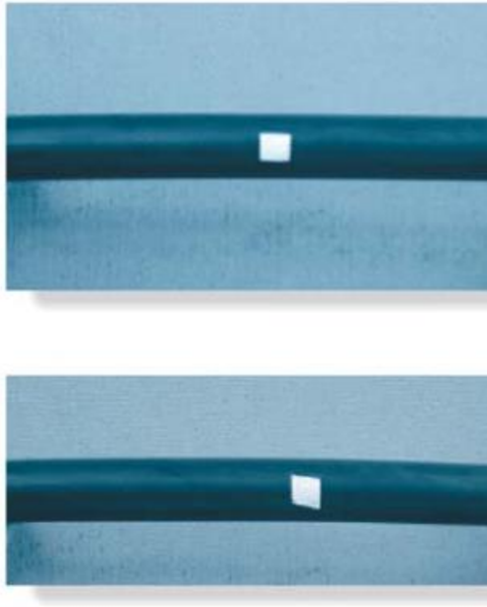
Angle of twist (Angle of rotation)

- The angle of twist changes along the axis of the bar



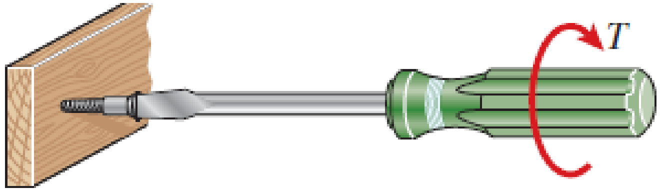


It is a pure shear!



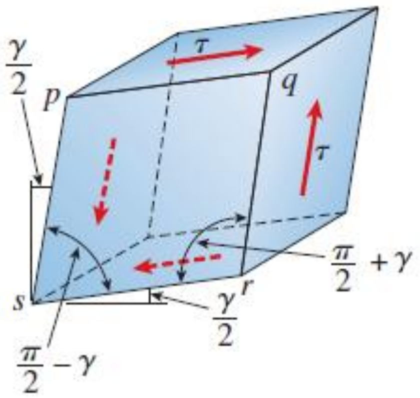
Torsion analysis

Torsion, T
Shear stress, $\boldsymbol{\tau}$



Compatibility

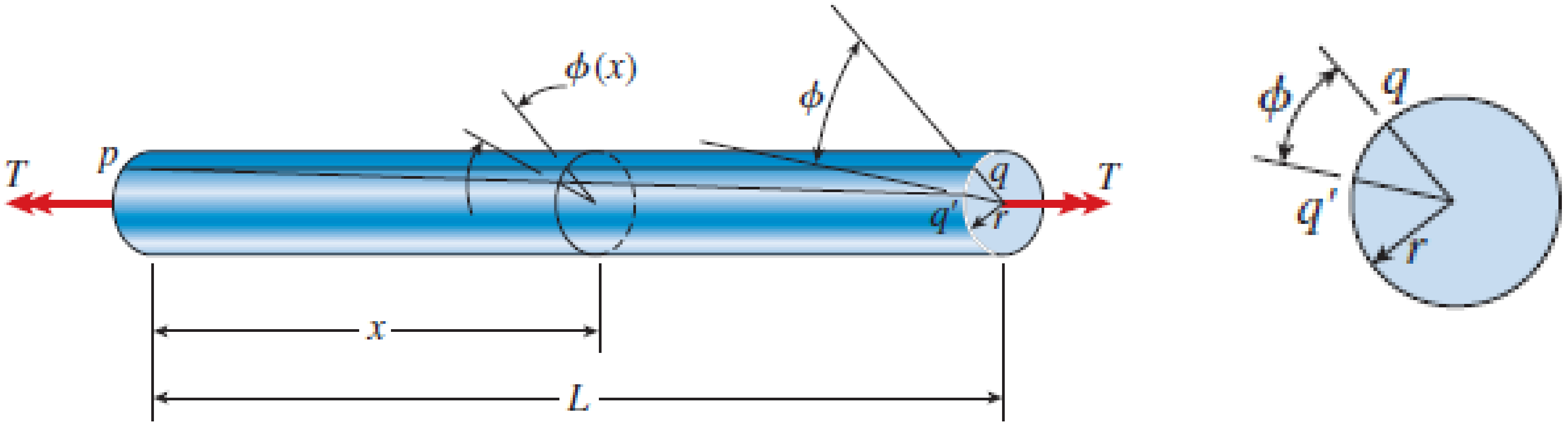
$$\tau = G\gamma$$



Shear strain, γ

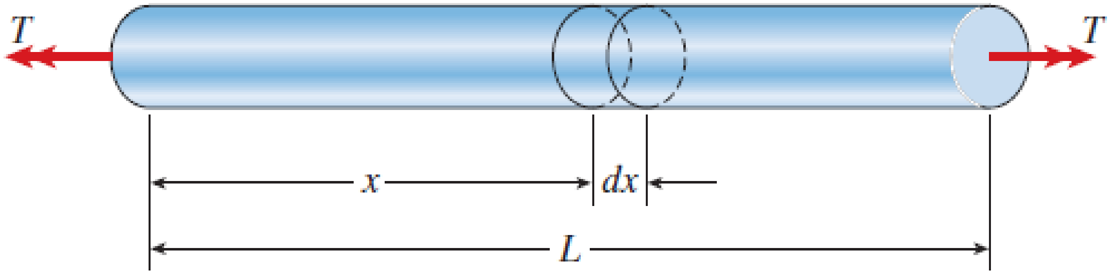
Torsional deformation of a circular bar

- Angle of twist (angle of rotation) $\phi(x)$

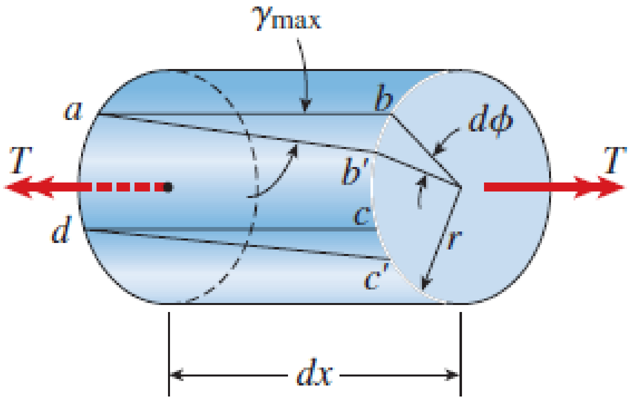


Shear strains at the outer surface

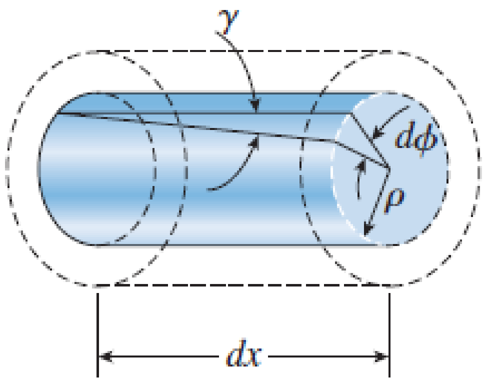
- A small element *abcd* twists to *ab′c′d*



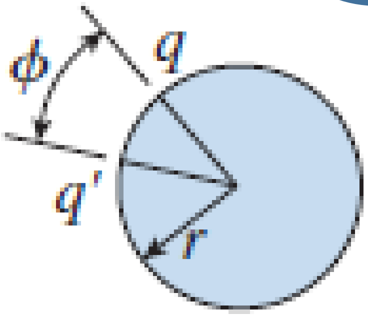
(a)



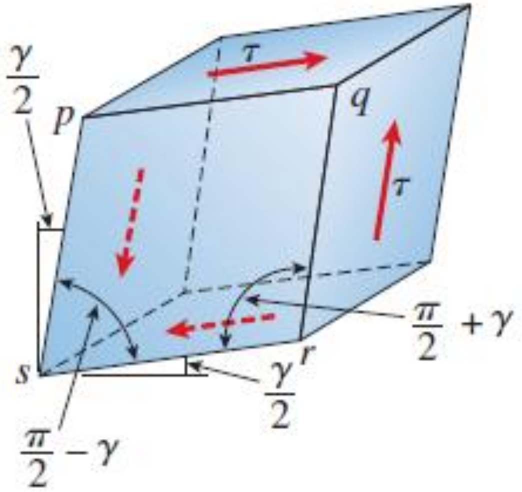
(b)



(c)

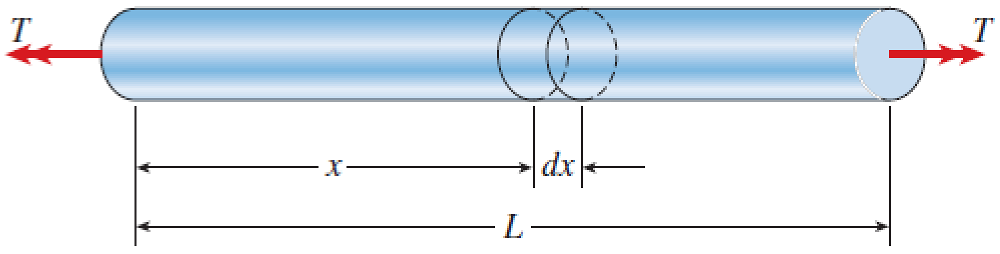


Shear strain under pure shear

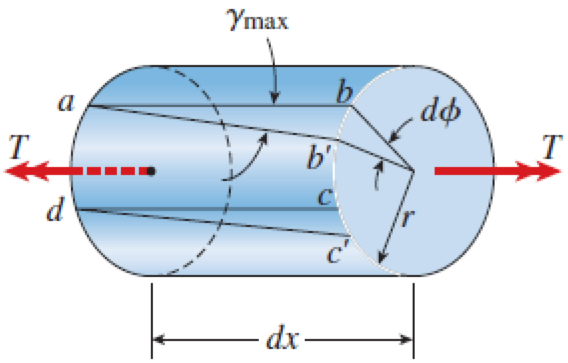


Shear strains at the outer surface

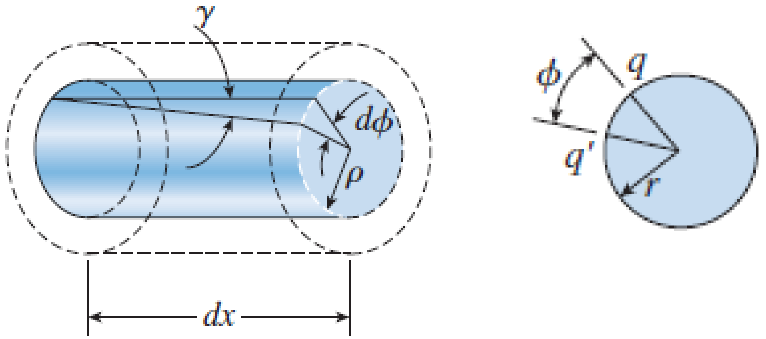
- A small element *abcd* twists to *ab'c'd*



(a)



(b)



(c)

Maximum shear strain on surface

$$\gamma_{\max} = \frac{bb'}{ab}$$

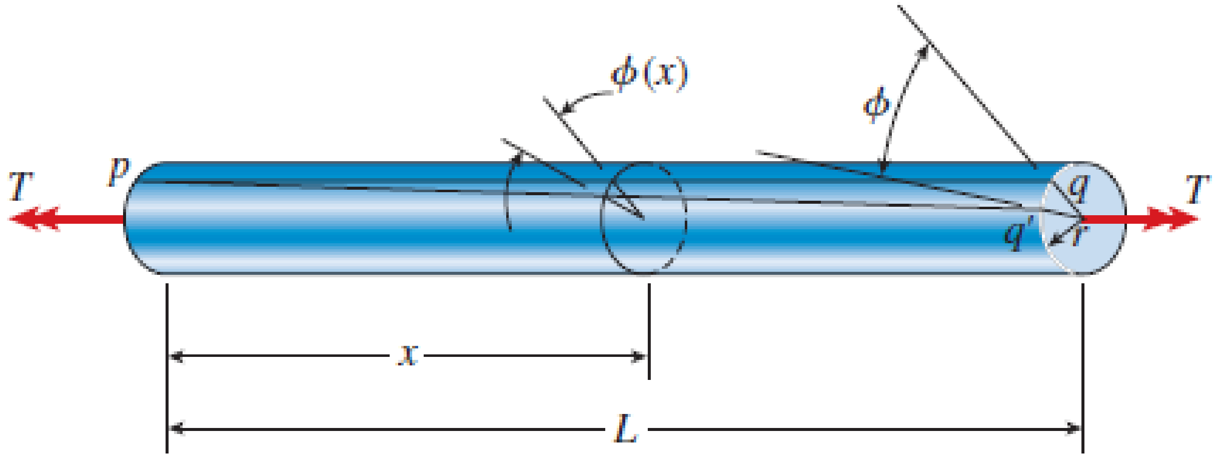
$$\gamma_{\max} = \frac{r\,d\phi}{dx}$$

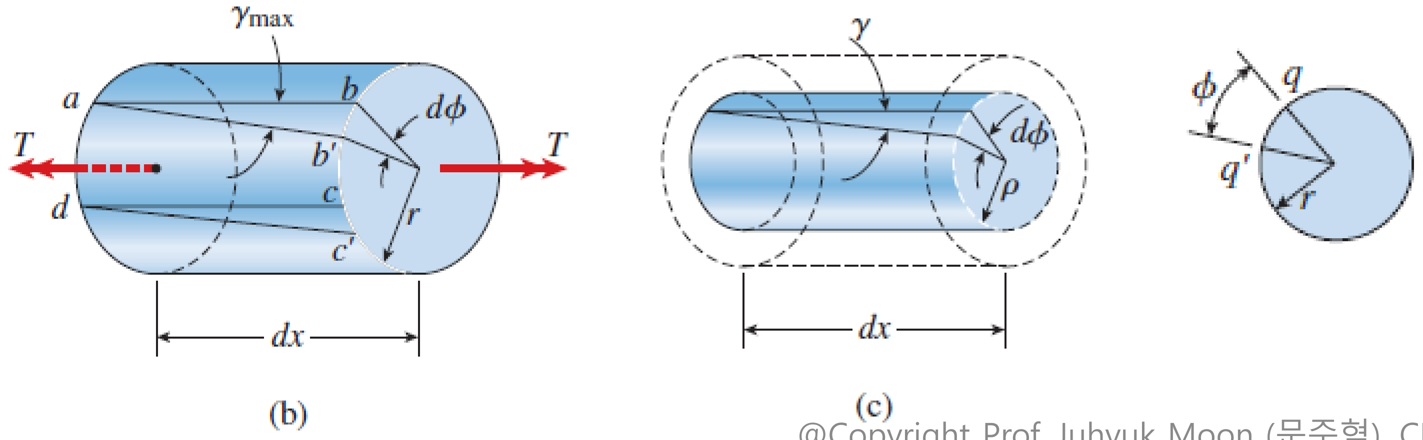
Rate of twist $\theta = \frac{d\phi}{dx}$

=Angle of twist per unit length

Shear strains at the outer surface

- A small element *abcd* twists to *ab′c′d*





Maximum shear strain on surface

$$\gamma_{max} = \frac{bb'}{ab}$$

$$\gamma_{max} = \frac{r d\phi}{dx}$$

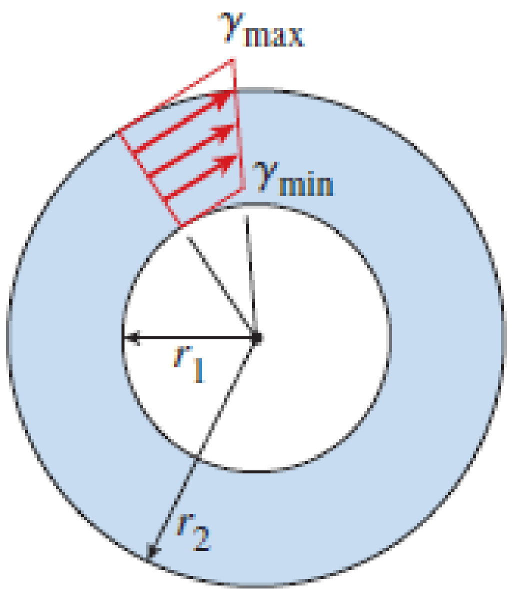
Rate of twist $\theta = \frac{d\phi}{dx}$

$$\gamma_{max} = \frac{r d\phi}{dx} = r\theta$$

$$\gamma_{max} = r\theta = \frac{r\phi}{L}$$

Shear strains within the bar

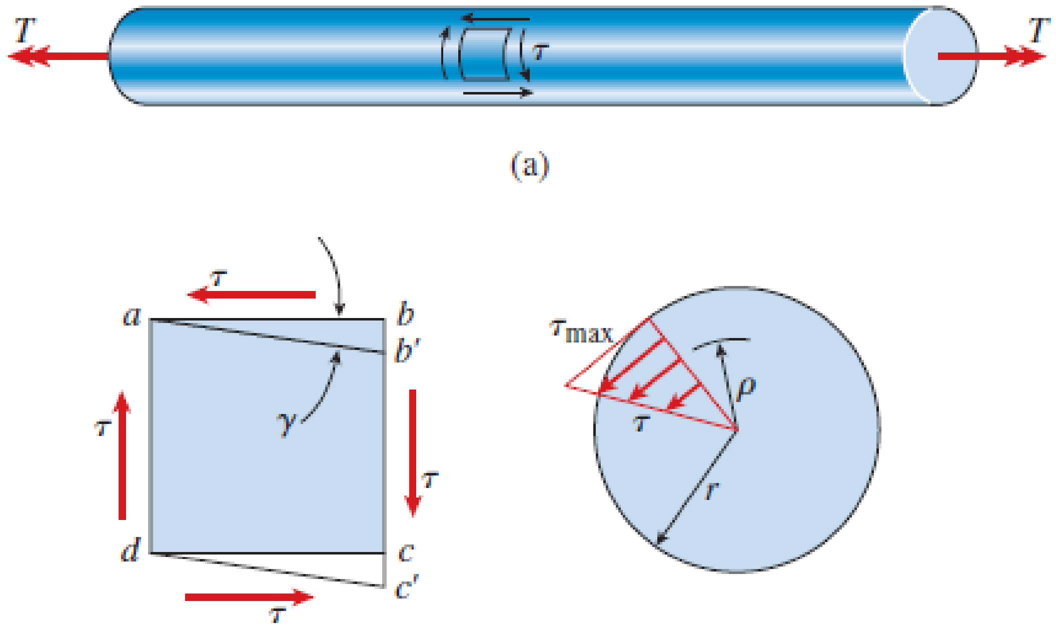
- The shear strain is a function of radius of the bar



$$\gamma_{max} = \frac{r_2 \phi}{L} \qquad \gamma_{min} = \frac{r_1}{r_2}\gamma_{max} = \frac{r_1 \phi}{L}$$

Link with shear stress

- Hooke's law in shear $\tau = G\gamma$



(a) (b) (c)

$$\gamma_{max} = r\theta = \frac{r\phi}{L}$$

$$\tau_{max} = Gr\theta$$

$$\tau = G\rho\theta = \frac{\rho}{r}\tau_{max}$$

The Torsion Formula

- Shear stress with Torsion force

$$dM = \tau\rho dA = \frac{\tau_{\max}}{r}\rho^2 dA$$

$$T = \int_A dM = \frac{\tau_{\max}}{r}\int_A \rho^2 dA = \frac{\tau_{\max}}{r} I_P$$

$$\tau_{\max} = \frac{Tr}{I_P}$$

Torsion formula

$$I_P = \int_A \rho^2 dA$$

Polar moment of inertia

$$I_P = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

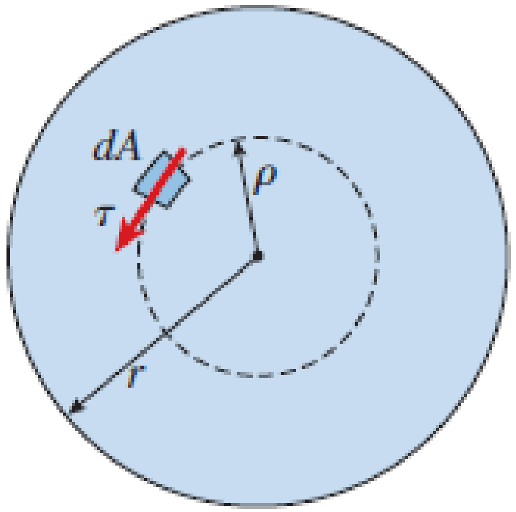


FIG. 3-9 Determination of the resultant of the shear stresses acting on a cross section

The Torsion Formula

- Generalized torsion formula

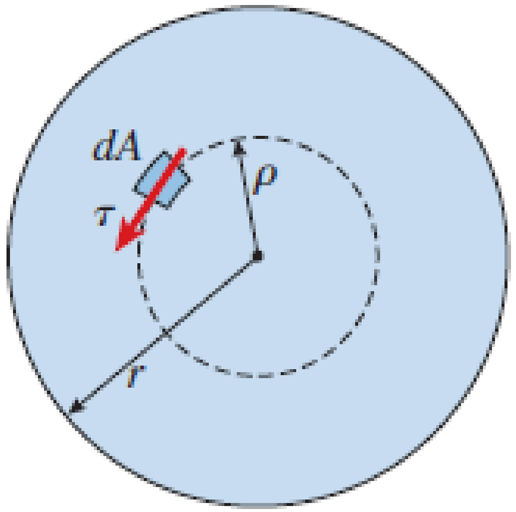


FIG. 3-9 Determination of the resultant of the shear stresses acting on a cross section

$$dM = \tau\rho dA = \frac{\tau_{max}}{r}\rho^2 dA$$

$$T = \int_A dM = \frac{\tau_{max}}{r}\int_A \rho^2 dA = \frac{\tau_{max}}{r} I_P$$

$$\tau_{max} = \frac{Tr}{I_P}$$

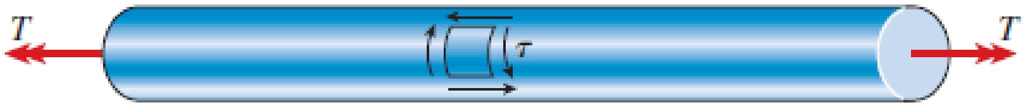
Torsion formula

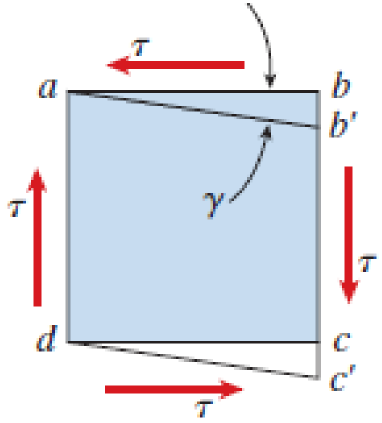
$$\tau = \frac{\rho}{r}\tau_{max} = \frac{T\rho}{I_P}$$

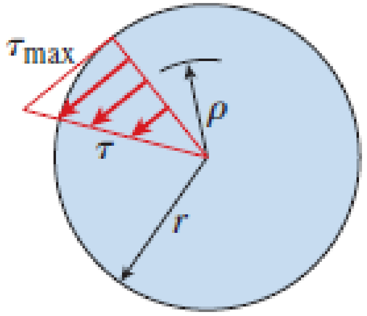
Generalized torsion formula

The Torsion Formula

- Angle of twist







$$\tau_{max} = \frac{Tr}{I_P}$$

Torsion formula

$$\tau_{max} = Gr\theta$$

$$\tau = G\rho\theta = \frac{\rho}{r}\tau_{max}$$

$$\theta = \frac{T}{GI_P}$$

Rate of twist
(Angle of twist per unit length)

$$\tau = \frac{\rho}{r}\tau_{max} = \frac{T\rho}{I_P}$$

Generalized torsion formula

$$\phi = \frac{TL}{GI_P}$$

Angle of twist

The Torsion Formula

- Shear stress with Torsion force

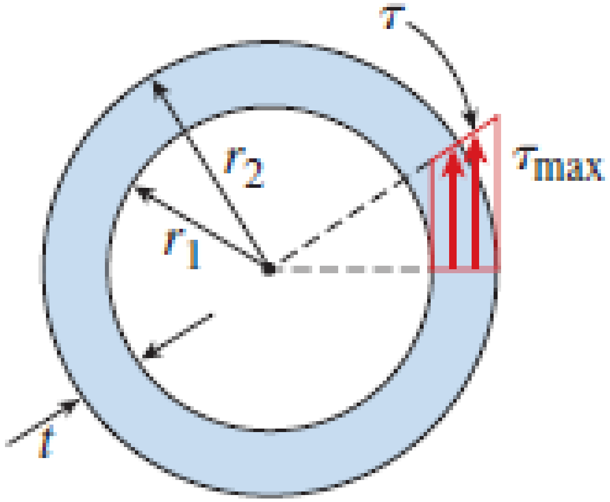


FIG. 3-10 Circular tube in torsion

$$I_P = \int_A \rho^2 dA$$

Polar moment of inertia

$$I_P = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

$$I_P = \frac{\pi}{2}(r_2^4 - r_1^4) = \frac{\pi}{32}(d_2^4 - d_1^4)$$

The Non-uniform Torsion

- Net torsion calculation

$$\phi = \sum_{i=1}^{n} \phi_i = \sum_{i=1}^{n} \frac{T_i L_i}{G_i (I_P)_i}$$

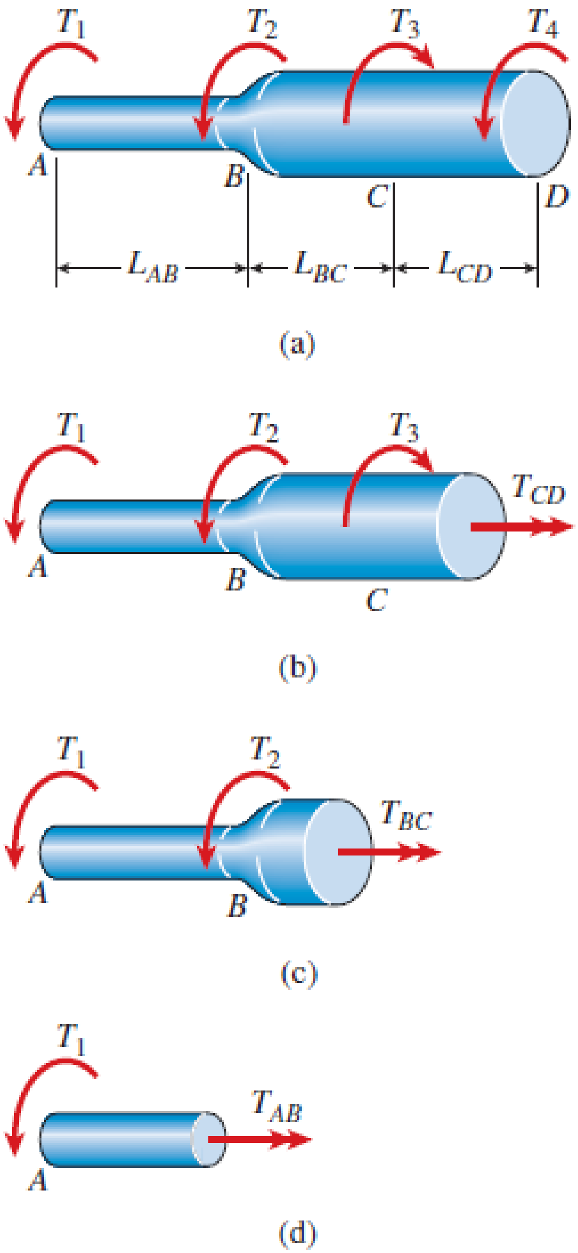
$$\phi = \int_0^L d\phi = \int_0^L \frac{T(x)\,dx}{GI_P(x)}$$

$$d\phi = \frac{T\,dx}{GI_P(x)}$$

$$T_{CD} = -T_1 - T_2 + T_3$$

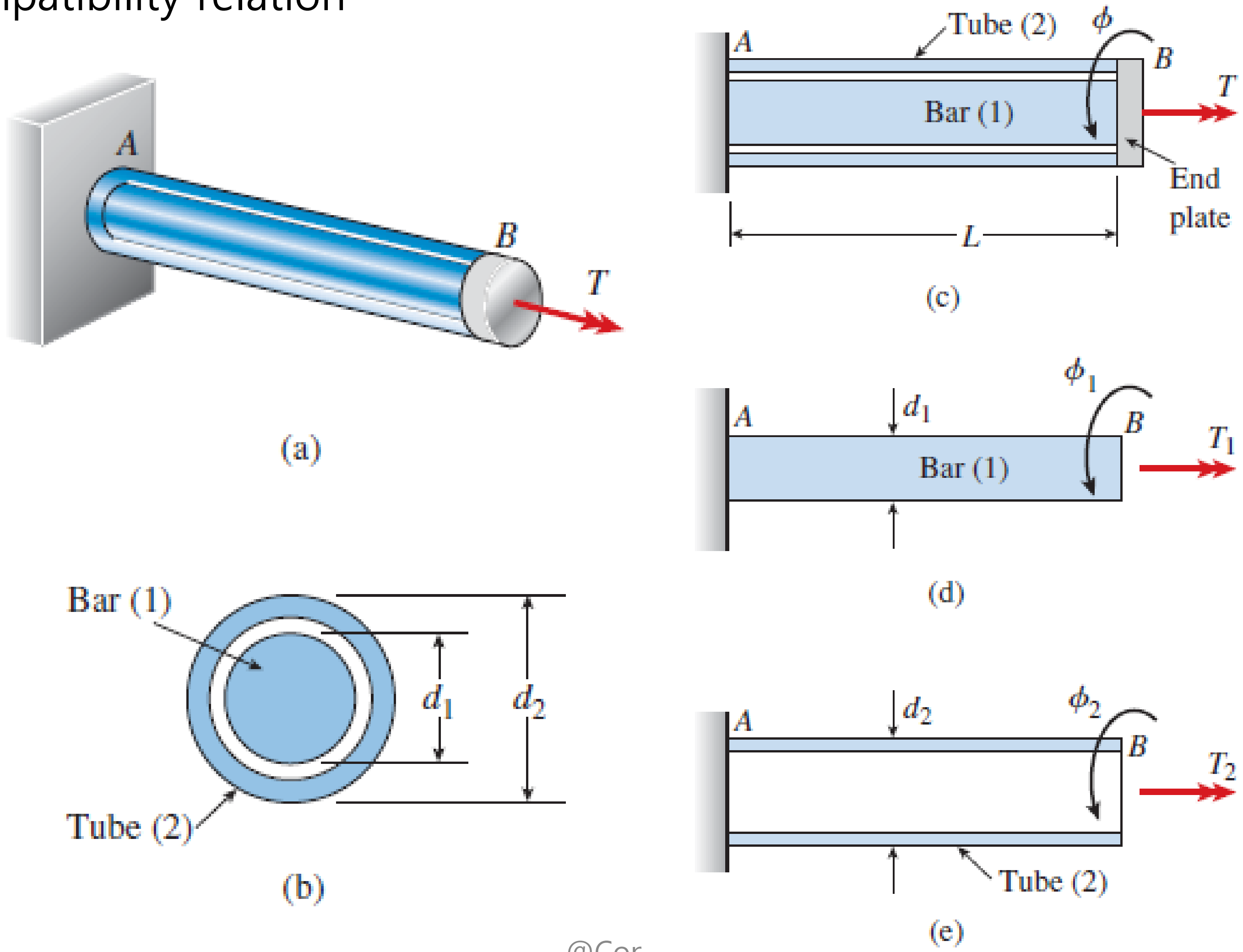
$$T_{BC} = -T_1 - T_2$$

$$T_{AB} = -T_1$$



Statically indeterminate torsional member

- Compatibility relation



Two unknowns T_1 T_2

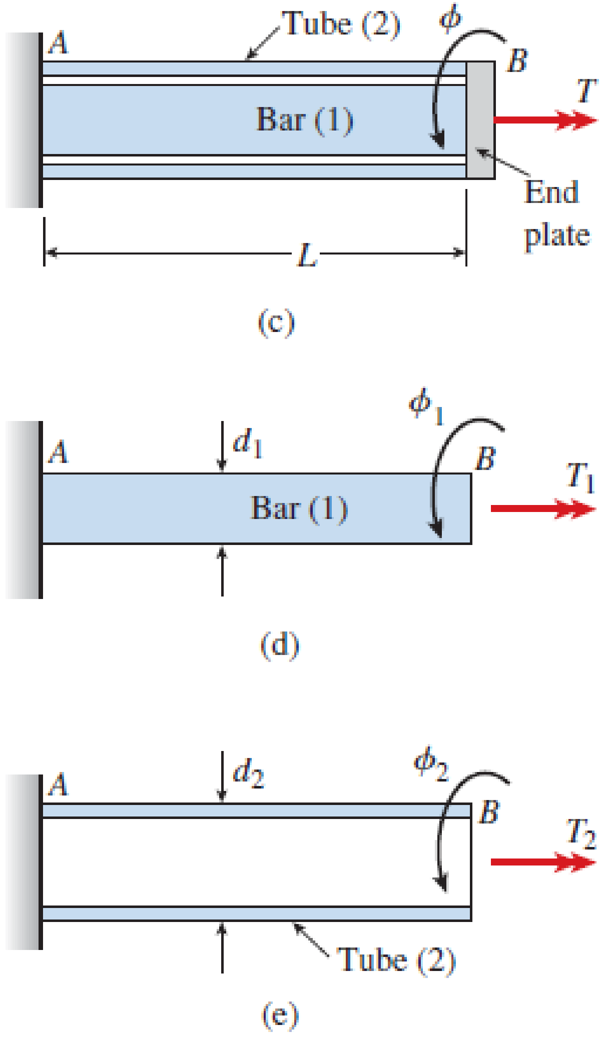
Equation of equilibrium

$$T_1 + T_2 = T$$

Equation of compatibility

$$\phi_1 = \phi_2$$

Statically indeterminate torsional member



Equation of compatibility

$$\phi_1 = \phi_2$$

$$\phi_1 = \frac{T_1 L}{G_1 I_{P1}} \qquad \phi_2 = \frac{T_2 L}{G_2 I_{P2}}$$

$$\frac{T_1 L}{G_1 I_{P1}} = \frac{T_2 L}{G_2 I_{P2}}$$

Equation of equilibrium

$$T_1 + T_2 = T$$

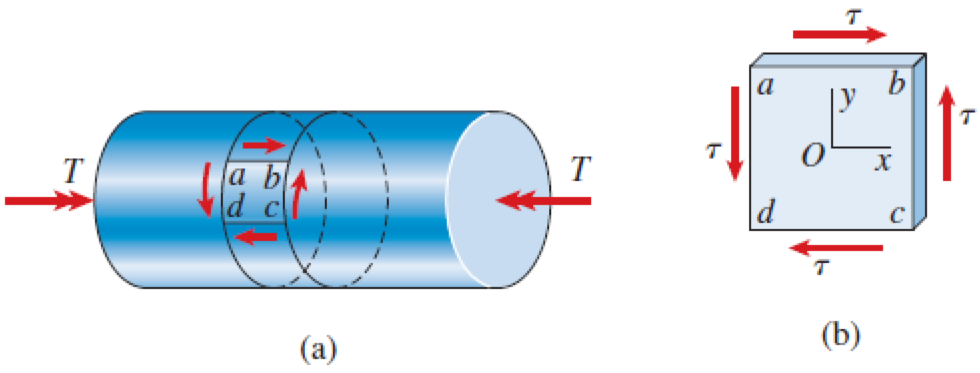
Solved!

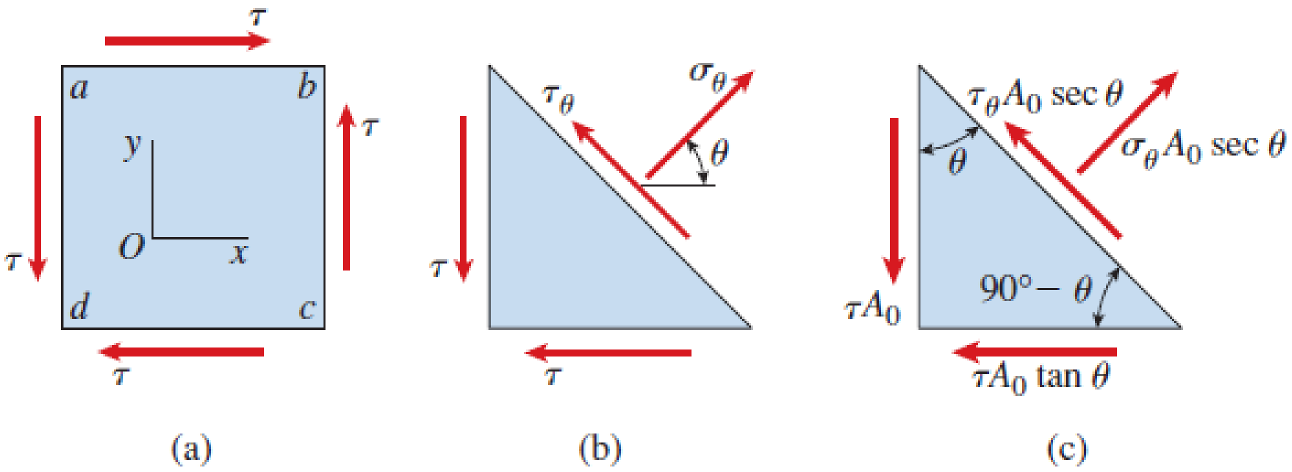
$$T_1 = T\left(\frac{G_1 I_{P1}}{G_1 I_{P1} + G_2 I_{P2}}\right)$$

$$T_2 = T\left(\frac{G_2 I_{P2}}{G_1 I_{P1} + G_2 I_{P2}}\right)$$

Stress and strain in pure shear

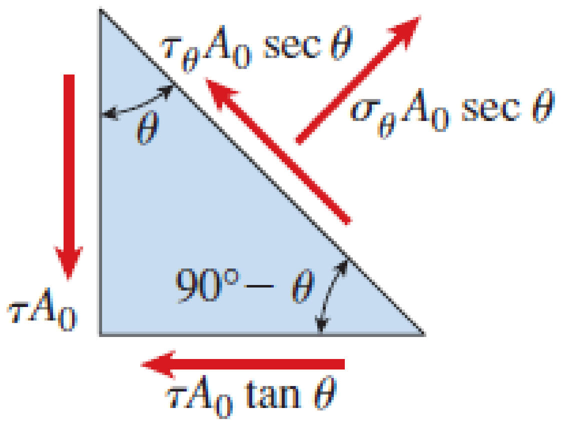
- Stresses on inclined planes





Stress and strain in pure shear

- Stresses on inclined planes



summing forces in the direction of σ_θ:

$$\sigma_\theta A_0 \sec\theta = \tau A_0 \sin\theta + \tau A_0 \tan\theta \cos\theta$$

$$\sigma_\theta = 2\tau \sin\theta \cos\theta$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\sigma_\theta = \tau \sin 2\theta$$

summing forces in the direction of τ_θ:

$$\tau_\theta A_0 \sec\theta = \tau A_0 \cos\theta - \tau A_0 \tan\theta \sin\theta$$

$$\tau_\theta = \tau(\cos^2\theta - \sin^2\theta)$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

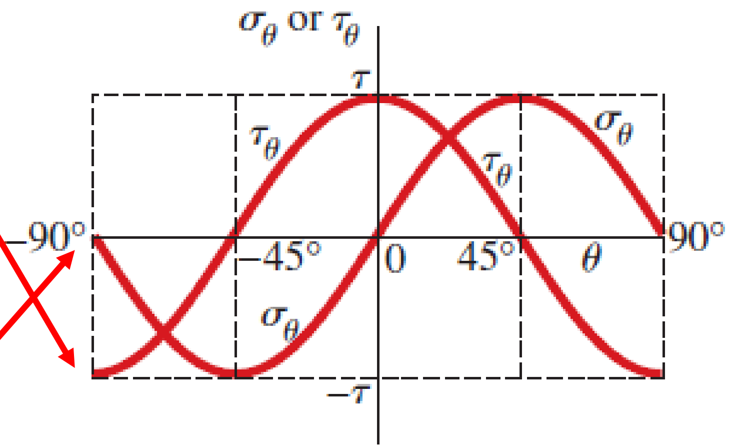
$$\tau_\theta = \tau \cos 2\theta$$

Stress and strain in pure shear

- Strains in pure shear

$$\sigma_\theta = \tau \sin 2\theta$$

$$\tau_\theta = \tau \cos 2\theta$$



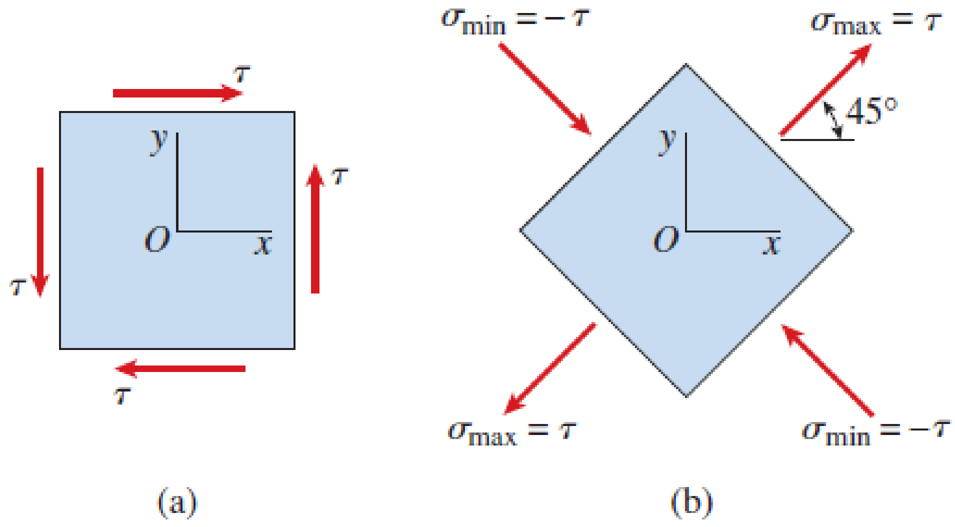


FIG. 3-23 Stress elements oriented at $\theta = 0$ and $\theta = 45°$ for pure shear

Relationship btw E and G

- Stresses on inclined planes

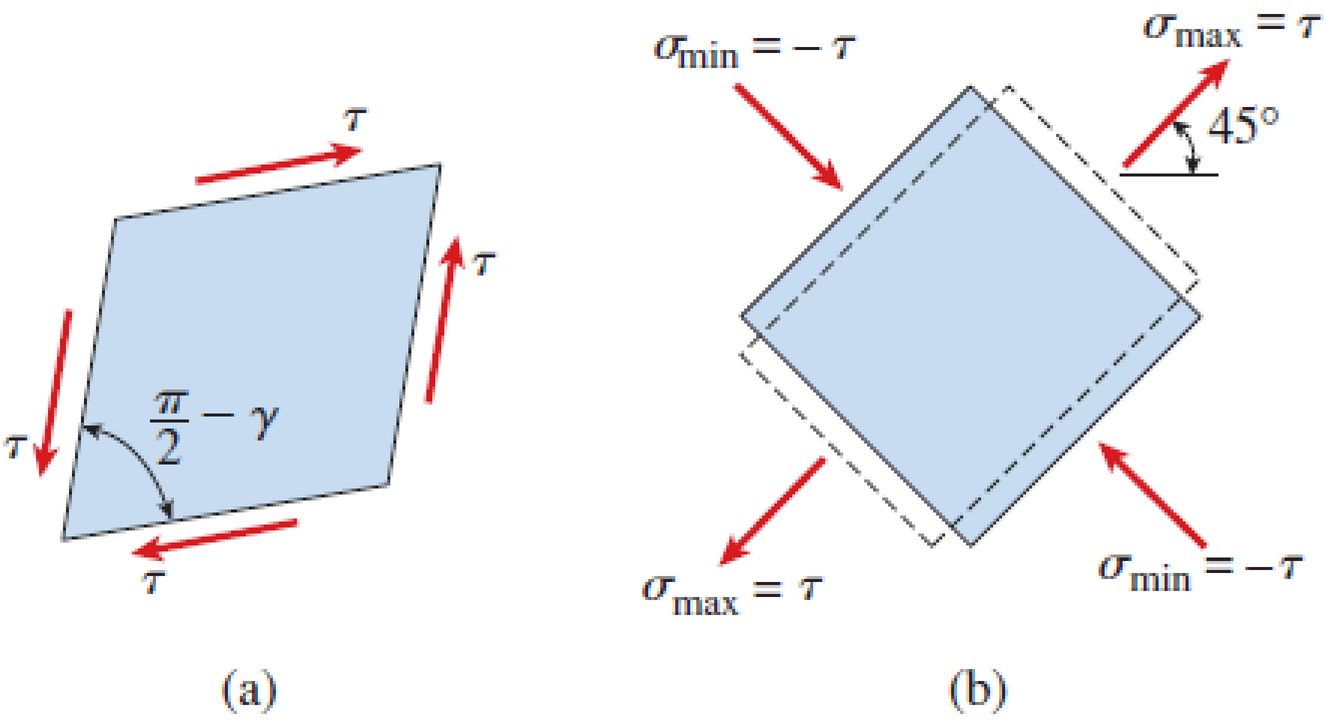
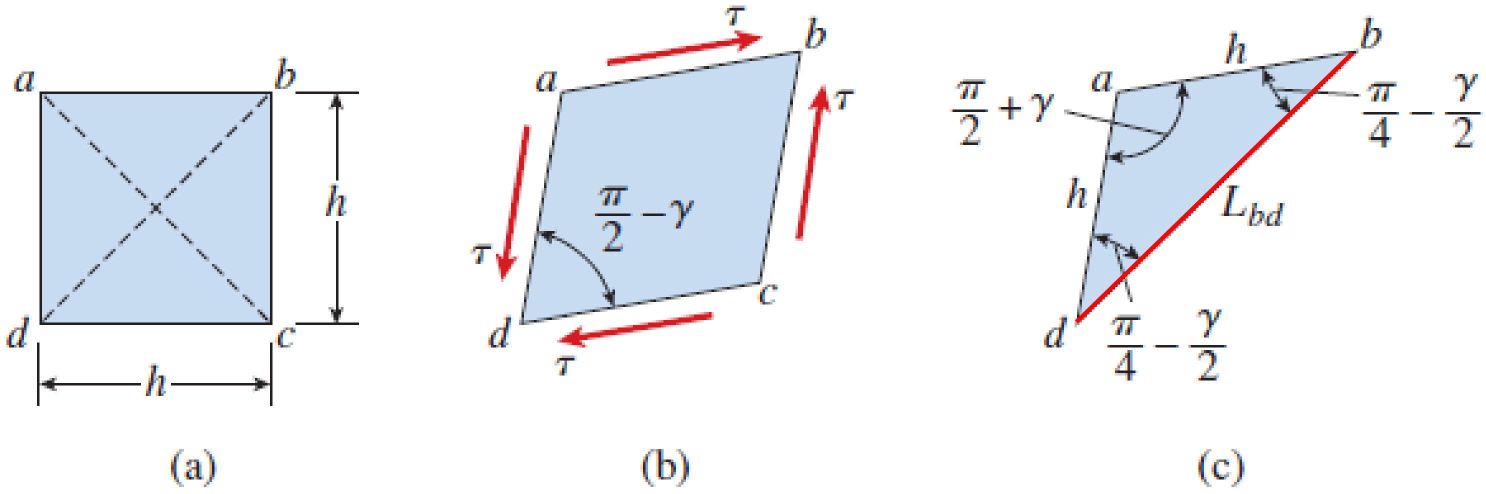


FIG. 3-25 Strains in pure shear: (a) shear distortion of an element oriented at $\theta = 0$, and (b) distortion of an element oriented at $\theta = 45°$

$$\epsilon_{max} = \frac{\tau}{E} + \frac{\nu\tau}{E} = \frac{\tau}{E}(1 + \nu)$$

Relationship btw E and G



$$L_{bd} = \sqrt{2}\, h(1 + \epsilon_{max})$$

=

$$L_{bd}^2 = h^2 + h^2 - 2h^2 \cos\left(\frac{\pi}{2} + \gamma\right)$$

FIG. 3-28 Geometry of deformed element in pure shear

Relationship btw E and G

$$L_{bd} = \sqrt{2}\, h(1 + \epsilon_{max})$$ **(=)** $$L_{bd}^2 = h^2 + h^2 - 2h^2 \cos\left(\frac{\pi}{2} + \gamma\right)$$

$$(1 + \epsilon_{max})^2 = 1 - \cos\left(\frac{\pi}{2} + \gamma\right)$$

$$\cos(\pi/2 + \gamma) = -\sin \gamma,$$

$$1 + 2\epsilon_{max} + \epsilon^2_{max} = 1 + \sin \gamma$$

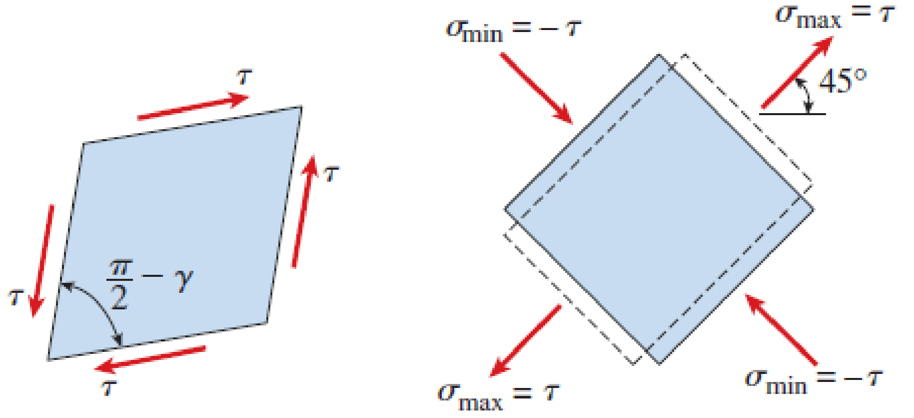
for very small strains

$$\varepsilon_{max}^2 \approx 0$$

$$sin\gamma \approx \gamma$$

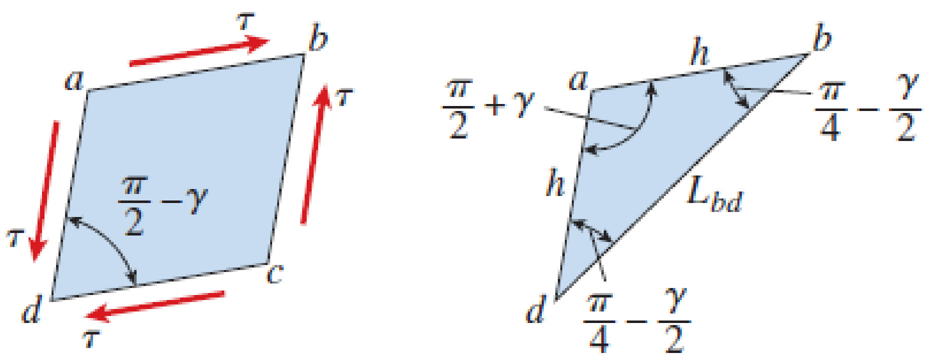
$$\epsilon_{max} = \frac{\gamma}{2}$$

Relationship btw E and G



(a) (b)

$$\epsilon_{max} = \frac{\tau}{E} + \frac{\nu\tau}{E} = \frac{\tau}{E}(1 + \nu)$$



(b) (c)

$$\epsilon_{max} = \frac{\gamma}{2}$$

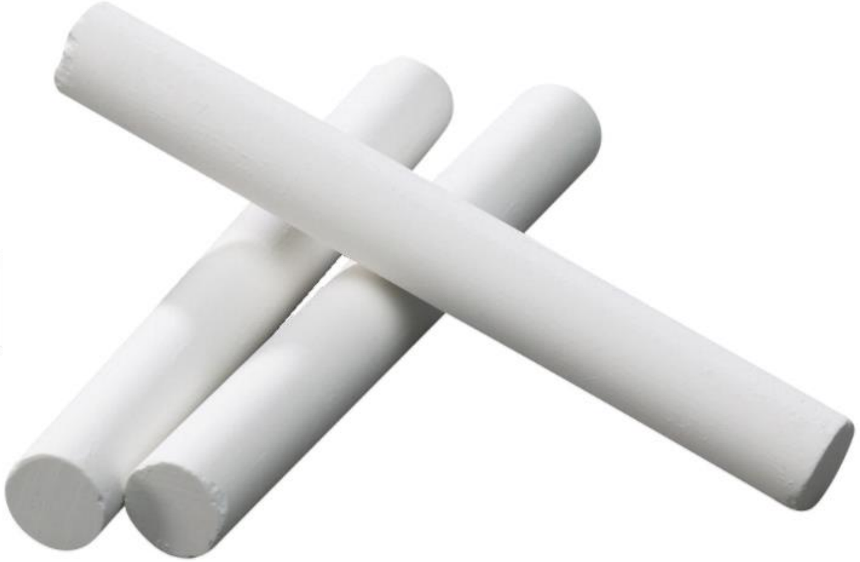
$$\gamma = \frac{\tau}{G}$$

$$G = \frac{E}{2(1 + \nu)}$$

Chalk failure pattern prediction

A chalk has below material properties

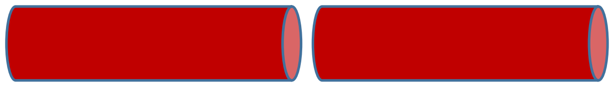
Allowable normal stress (compression) = **8 MPa**
Allowable normal stress (tension) = **2 MPa**
Allowable shear stress = **0.5 MPa**



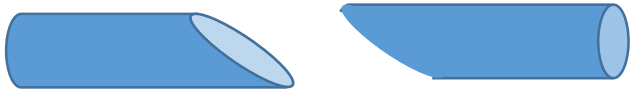
A chalk has below material properties

Allowable normal stress (compression) = **8 MPa**
Allowable normal stress (tension) = **2 MPa**
Allowable shear stress = **0.5 MPa**

Vertical plane failure



Sharply inclined plane failure with 45 degree



Curved inclined plane with 45 degree crack line