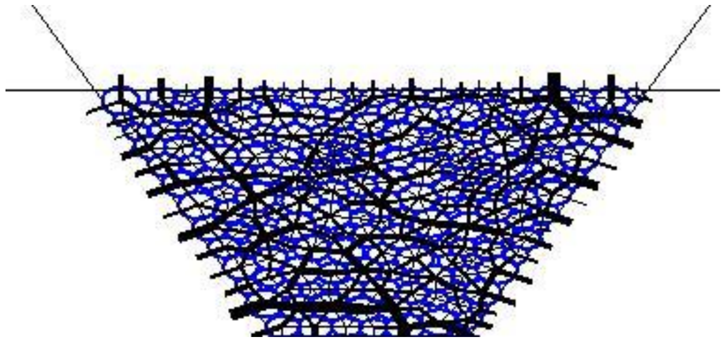
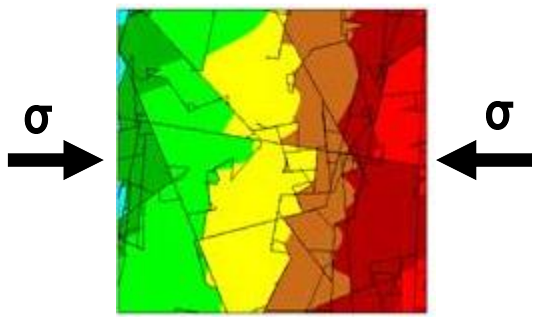
Numerical Methods in Rock Engineering
- Introduction to Discrete Element Method

(Week 8, 9, 10, 11) (26 April, 3, 10, 17 May 2021)





Ki-Bok Min, PhD

Professor
Department of Energy Resources Engineering
Seoul National University

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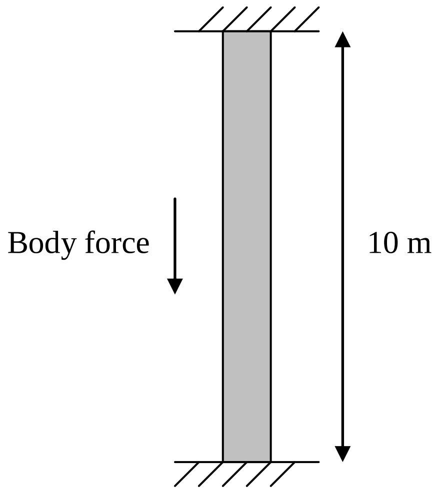
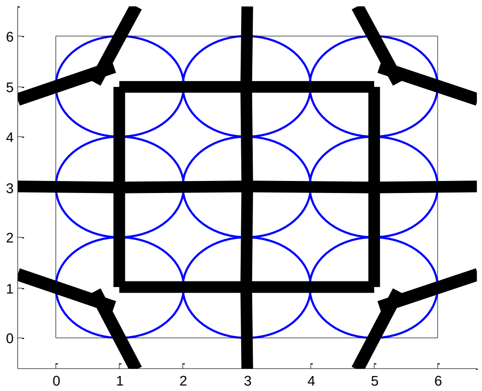
Introduction
Home Assignments (40%)

- #1 1 page summary of selected papers on numerical analysis
- #2 1D(or 2D) coding of FEM/FDM (use excel, matlab, or other codes)
- #3 Exercise with FEM code (comsol multiphysics)
- #4 Paper reading (DEM) – classical paper of your choice
- #5 Exercise with UDEC/PFC





Introduction
Term Project (20%)

- Select a subsurface engineering problem of your interest and conduct a numerical analysis using any available codes.
- Term paper must include;
  - Clear objectives
  - One or two verification cases
  - Thorough formulation of the chosen numerical method
  - Concise presentation and discussion on the results
- Timeline
  - 31 May — Proposal (1 page) & 10 minutes presentation
  - 7 June — Consultation with instructor
  - 14 June — Presentation and submission of Term Paper

- A list of example topics
  - Reproduction of published landmark papers
  - Borehole Stability problem in Anisotropic Media (FEM or FDM)
  - Fracture propagation in petroleum/geothermal reservoir (BEM or DEM)
  - Calibration of micromechanical parameters for transversely isotropic rock rock (DEM)
  - Coupled (thermo) hydromechanical analysis in porous medium
  - CO2 injection in saline formation
  - Thermomechanical analysis for geological repository of nuclear waste
  - Slope Stability in fractured or continuum rock
  - Reinforcement of tunnel
  - Determination of equivalent properties of fractured rock mass (DEM)

Term Project

- Presentation
  - Presentation is an extremely important part of your professional life. Therefore, you have a good reason to be serious about this.
  - 10 minutes + 5 min (questions)

- Your term papers will be published as proceedings.
- Your term papers may be developed into journal papers in the future.

Proceedings of

2011 SNU Student Conference

- Numerical Analysis in Rock Engineering -

Editor : Ki-Bok Min

Department of Energy Resources Engineering

Seoul National University

Outline

- Introduction
- Solution techniques - Governing Equations
- Implementation issues – Numerical Stability, Damping, Contact Detection,
- Explicit DEM
  - Unbonded particulate system
  - Bonded particulate system
  - Blocky system
- Hydraulic processes – fluid flow in fractured rock
- Examples

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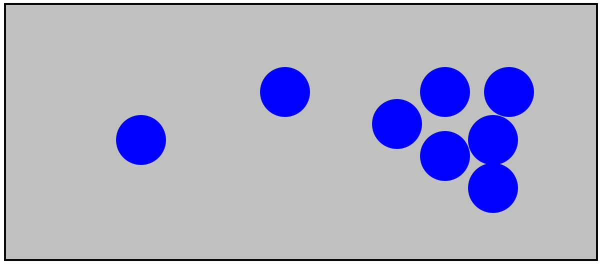
- Up to this point, we were able to model various physical problems using FEM and FDM. Essentially we solved a single or a coupled set of partial differential equation in *continuum*.

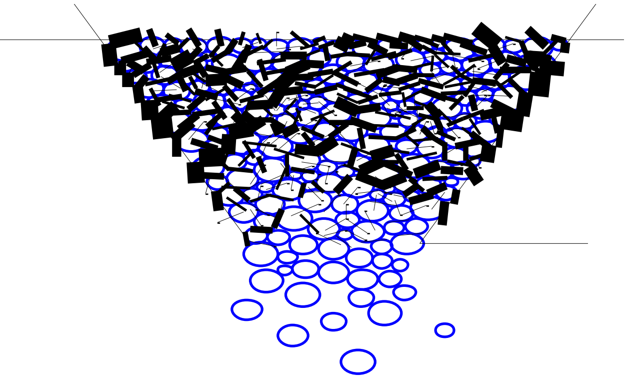
- Can we solve following problem using FEM or FDM? – a system of particles interacting each other.



Explicit Discrete Element Method

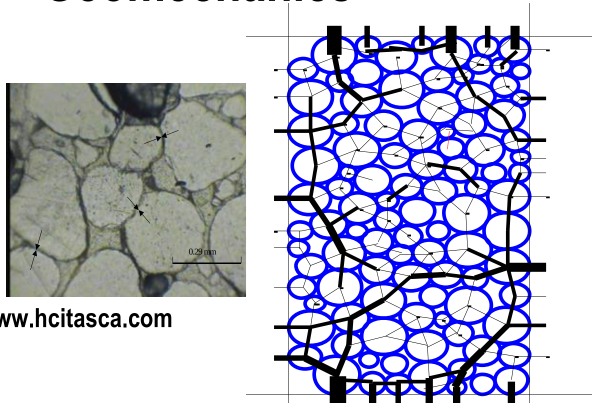
Introduction – particulate system

Powder technology,
chemical engineering



Powder or gravels in a hopper. Thickness of lines indicate forces.

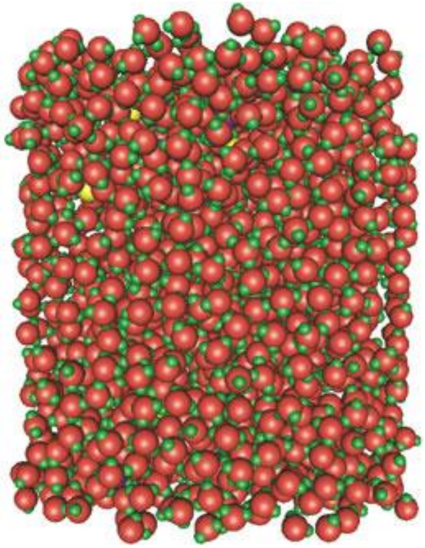
Geomechanics



www.hcitasca.com

Grains composing rock/soil

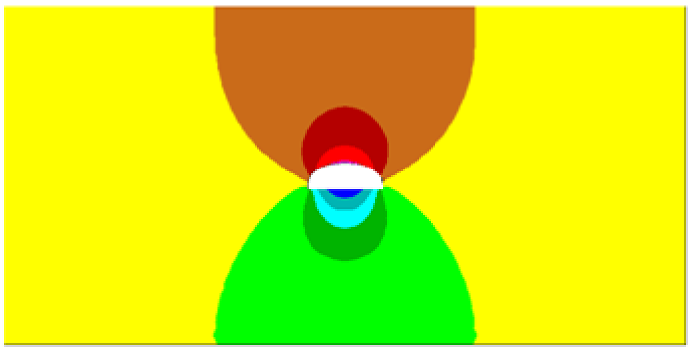
Molecular dynamics



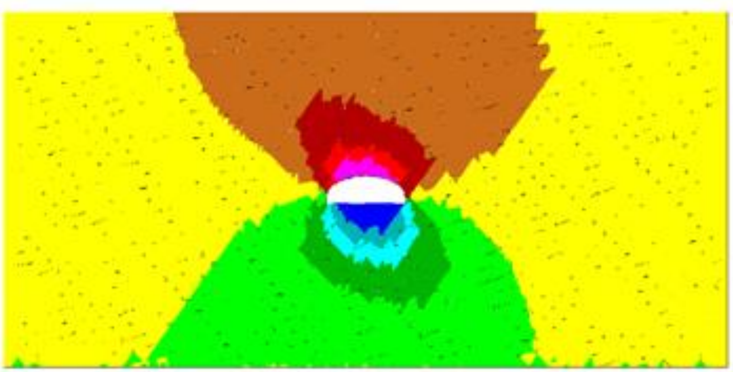
And, of course, billiards…

Sodium chloride aqueous solution, each particle represent sodium, hydrogen, oxygen and chloride.
http://www.ornl.gov/info/ornlreview/v34_2_01/p23b.jpg

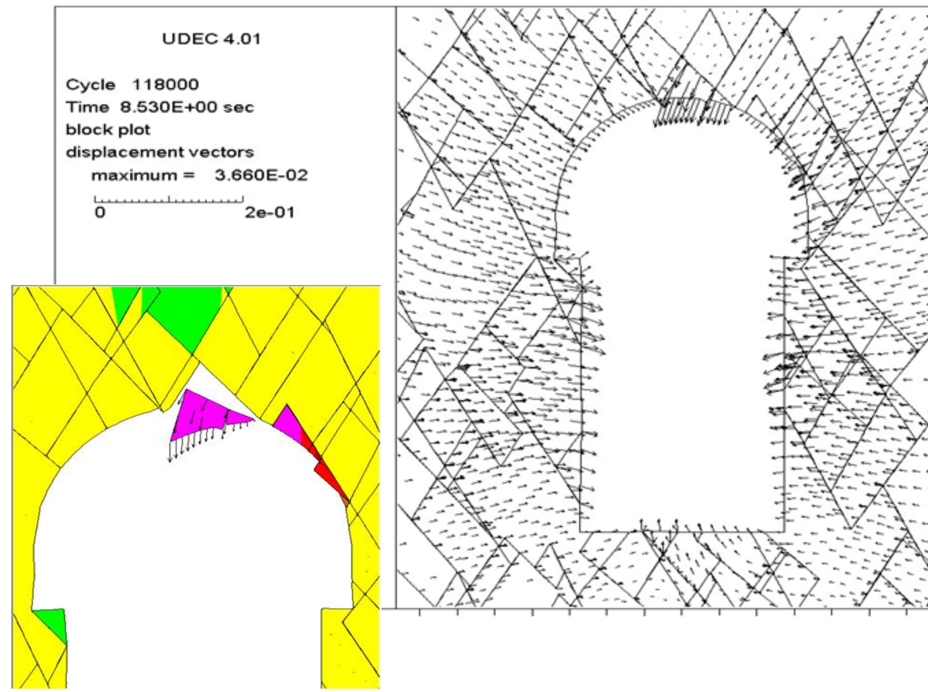
Explicit Discrete Element Method
Introduction – blocky system



Continuum



Discontinuum



Rock Fall Analysis

Discrete Element Method

- The name 'discrete element method' can be used only if it
  - Allows finite displacements and rotations of discrete bodies; including complete detachment
  - Recognizes new interactions (contact) automatically as calculation progresses
- A DEM code will embody an efficient algorithm for detecting and classifying contacts. It will maintain a data structure and memory allocation scheme that an handle many discontinuities or contacts.
- Two types of DEM
  - Explicit DEM (often called distinct element method): use explicit FDM for solution. Ex) PFC, UDEC, FLOBALL
  - Implicit DEM: similar to FEM solution technique. Matrix is formed. Ex) DDA

Explicit Discrete Element Method
Introduction

- Explicit Discrete Element Method (DEM) solves a motion of interacting particles or blocks (in bonded or unbonded form).
  - DEM recognize new contacts within internal algorithm.
  - Applications - rock mechanics, powder mechanics, granular materials.
  - Finite Difference Method is used to integrate the equation – we replace the time derivative with difference equation in finite time interval.

- DEM is very similar to Molecular Dynamics (MD) which has applications in biophysics, material science and biochemistry - atoms and molecule are used for particles. MD simulate material properties, for examples.
- This lecture is focused on DEM, however, large portions of DEM principle is directly applicable for MD.

Explicit Discrete Element Method
Introduction – a seminal paper

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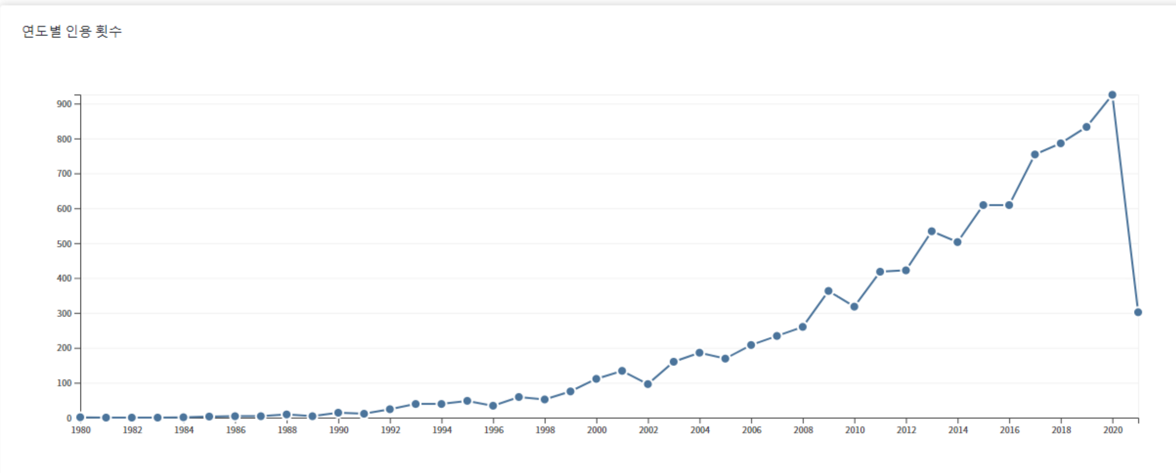
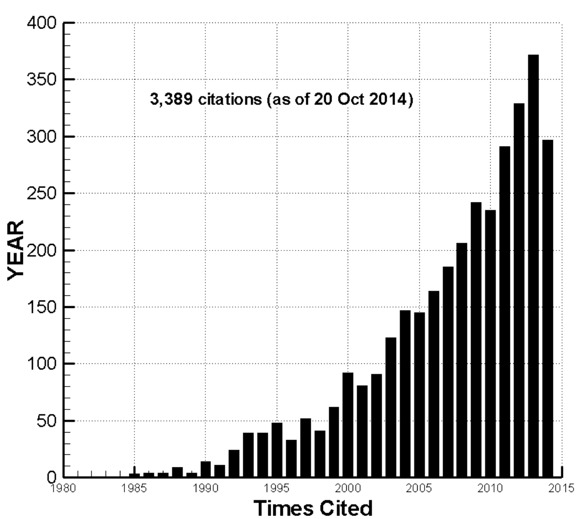
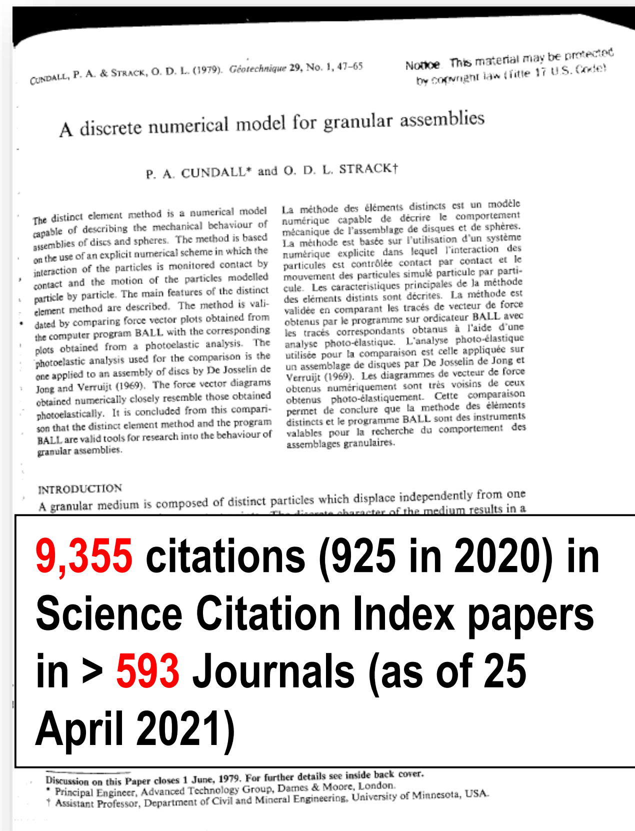
Cundall, P.A. and O.D.L. Strack, Discrete Numerical Model for Granular Assemblies. Geotechnique. 1979. 29(1): p. 47-65







CUNDALL, P. A. & STRACK, O. D. L. (1979). *Géotechnique* 29, No. 1, 47–65

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A discrete numerical model for granular assemblies

P. A. CUNDALL* and O. D. L. STRACK†

The distinct element method is a numerical model capable of describing the mechanical behaviour of assemblies of discs and spheres. The method is based on the use of an explicit numerical scheme in which the interaction of the particles is monitored contact by contact and the motion of the particles modelled particle by particle. The main features of the distinct element method are described. The method is validated by comparing force vector plots obtained from the computer program BALL with the corresponding plots obtained from a photoelastic analysis. The photoelastic analysis used for the comparison is the one applied to an assembly of discs by De Josselin de Jong and Verruijt (1969). The force vector diagrams obtained numerically closely resemble those obtained photoelastically. It is concluded from this comparison that the distinct element method and the program BALL are valid tools for research into the behaviour of granular assemblies.

La méthode des éléments distincts est un modèle numérique capable de décrire le comportement mécanique de l'assemblage de disques et de sphères. La méthode est basée sur l'utilisation d'un système numérique explicite dans lequel l'interaction des particules est contrôlée contact par contact et le mouvement des particules simulé particule par particule. Les caracteristiques principales de la méthode des eléments distints sont décrites. La méthode est validée en comparant les tracés de vecteur de force obtenus par le programme sur ordicateur BALL avec les tracés correspondants obtanus à l'aide d'une analyse photo-élastique. L'analyse photo-élastique utilisée pour la comparaison est celle appliquée sur un assemblage de disques par De Josselin de Jong et Verruijt (1969). Les diagrammes de vecteur de force obtenus numériquement sont très voisins de ceux obtenus photo-élastiquement. Cette comparaison permet de conclure que la methode des éléments distincts et le programme BALL sont des instruments valables pour la recherche du comportement des assemblages granulaires.

INTRODUCTION

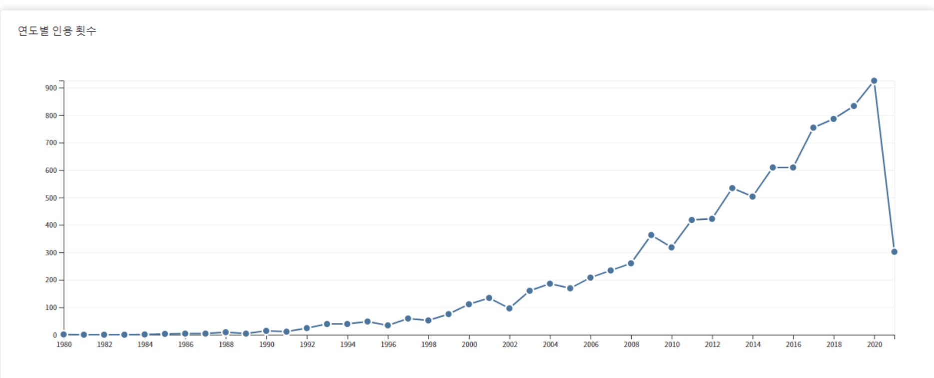
A granular medium is composed of distinct particles which displace independently from one

9,355 citations (925 in 2020) in Science Citation Index papers in > 593 Journals (as of 25 April 2021)

Discussion on this Paper closes 1 June, 1979. For further details see inside back cover.
* Principal Engineer, Advanced Technology Group, Dames & Moore, London.
† Assistant Professor, Department of Civil and Mineral Engineering, University of Minnesota, USA.

Explicit Discrete Element Method
Introduction – a seminal paper

	citations
1. POWDER TECHNOLOGY	(798)
2. GRANULAR MATTER	(353)
3. CHEMICAL ENGINEERING SCIENCE	(275)
4. PHYSICAL REVIEW E	(272)
5. computer and Geotechnics	(195)
…	
13. Int J Rock Mech Min Sci	(80)
16. Geotechnique	(71)



- Drastic increase of citations in the past 25 years
- Appreciation in various fields - proves DEM is a truly interdisciplinary science!
- What is going to happen in the next 10-20 years???

Explicit DEM
Introduction – a seminal paper

- …rock mechanics has traditionally attempted to use procedures from other branches of mechanics; however, the developments by Dr Cundall and his colleagues in the modelling of the deformation behaviour of blocky or particulate systems are now attracting interest from these other branches. It appears that there are many fields of study where a discrete or discontinuum approach can provide illuminating insights into the mechanics of deformation--insights and behaviours that may be obscured by classical continuum analyses. (Charles Fairhurst, 1988, IJRMMS foreword)

- DEM is probably the first export product from rock mechanics community to other branches of science/engineering

Explicit DEM
Introduction – overview

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Distinct Element Method

-Theory (F=ma, F=Kδ)

-Contact detection

-Time-step

-damping

Very similar

Molecular dynamics

-Theory (F=ma, $f = -\nabla V$)

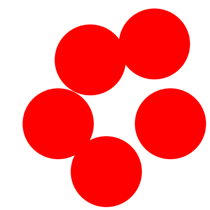
-Contact detection

-Time-step, damping

-…

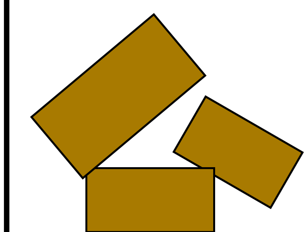
Particulate system – PFC2D/3D, FLOBALL

- Rigid particle
- bonded material
- fluid flow/heat transfer
- frictional material, mineral processing



Blocky system – UDEC/3DEC, LDEC

- Rigid/Deformable block (elastic, elastoplastic)
- Joint model (linear, nonlinear, Barton-Bandis)
- fluid flow through joint/heat transfer
- jointed (fractured) rock



Search through ‘molecular dynamics’!

www.ks.uiuc.edu/Research/vmd/

Explicit DEM
Introduction

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- Damage, and its evolution, is explicitly represented in the model; no empirical relations are needed to define damage or quantify its effect upon material behavior.
- Microcracks form and coalesce “automatically” without the need for grid reformulation.
- Complex nonlinear behaviors arise as emergent features, given simple behavior at particle level.
  - dependence of strength on confining stress
  - Dilatancy
  - evolution of material anisotropy
- Secondary phenomena, such as acoustic emission, occur in the DEM model without additional assumptions.

Explicit DEM
Equation of motion – a governing equation

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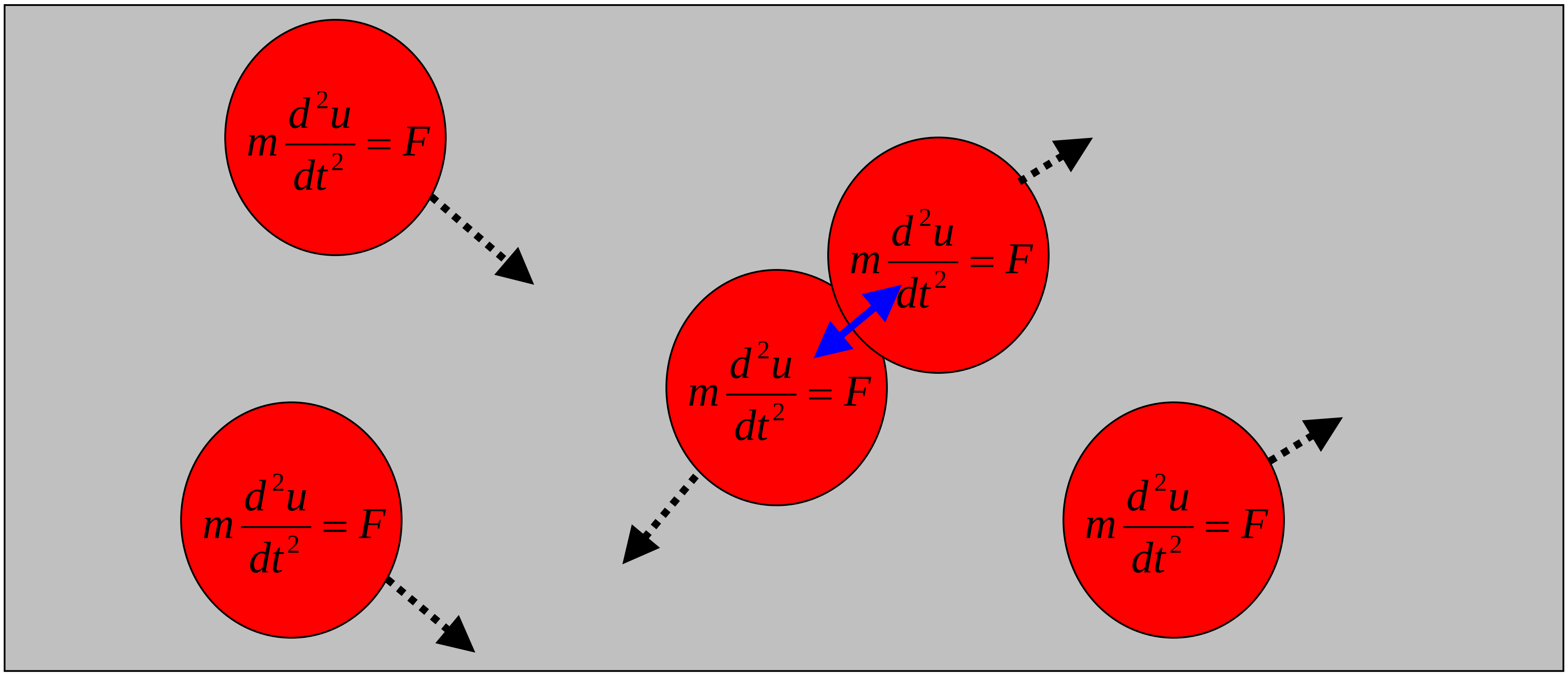

- How do we model this system?
  - Equation of motion and interacting force

Explicit DEM
Equation of motion – a governing equation

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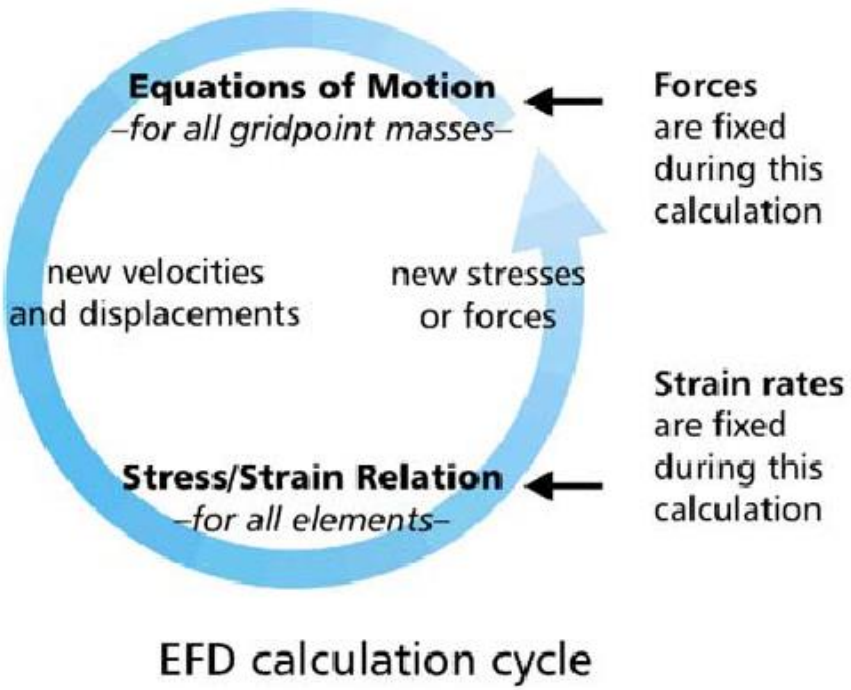
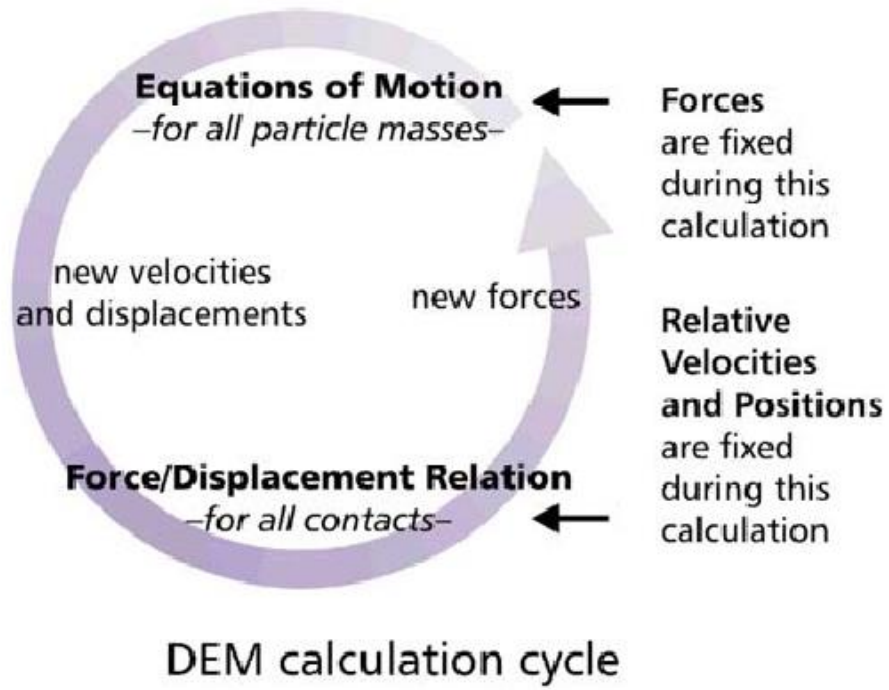




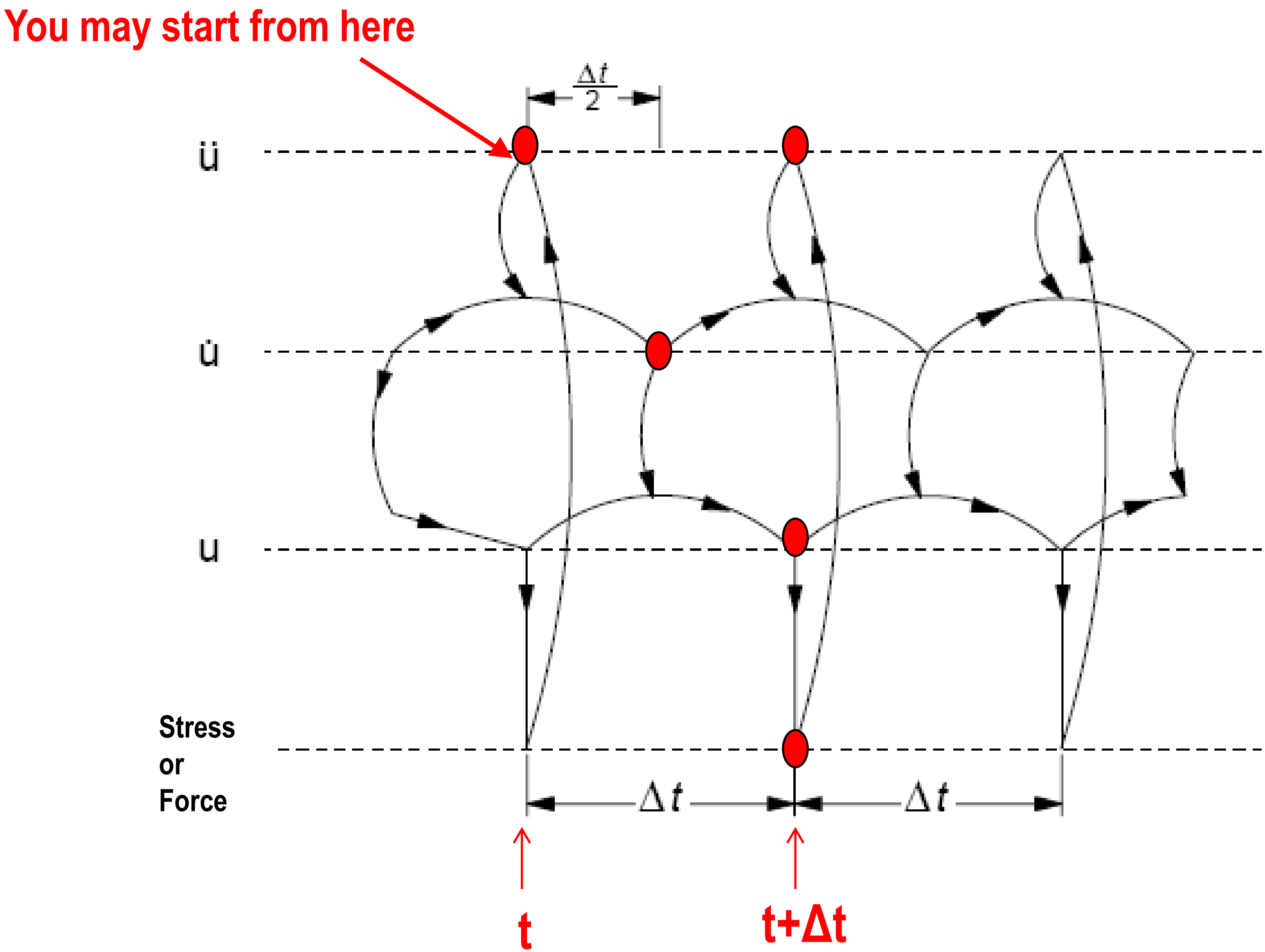
Explicit Finite Difference Method (such as FLAC) and Explicit DEM (such as PFC and UDEC) essentially have the same calculation scheme – i.e., FDM

Hart R, IJRMMS 2003;40:1089-1097

Explicit DEM
Equation of motion – a governing equation

- Let's now talk about the technique to integrate the equation of motions and calculate forces.
  - For integration two methods will be explained – interlaced central difference method (leapfrog method) and Verlet algorithm.
  - For force calculation, spring model will be used

Explicit DEM
Equation of motion – a governing equation



UDEC manual

Explicit DEM
Equation of motion – a governing equation

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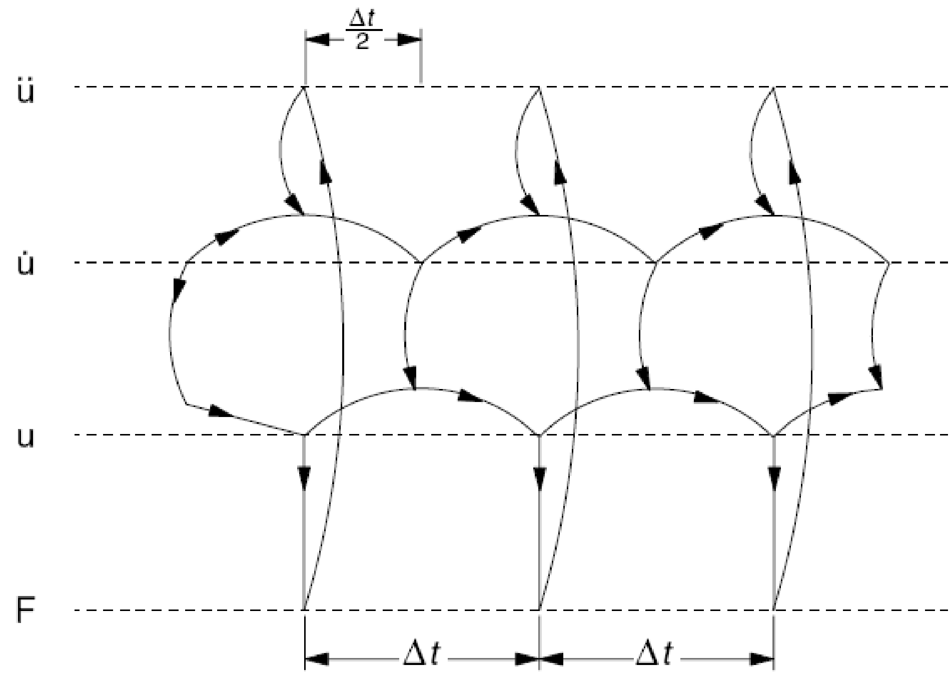
- From central difference scheme

$$\dot{u}^{(t+\Delta t/2)} = \frac{u(t+\Delta t) - u(t)}{\Delta t}$$

$$\dot{u}^{(t+\Delta t/2)} = \dot{u}^{(t-\Delta t/2)} + \frac{F^{(t)}}{m}\Delta t$$

$$u^{(t+\Delta t)} = u^{(t)} + \dot{u}^{(t+\Delta t/2)}\Delta t$$



UDEC manual (2004)

- the same as Leapfrog algorithm (Frenken & Smith, 2002)

Governing Equation

Time integration of equation of motion (2) – Verlet algorithm (Verlet, 1967*)

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$$u(t+\Delta t) = u(t) + \dot{u}(t)\Delta t + \frac{\ddot{u}(t)}{2}\Delta t^2 + \frac{\dddot{u}(t)}{3!}\Delta t^3 + O(\Delta t^4) \longleftarrow$$

Taylor expansion

$+$

$$u(t-\Delta t) = u(t) - \dot{u}(t)\Delta t + \frac{\ddot{u}(t)}{2}\Delta t^2 - \frac{\dddot{u}(t)}{3!}\Delta t^3 + O(\Delta t^4)$$

$$u(t+\Delta t) + u(t-\Delta t) = 2u(t) + \ddot{u}(t)\Delta t^2 + O(\Delta t^4)$$

$$\boxed{u(t+\Delta t) = 2u(t) - u(t-\Delta t) + \frac{f(t)}{m}\Delta t^2}$$

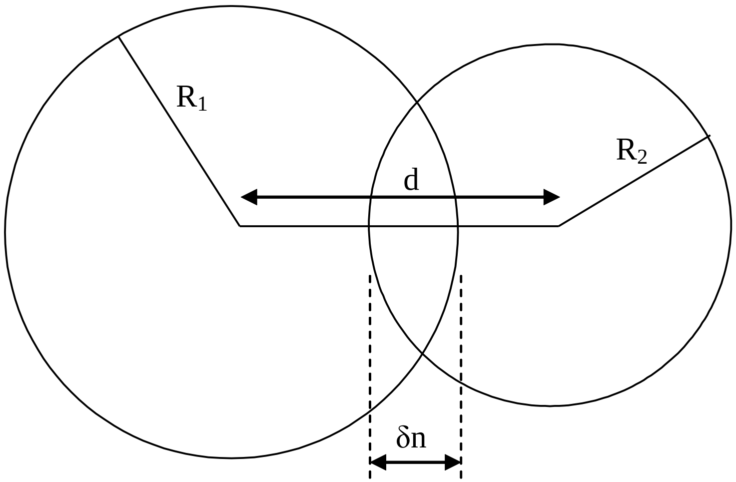
Verlet does not use the velocity to compute the new position.
However, velocity can be calculated as follows,

$$\boxed{\dot{u}(t) = \frac{u(t+\Delta t) - u(t-\Delta t)}{2\Delta t}}$$

Verlet algorithm is the most widely used in MD simulations

*Verlet L, 1967, Computer 'experiemtns' on classical fluids. I. Thermodynamical Properties of Lennard-Jones Molecules, Phys Rev 159(1), 98-103,

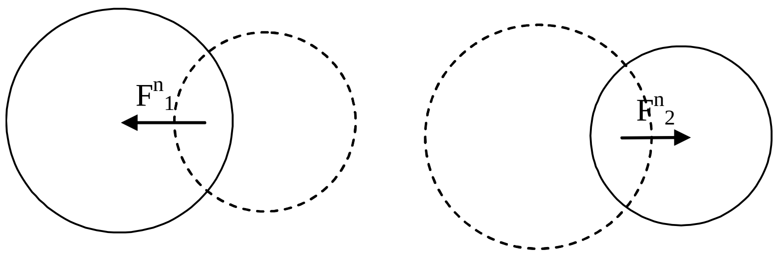
Governing Equation
Contact Force



$$F = K_n \delta_n$$

$$K_n = normal\ \ stiffness$$

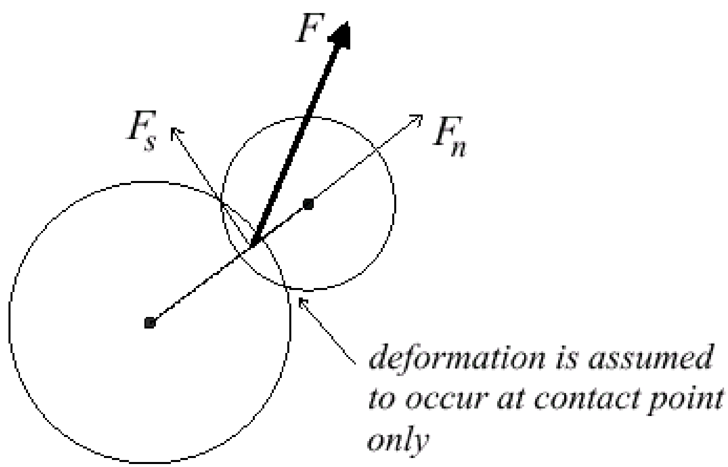
$$\delta_n = overlap$$



Directions of F in each particle

F can be defined differently depending on the nature of the problem (e.g., gradient of potential energy, Frenkel and Smit, 2002).

Governing Equation
Contact Force



Linear contact law

$$F_n = k_n U_n$$

$$\Delta F_s = k_s \Delta U_s$$

From translation & rotation

3 microproperties:

kn, ks, μ

Slip condition

friction coefficient

$$F_s \leq \mu F_n$$

From HCItasca.com

Explicit DEM
Calculation Cycle – rotation, angular velocity, acceleration

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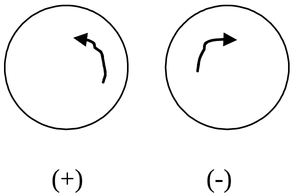
$$\dot{\theta}^{(t+\Delta t/2)} = \dot{\theta}^{(t-\Delta t/2)} + \left(\frac{\sum M^{(t)}}{I} \right) \Delta t$$

$$\theta^{(t+\Delta t)} = \theta^{(t)} + \dot{\theta}^{(t+\Delta t/2)} \Delta t$$



Sign convention of rotation

$$\theta: rotation,\ \dot{\theta}: angular\ velocity,\ \ddot{\theta}: angular\ acceleration$$

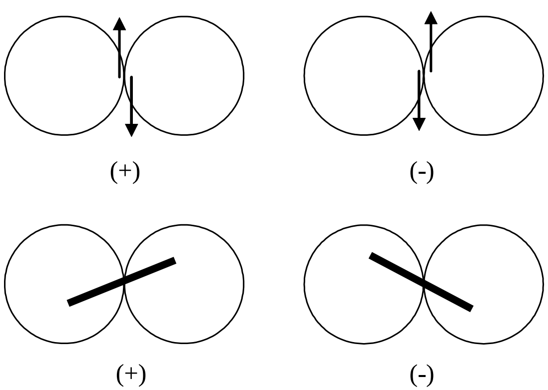
$$M: moment,\ I: moment\ of\ inertia$$

- tangential force (F_s)

$$F_s = K_s \delta_s$$

$$K_s: shear\ stiffness$$

$$\delta_s: shear\ displacement (when\ overlapped)$$

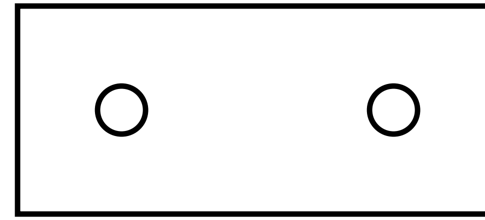


- Slip condition

$$F_s \le \mu K_n$$

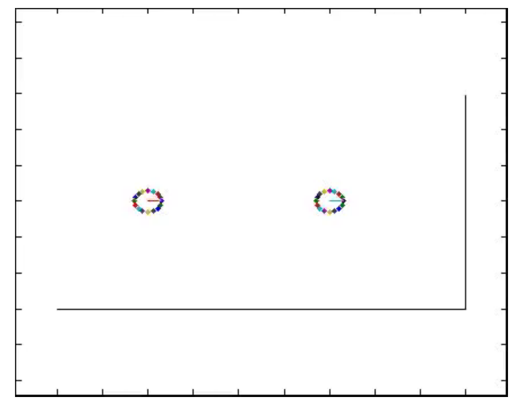
$$\mu: friction\ coefficient$$

Explicit DEM
Calculation Cycles



$$\sum F_i = m\frac{d^2u}{dt^2} \qquad \sum M_i = I\frac{d^2\theta}{dt^2}$$

$$\dot{u}^{(t+\Delta t/2)} = \dot{u}^{(t-\Delta t/2)} + \left(\frac{\sum F_i^{(t)}}{m} + g_i\right)\Delta t \qquad \dot{\theta}^{(t+\Delta t/2)} = \dot{\theta}^{(t-\Delta t/2)} + \left(\frac{\sum M^{(t)}}{I}\right)\Delta t$$



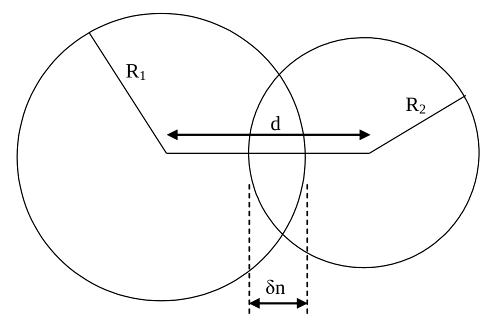
$$u^{(t+\Delta t)} = u^{(t)} + \dot{u}^{(t+\Delta t/2)}\Delta t \qquad \theta^{(t+\Delta t)} = \theta^{(t)} + \dot{\theta}^{(t+\Delta t/2)}\Delta t$$

update particle + wall positions and set of contacts

Law of Motion
(applied to each particle)
• resultant force + moment

Force–Displacement Law
(applied to each contact)
• relative motion
• constitutive law

contact forces



From Itasca, PFC manual

$$F = K_n\delta_n \qquad F = K_s\delta_s$$

Explicit DEM
Implementing issues

- We covered the 1) solution method of equation of motion and 2) force calculation
- There are a couple of implementing issues
  - Stability of solution
  - Contact detection
  - Extension to 2D, 3D
  - Damping scheme

Implementation issues for explicit DEM
Stability of solution

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- There are two important considerations with dynamic relaxation:
  - Choice of time step
  - Effect of damping

Implementation issues for explicit DEM
Stability of solution

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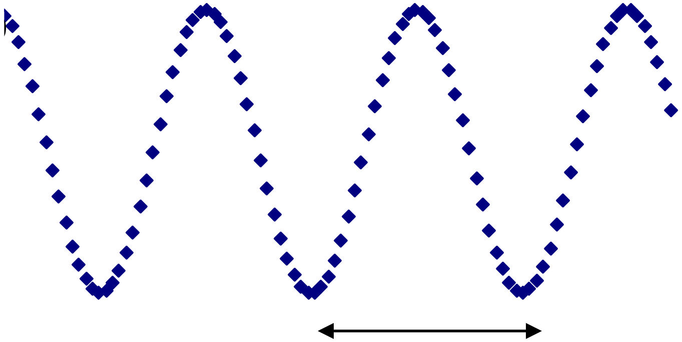
- Explicit schemes are only conditionally stable – have to use small enough time step (Δt)

$$my'' = -ky$$

$$y(t) = A\cos w_0 t + B\sin w_0 t, \quad w_0 = \sqrt{k/m}$$



$$T = 2\pi\sqrt{m/k}$$

- Stable condition for explicit FDM for a single degree of freedom (Cundall and Strack, 1979),

$$\Delta t < \frac{2}{w} = 2\sqrt{m/k}$$

Implementation issues for explicit DEM
Stability of solution

- Time step must be lower than the period of the system by some amount
- This Δt is called critical Δt
- DEM code can automatically determine the Δt but this can also be specified.

$$t_{\text{crit}} = \begin{cases} \sqrt{m/k^{\text{tran}}}, & \text{(translational motion)} \\ \sqrt{I/k^{\text{rot}}}, & \text{(rotational motion)} \end{cases}$$

Implementation issues for explicit DEM
Stability of solution

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- **Time step must be lower than the period of the system by some amount.**

$$\Delta t_c = \alpha \sqrt{\frac{m}{k}}$$

α = 0.1 usually gives a stable results

Implementation issues for explicit DEM
Dynamic Relaxation – Effect of Damping

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- Damping is necessary to dissipate the kinetic energy, e.g., static problem
- damping force acts opposite to current velocity
- damping force magnitude proportional to out-of-balance force – there are other way of applying damping force, e.g., proportional to velocity magnitude

$$F + F_d = m\ddot{u}$$

$$F_d = -\alpha \left| F \right| sign\left| \dot{u} \right|$$

Implementation issues for explicit DEM
Dynamic Relaxation – Effect of Damping

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- Velocity-proportional damping introduces body forces that can affect the solution.
- Local damping is used in FLAC --- The damping force at a gridpoint is proportional to the magnitude of the unbalanced force with the sign set to ensure that vibrational modes are damped:
- Damping forces are introduced to the equations of motion:

$$\Delta \dot{u}_i = \left[\Sigma F_i - \alpha \,|\Sigma F_i| \operatorname{sgn}(\dot{u}_i)\right]\frac{\Delta t}{m} \qquad F_d = -\alpha \left|\Sigma F_i\right| \operatorname{sgn}(\dot{u}_i)$$

- In *FLAC* the unbalanced force ratio (ratio of unbalanced force, F_i, to the applied force magnitude, F_m) is monitored to determine the static state.
- By default, when $F_i / F_m < 0.001$, then the model is considered to be in an equilibrium state.

Explicit DEM
Examples (1) – Falling Ball under gravity

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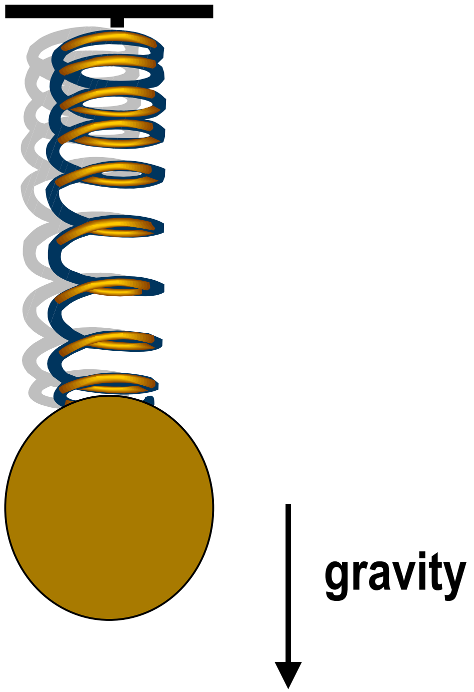
Mass = 1 kg

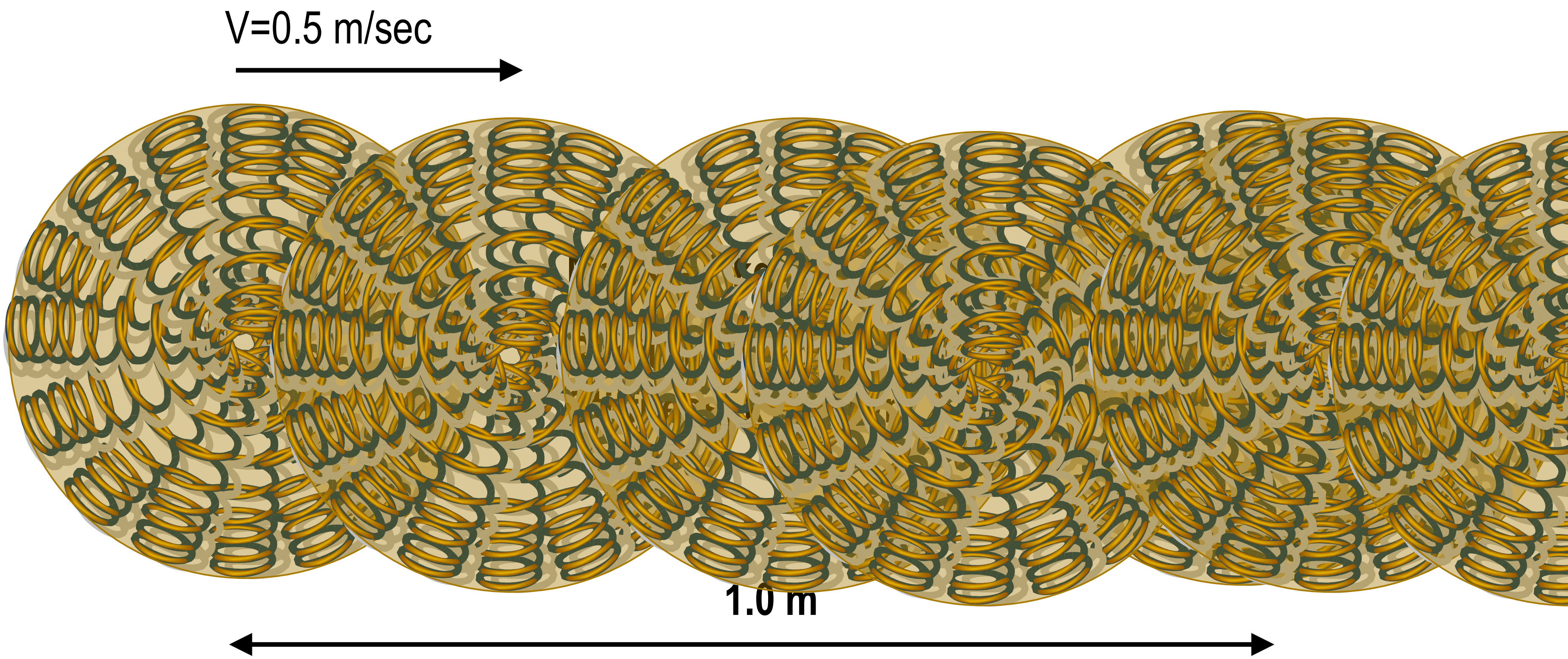
Stiffness, k = 1 N/m

g = 10 m/sec^2



- Step-by-step calculation of velocity, displacement, Force and acceleration
- Implementation of gravity and damping
- Appreciation of the ability to model the dissipation of kinetic energy

Explicit DEM
Examples (2) – Colliding two balls



- Contact detection between two balls

Governing Equation
Extension of model to 2D & 3D

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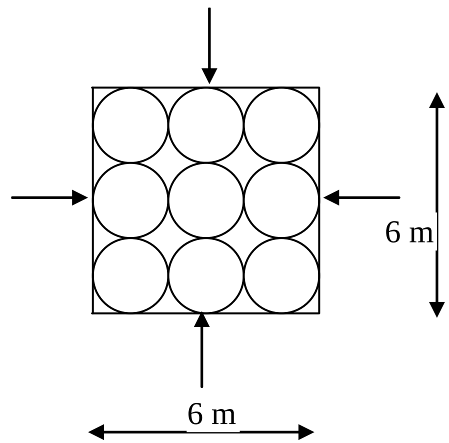
- 2D, 3D extension and inclusion of gravity

$$\dot{u}_i^{(t+\Delta t/2)} = \dot{u}_i^{(t-\Delta t/2)} + \left(\frac{\left(\sum F^{(t)}\right)_i}{m} + g_i \right) \Delta t$$

$$u_i^{(t+\Delta t)} = u_i^{(t)} + \dot{u}_i^{(t+\Delta t/2)} \Delta t$$

  - Can be achieved straightforwardly

Explicit DEM
Examples (3) – Cundall's nine disc test



- Static problem using dynamic formulations
- Biaxial loading with constant velocity, 0.001 m/s
- Loading stopped after 300 cycles
- Ball & Wall stiffness: 1e5 N/m
- Friction coefficient 0.5, damping coefficient 0.7

*Cundall PA, Strack ODL, discrete numerical model for granular assemblies, Geotechnique 29 (1):47-65, 1979

Explicit DEM
Examples (3) – Cundall's nine disc test

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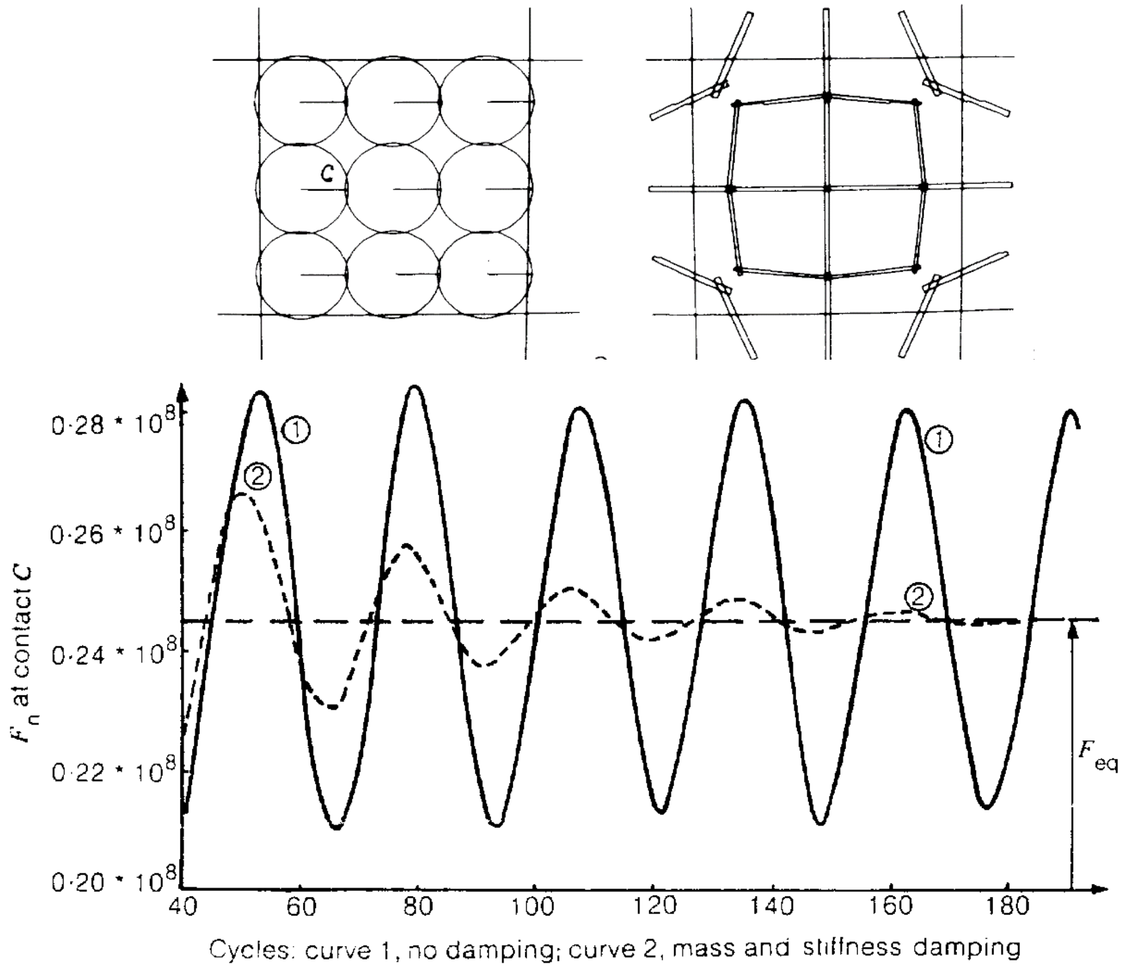
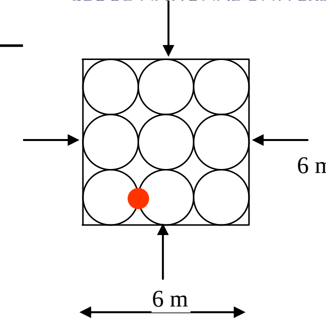
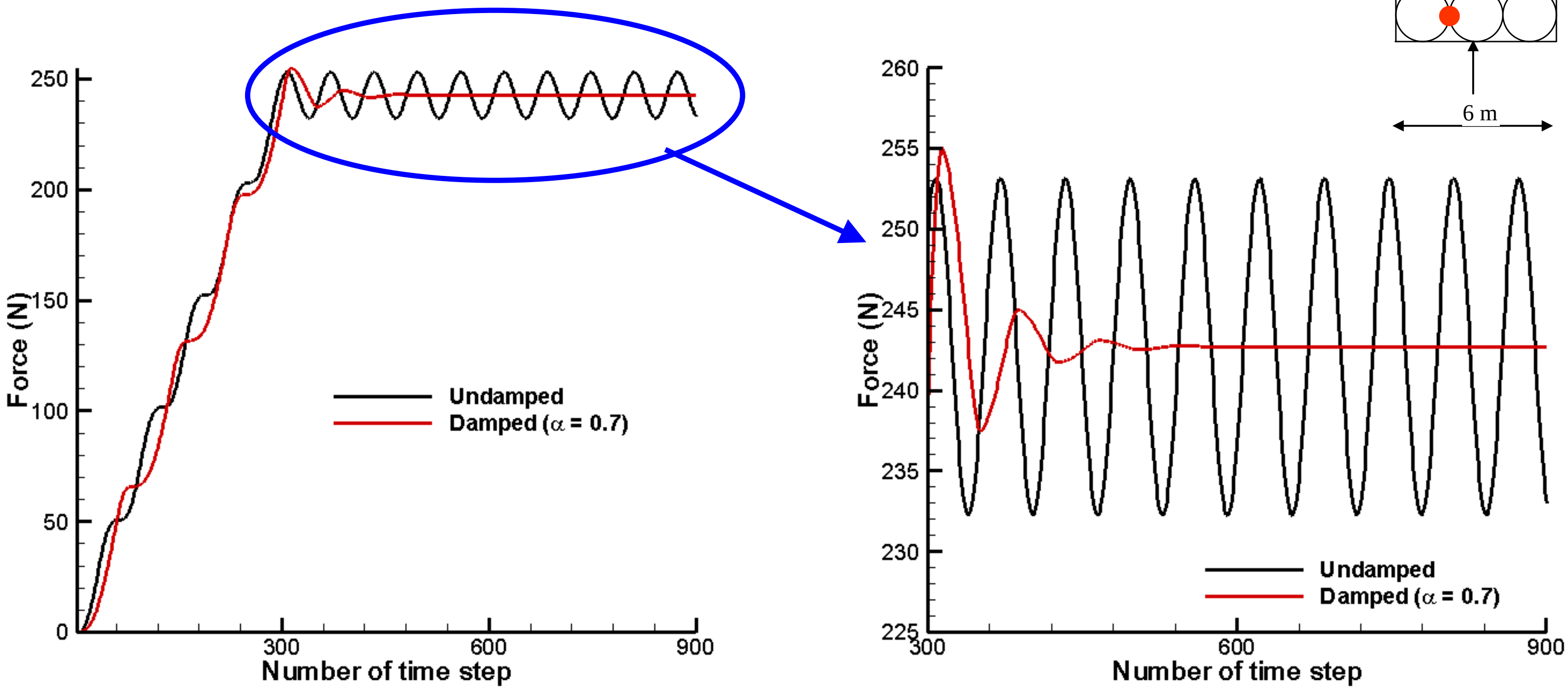
- Original presentation (Cundall & Strack, 1979)



*Cundall PA, Strack ODL, discrete numerical model for granular assemblies, Geotechnique 29 (1):47-65, 1979

Explicit DEM
Examples (3) – Cundall’s nine disc test





- Static analysis possible by solving dynamic equations with damping

Implementation issues
Contact detection - Cell-mapping

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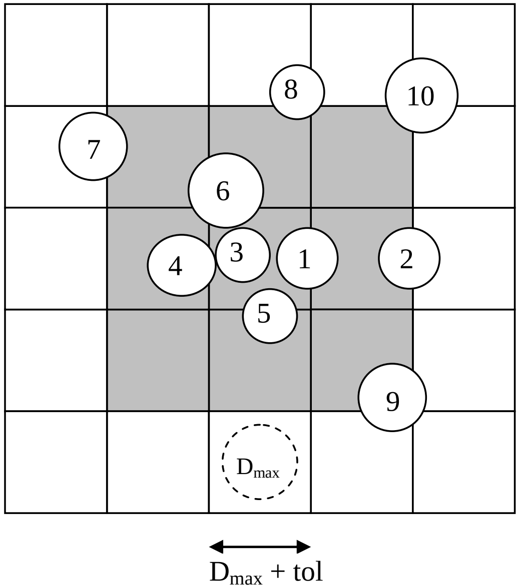
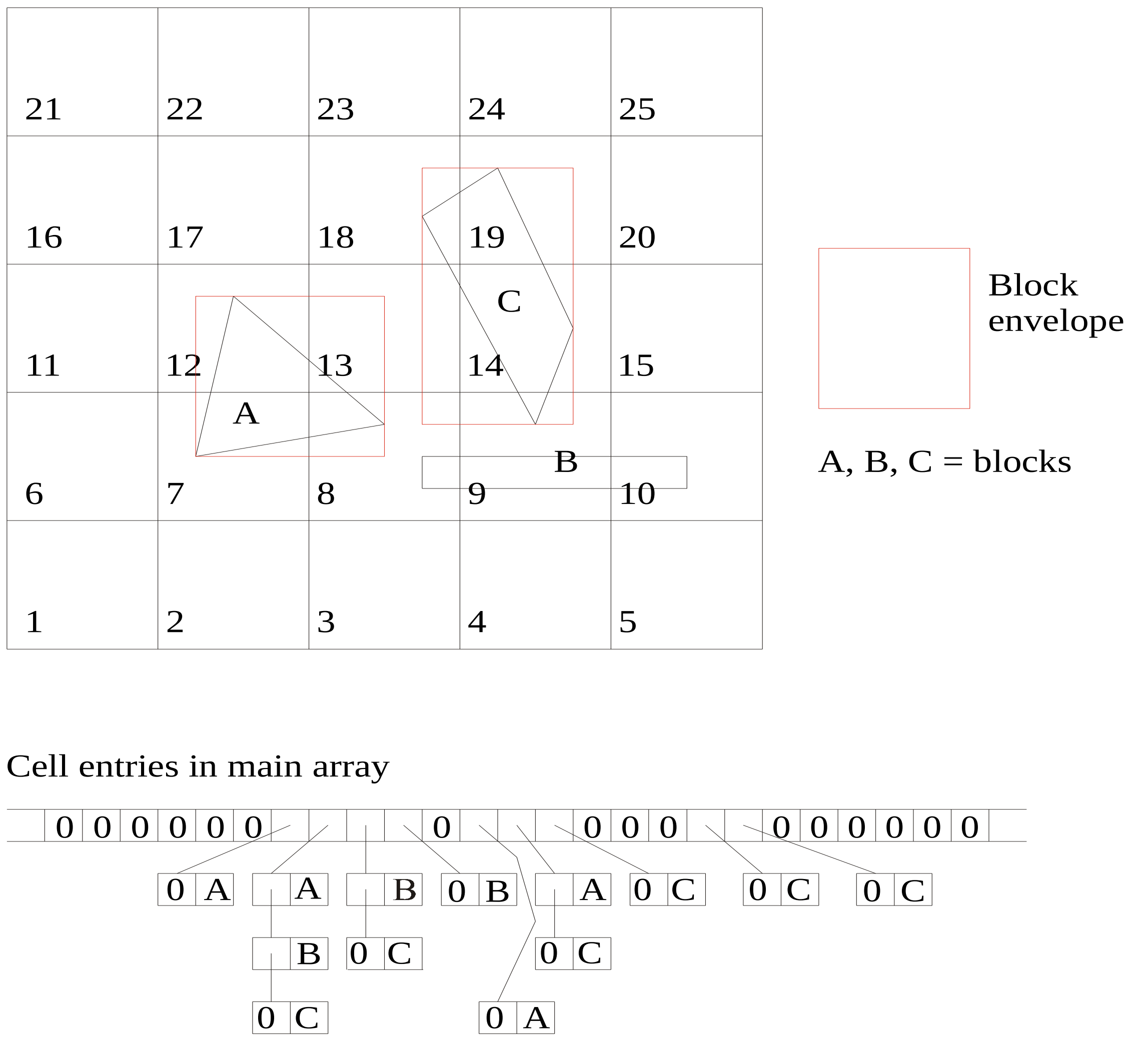
Why is this important? - Imagine we run simulation with 10,000 particles and try to check forces between them.

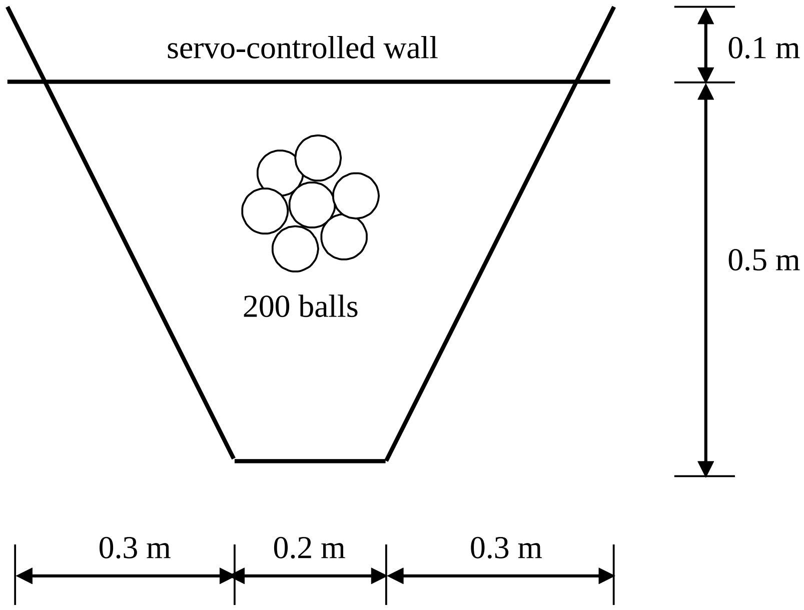


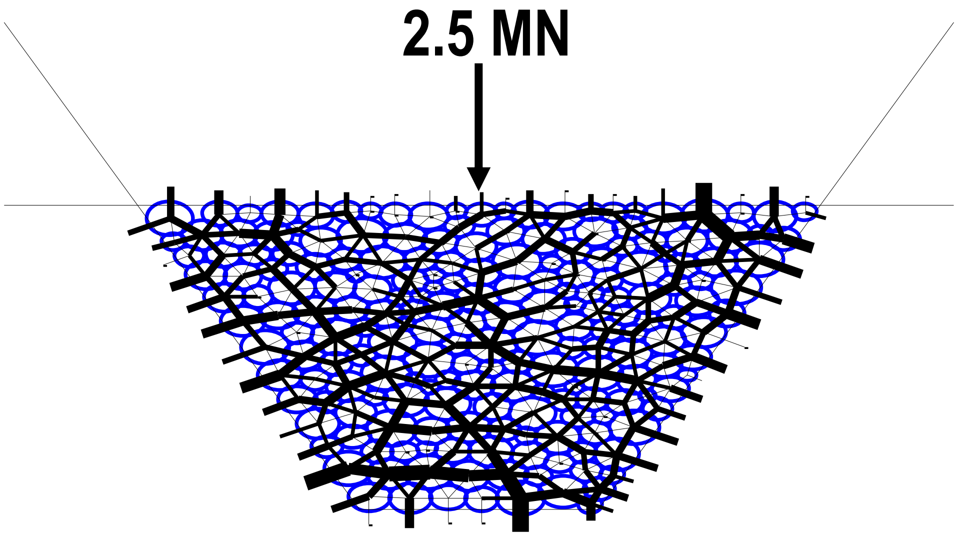
- Calculation time: Naïve calculation of all contacts, ${}_nC_2 \sim n^2$
- Mapping cell logic $\sim \alpha n$

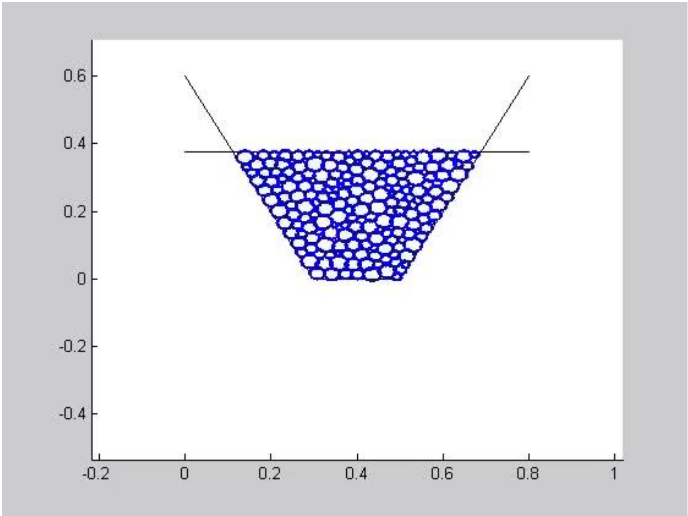
Implementation issues
Contact detection - Cell-mapping



Explicit DEM
Examples (4) – Granular flow in a hopper

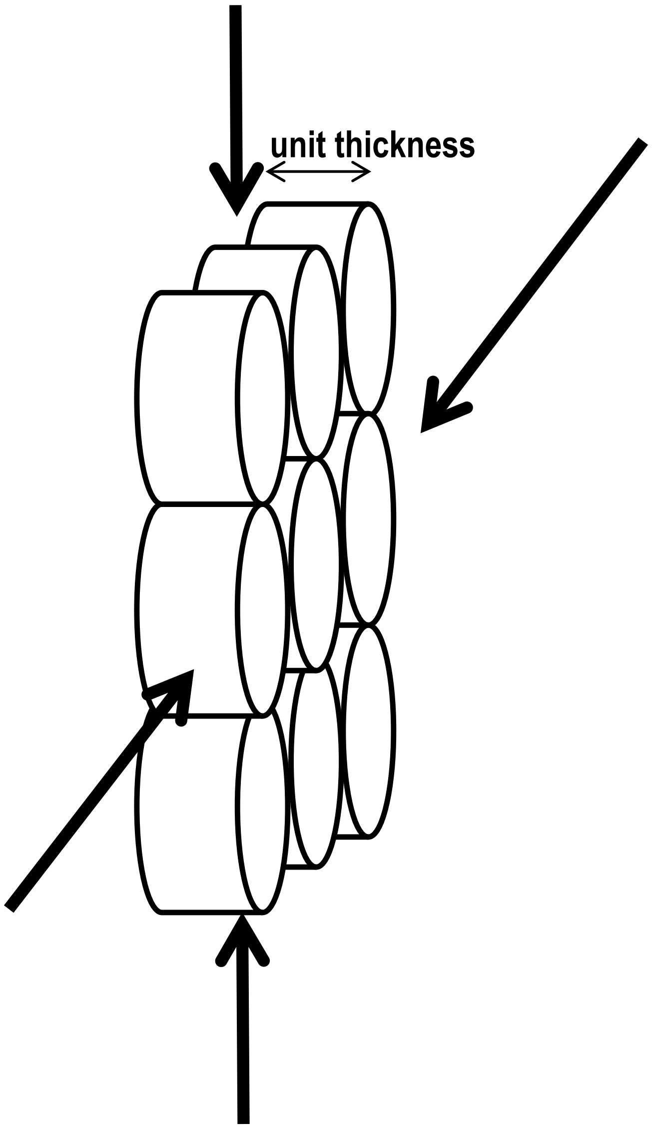






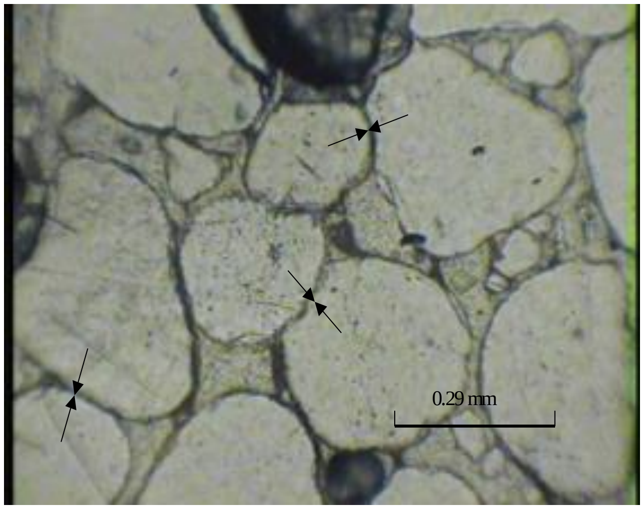
- Servo-controlled wall implemented
- $K_n=K_s=1e8$ N/m, $\mu=2.5$
- Can complement expensive large scale physical test

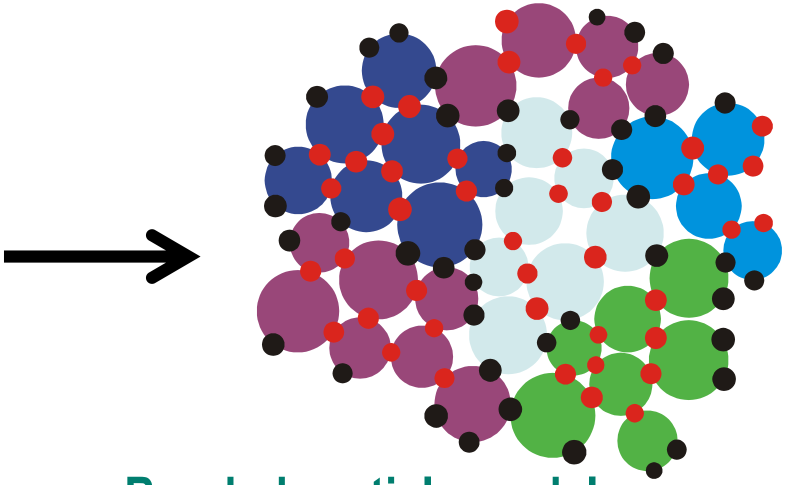
Physical model for 2D DEM



- Conceptually, 2D DEM can be said to be ‘plane stress’ on ‘rigid’ ‘cylindrical’ particle with unit thickness (hence, disc) as shown here.
- Stress (actually force) is in 2D and there is no third directional deformation due to Poisson’s effect.

Explicit DEM – Bonded Particulate system
Motivation





- **Rock** behaves like a cemented granular material of complex-shaped grains in which both the grains and the cement are deformable and may break.

- **Bonded-particle model**
  - Fundamental particle is circular or spherical, but complex "grains" produced by bonding particles.
  - Damage occurs by bond breakages, material evolves from solid to granular.
  - Exhibits rich set of emergent behaviors similar to crystalline rock.

Explicit DEM – Bonded Particulate system
Bonding logic

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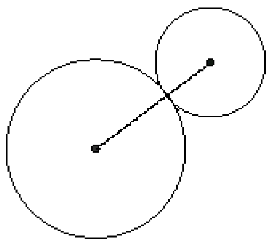
contact bond

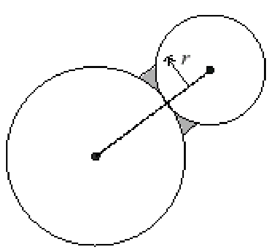


2 microproperties:

normal & shear strengths

models adhesion over vanishingly small area of contact point
(does not resist moment)
breaks if normal or shear force exceeds bond strength

parallel bond



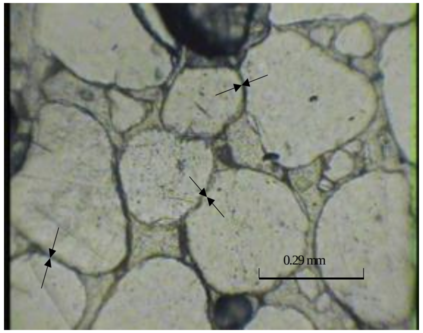
5 microproperties:

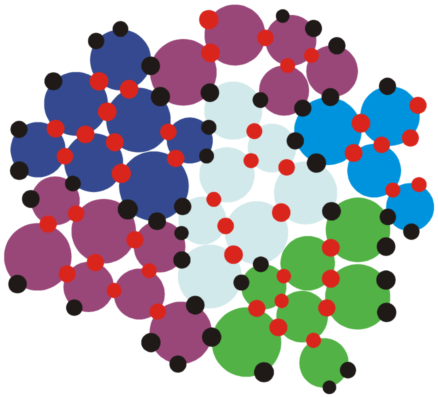
bond radius

normal & shear stiffnesses

normal & shear strengths

models additional material deposited after balls are in contact
(does resist moment)
breaks if normal or shear stress exceeds bond strength

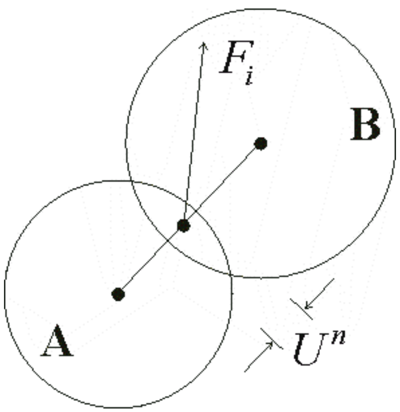


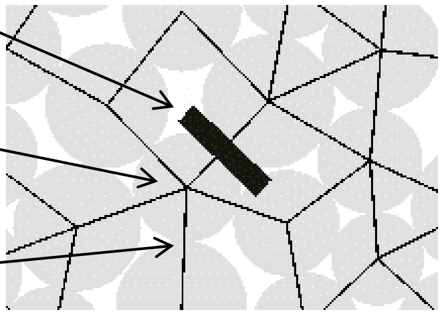


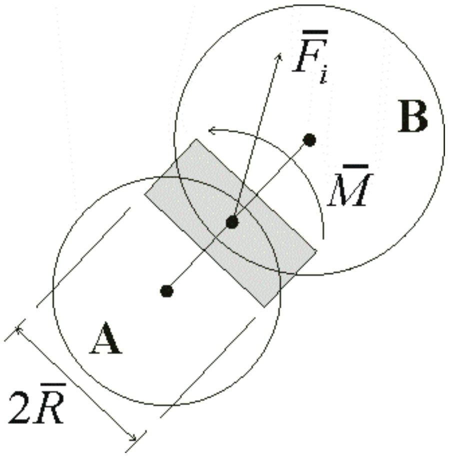
from www.HCItasca.com

This is definitely not for billiards balls!

Explicit DEM – Bonded Particulate system
Grain-cement microproperties







simulated via distinct-element method

$$F^n = K^n U^n$$

$$\Delta F^s = k^s \Delta U^s$$

deformability

$$\Delta \bar{F}^n = \bar{k}^n A \Delta U^n$$

$$\Delta \bar{F}^s = \bar{k}^s A \Delta U^s$$

$$\Delta \bar{M} = \bar{k}^n I \, \Delta \theta$$

$$F^s \leq \mu F^n$$

strength

$$\bar{\sigma}^{\max} = \frac{-\bar{F}^n}{A} + \frac{|\bar{M}|\bar{R}}{I} < \bar{\sigma}_c$$

$$\bar{\tau}^{\max} = \frac{|\bar{F}^s|}{A} < \bar{\tau}_c$$

grain behavior

cement behavior

Explicit DEM – Bonded Particulate system
Overall behavior of bonded system

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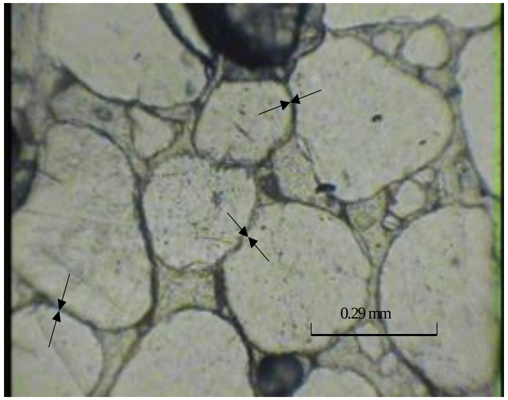
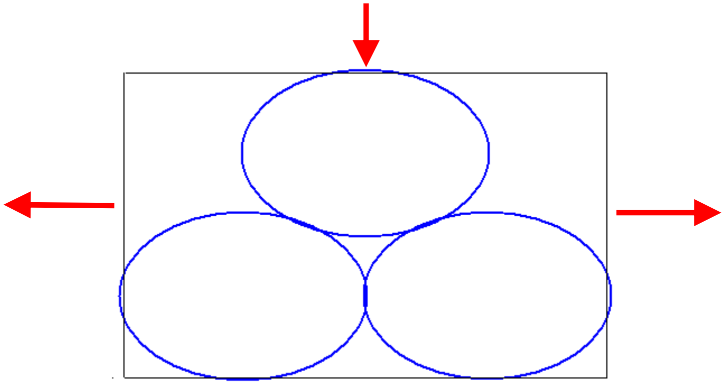
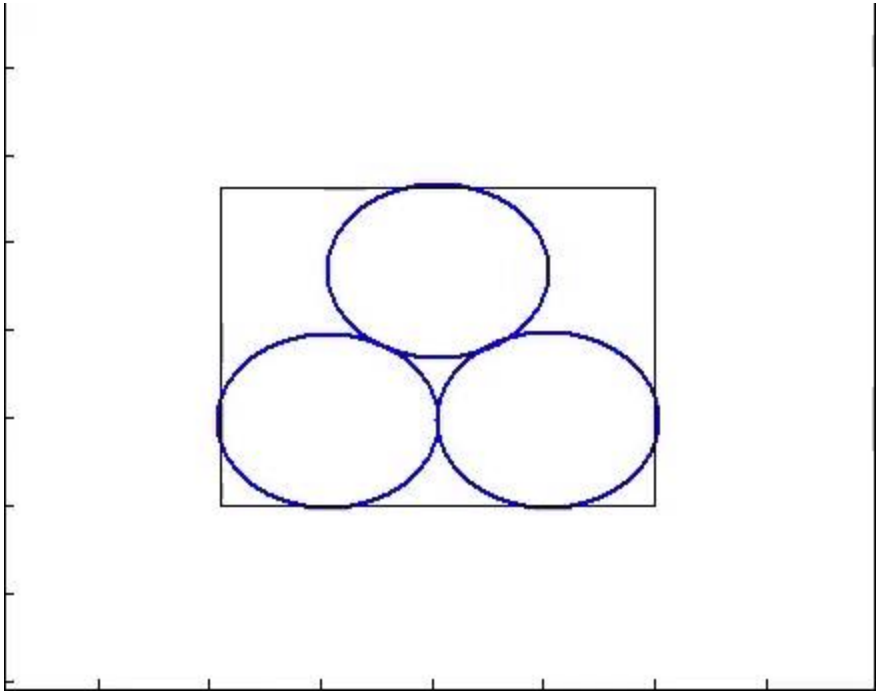
From HCItasca.com







- System behavior affected by:
  - grain shape
  - grain size distribution
  - grain packing
  - grain-cement microproperties
  - material-genesis procedure

Explicit DEM – Bonded Particulate system
Overall behavior of bonded system

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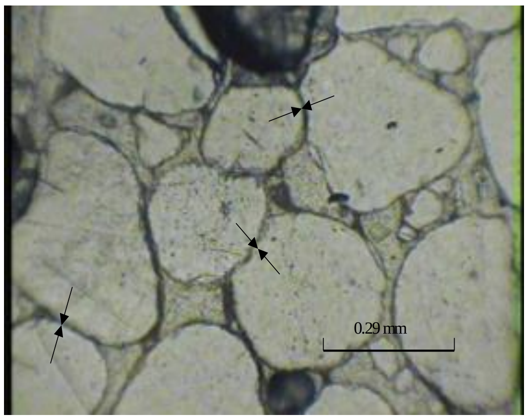
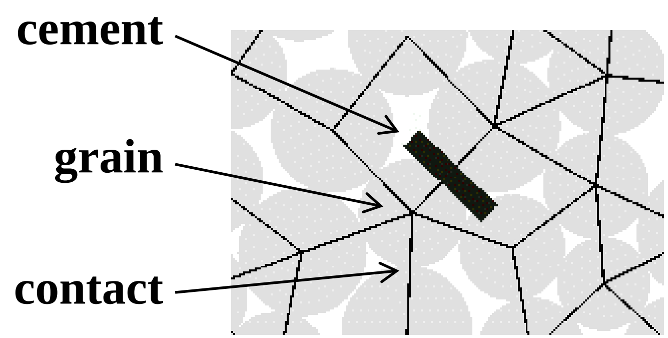




- System behavior affected by:
  - grain shape — circular disks in 2D
  - grain size distribution — uniform $[D_{\min}, D_{\max}]$
  - grain packing — arbitrary & isotropic
  - grain-cement microproperties — deformability & strength
  - material-genesis procedure — material vessel

From CItasca.com

Explicit DEM – Bonded Particulate system
Calibration – determination of microparameter

micro-properties = ? —— to predict / perform simulation ——> macro-response = ?

grains & glue

circular balls & parallel bonds

- deformability
- strength

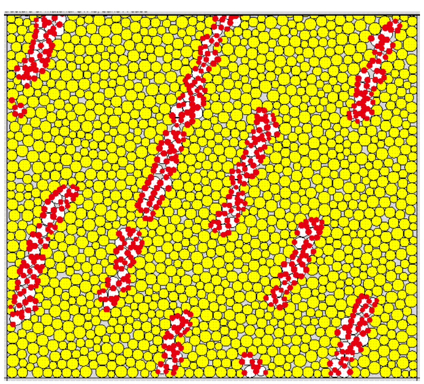
grain shape / **grain packing**

represent microstructure

- clusters
- clumps

- porosify

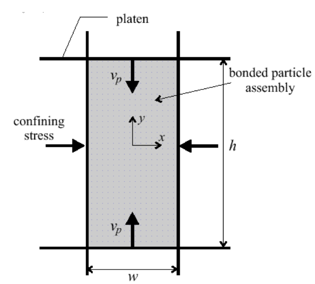
anisotropic properties



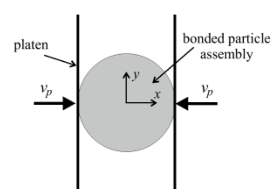
Weaker and less stiff biotite bands within rock matrix

lab scale (properties)

biaxial tests



Brazilian tests



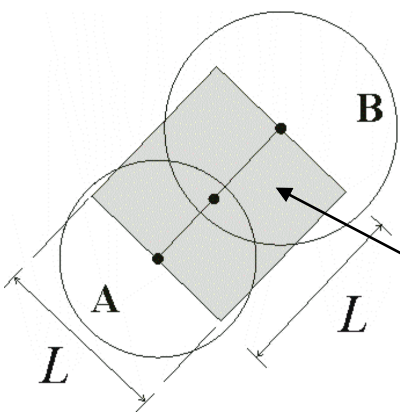
field scale (bulk behavior & relevant mechanisms)

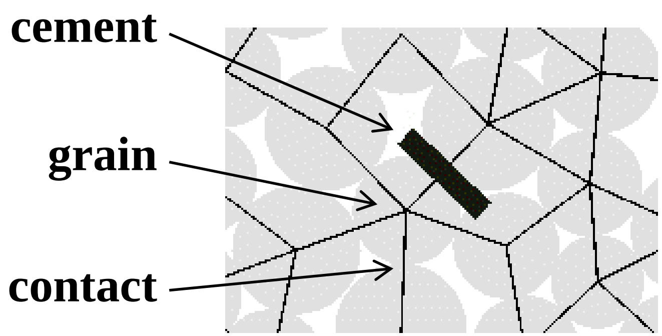
mining : excavation damage & stability

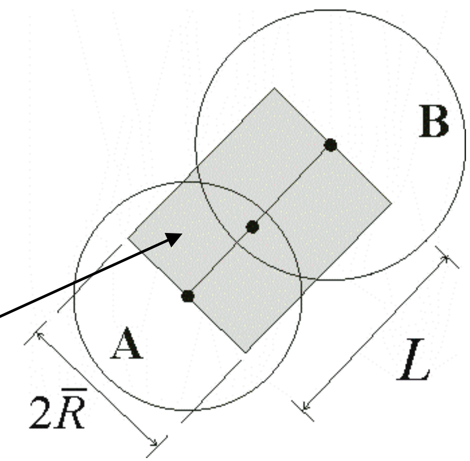
rock cutting : chip formation

liner anchor : pullout strength

Explicit DEM – Bonded Particulate system
Calibration – determination of microparameter







equivalent continuum

$$E_c = \frac{k_n}{2t}, \quad t = 1$$

deformability

$$\bar{E}_c = \bar{k}^n \left(R^{(A)} + R^{(B)} \right)$$

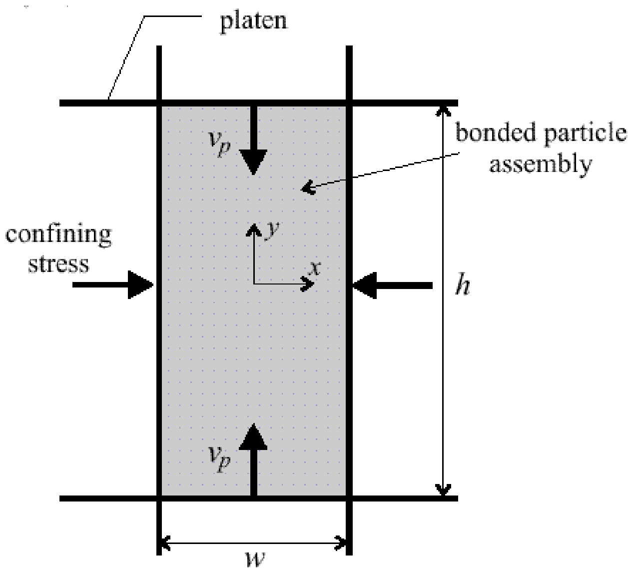
grain behavior

$$E \propto \left\{ E_c, \quad \bar{E}_c \right\}$$

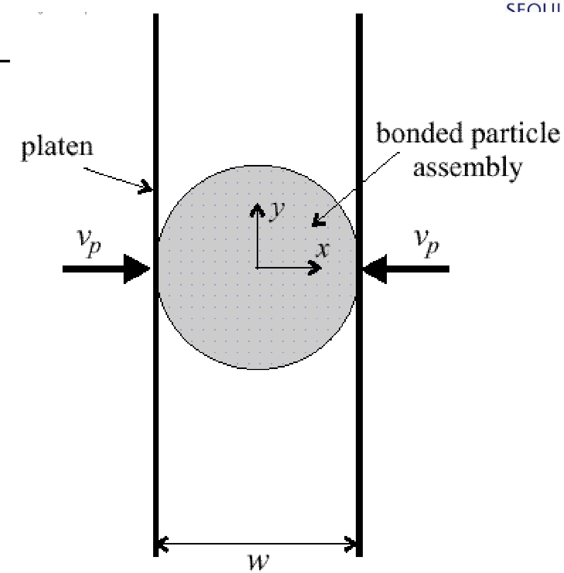
$$\nu \propto \left\{ \left(k_n / k_s \right), \quad \left(\bar{k}^n / \bar{k}^s \right) \right\}$$

cement behavior

Macroproperty characterization



Biaxial-test environment



Brazilian-test environment

- elastic constants E and ν
- Brazilian tensile strength
- unconfined compressive strength
- crack-initiation stress
- strength envelope:
  linearize by: friction angle
  cohesion

material-genesis procedure

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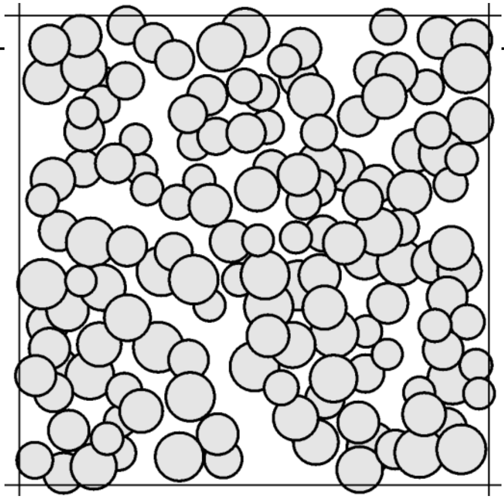
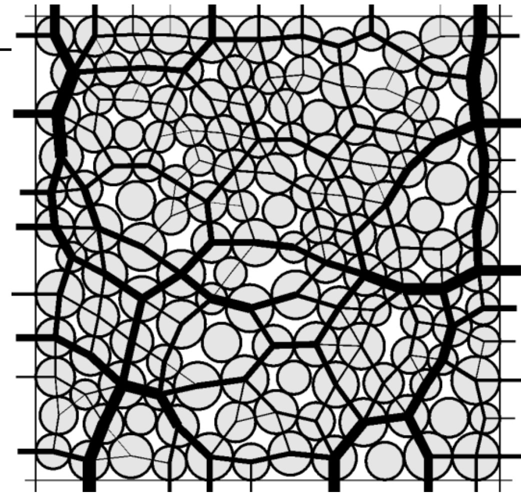
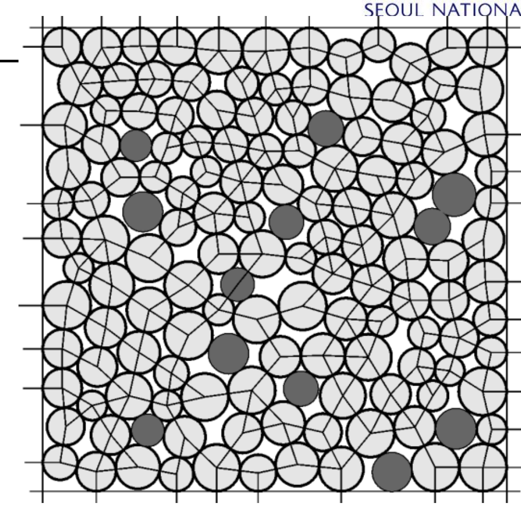


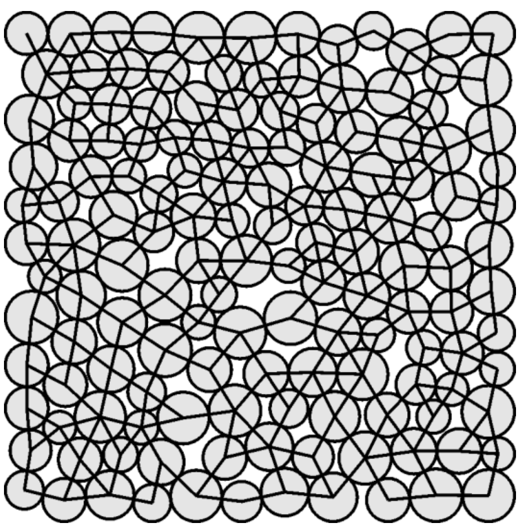
1. Compact initial assembly.



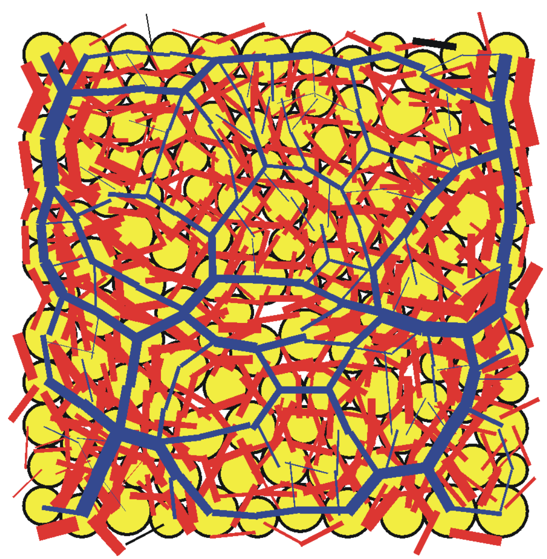
2. Install isotropic stress, sig0.



3. Remove “floating” particles.



4. Install parallel bonds.

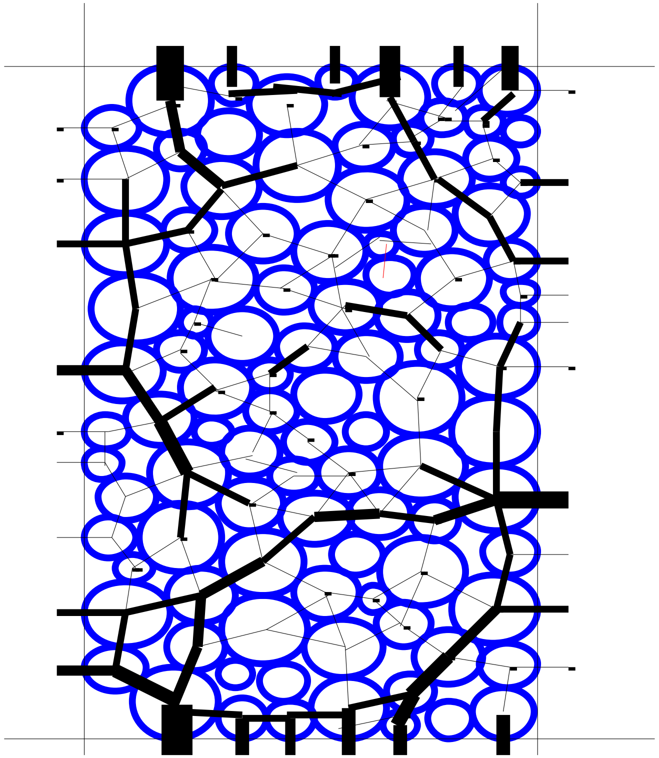


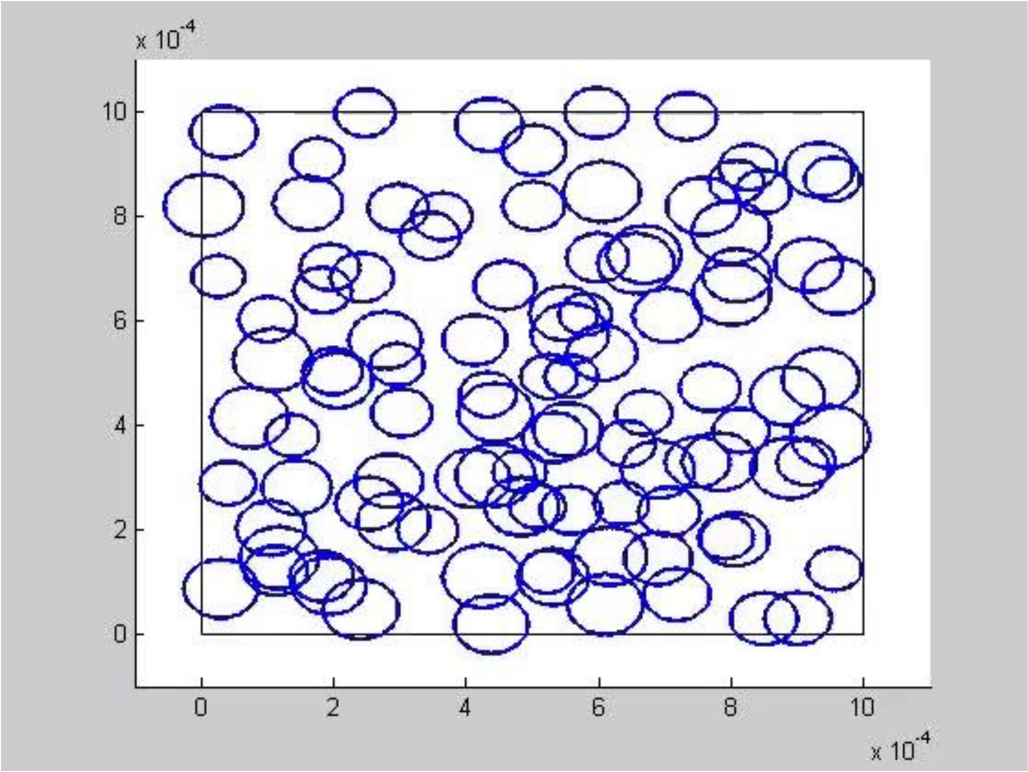
5. Remove from material vessel.

Locked-in forces (red-tension, blue-compression).

Magnitude is small relative to UCS.

Example of initial packing



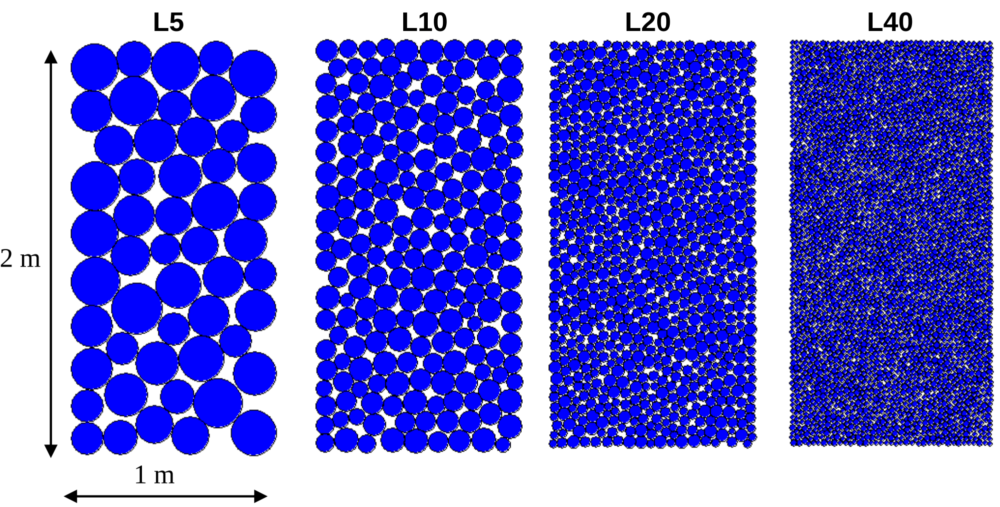


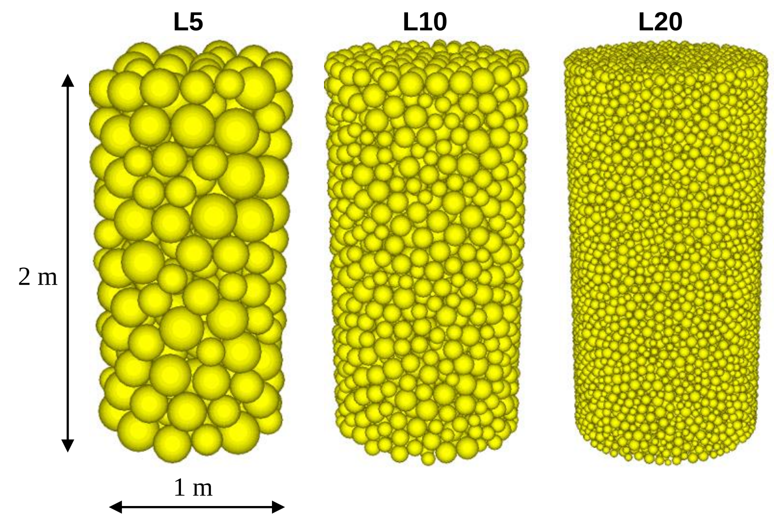
- Initial packing is an important issue for PFC type modeling
- In this example, radius of balls are fixed and random locations are selected via a method similar to Monte Carlo Method.

Examples

- Implementation examples
  1) Falling ball under gravity
  2) Colliding two balls
  3) Static loading of a nine disc system
- Application examples
  4) Gravel or powder in a hopper
  5) Uniaxial strength test of a material
  6) Uniaxial strength test of a material with a crack
  7) Modeling of transversely isotropic rock

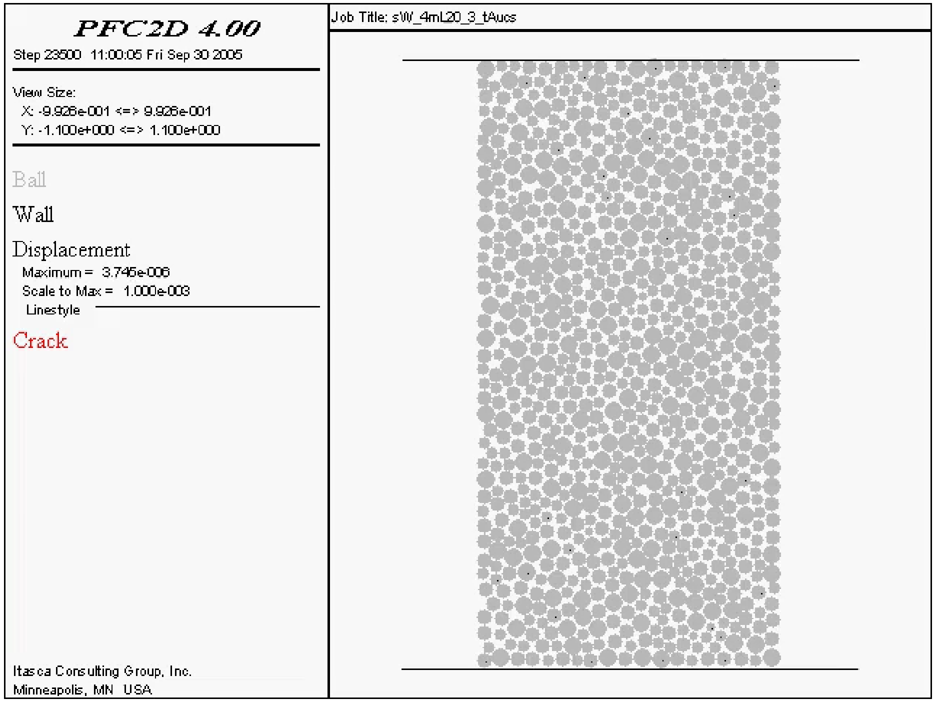
Explicit DEM
Examples (5) – Uniaxial Strength Test

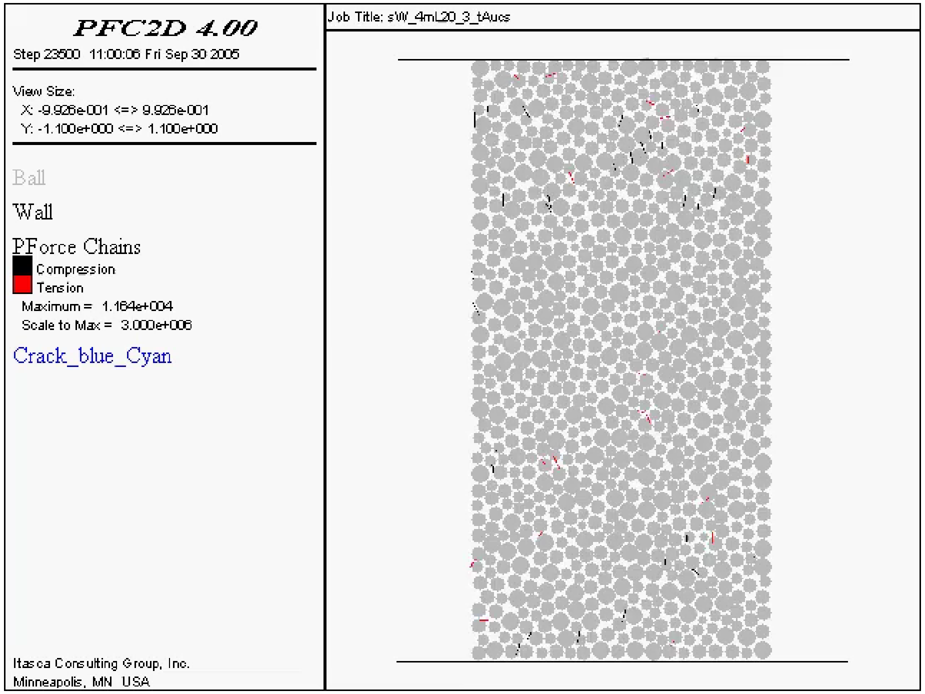




- Samples were generated through genesis scheme
- Stress and strain were monitored in the sample
- Elastic modulus, Poisson's ratio and strength can be measured by numerical experiment - this needs to be compared with actual(real) measurement.
- Different resolution (av. # of balls/section)
- Min. ball D ≈ 2 cm (L40) ~ 15 cm (L5), D_{max}/D_{min}=1.66

Explicit DEM
Examples (5) – Uniaxial Strength Test





Displacement + cracking

Parallel bond force + cracking

Explicit DEM

Examples (5) – Uniaxial Strength Test Obtained σ-ε (stress-strain) curve

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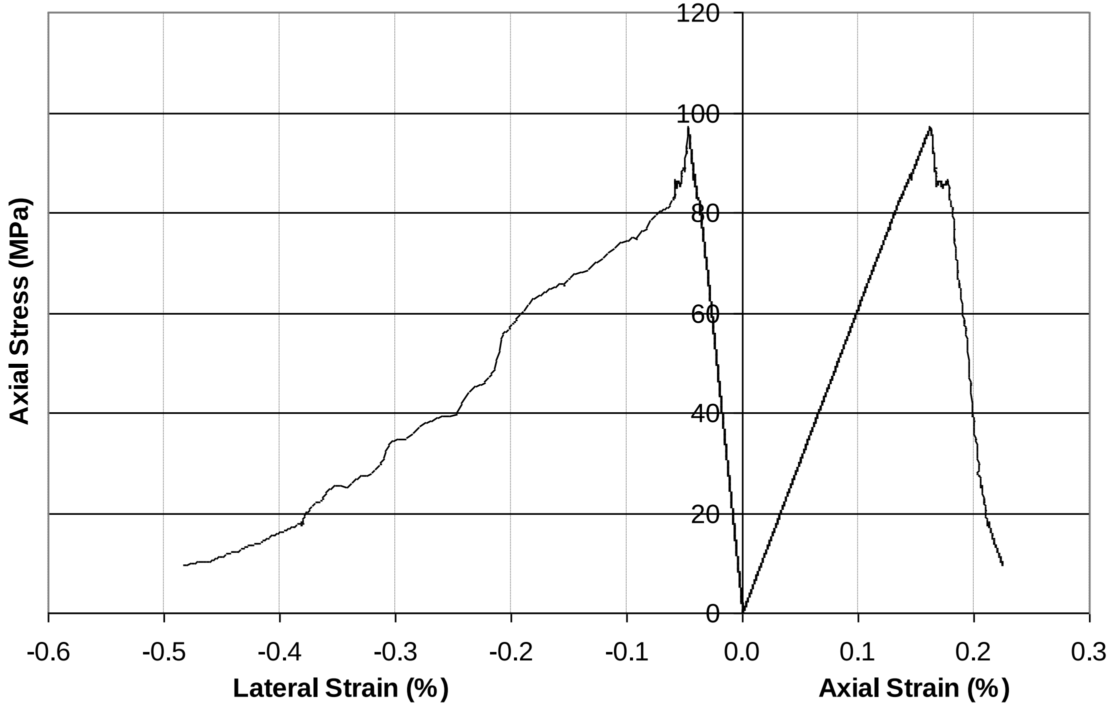

- Very similar observation to actual experiments can be obtained.

Explicit DEM Examples (6) - Rock Mass Strength
– Uniaxial Strength Test (with a internal fracture)

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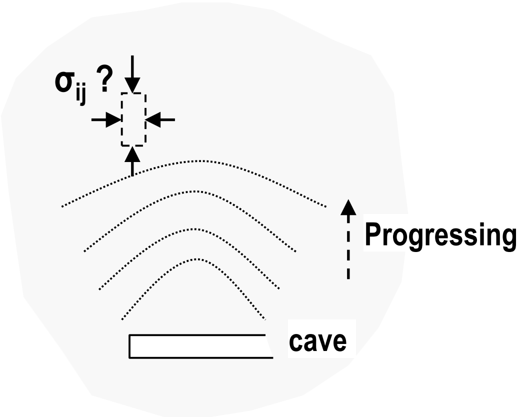
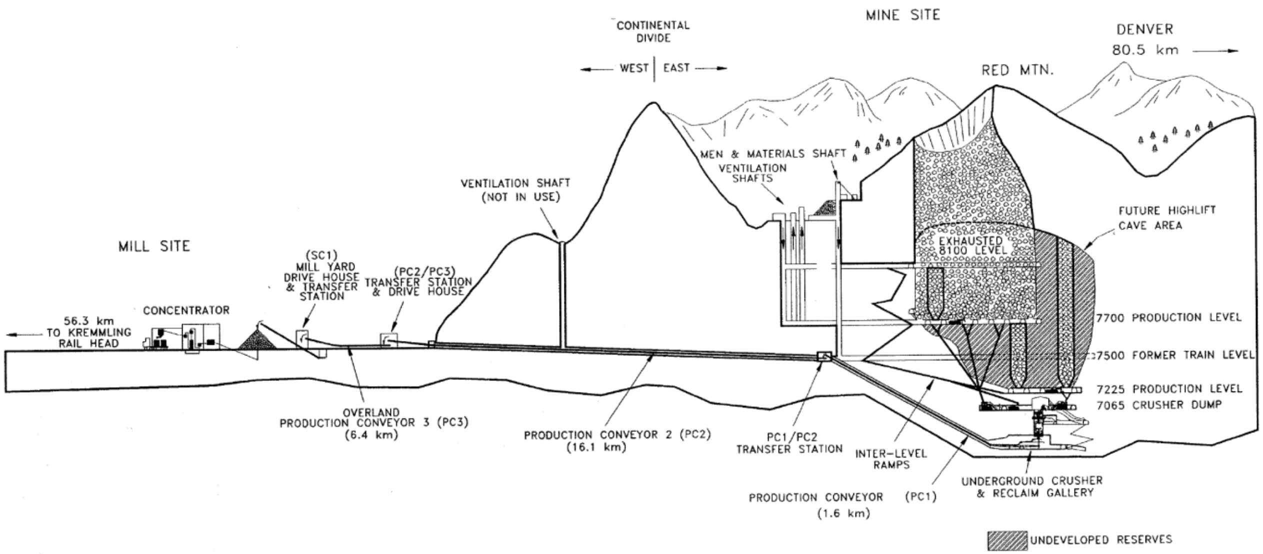
- To understand the nature of fracturing around a cave
- Use numerically based analysis for caving prediction





Hustrulid & Bullock, 2001

Hendersen Mine, 2006 (Min)

Explicit DEM Examples (6) - Rock Mass Strength – Uniaxial Strength Test (with a internal fracture)

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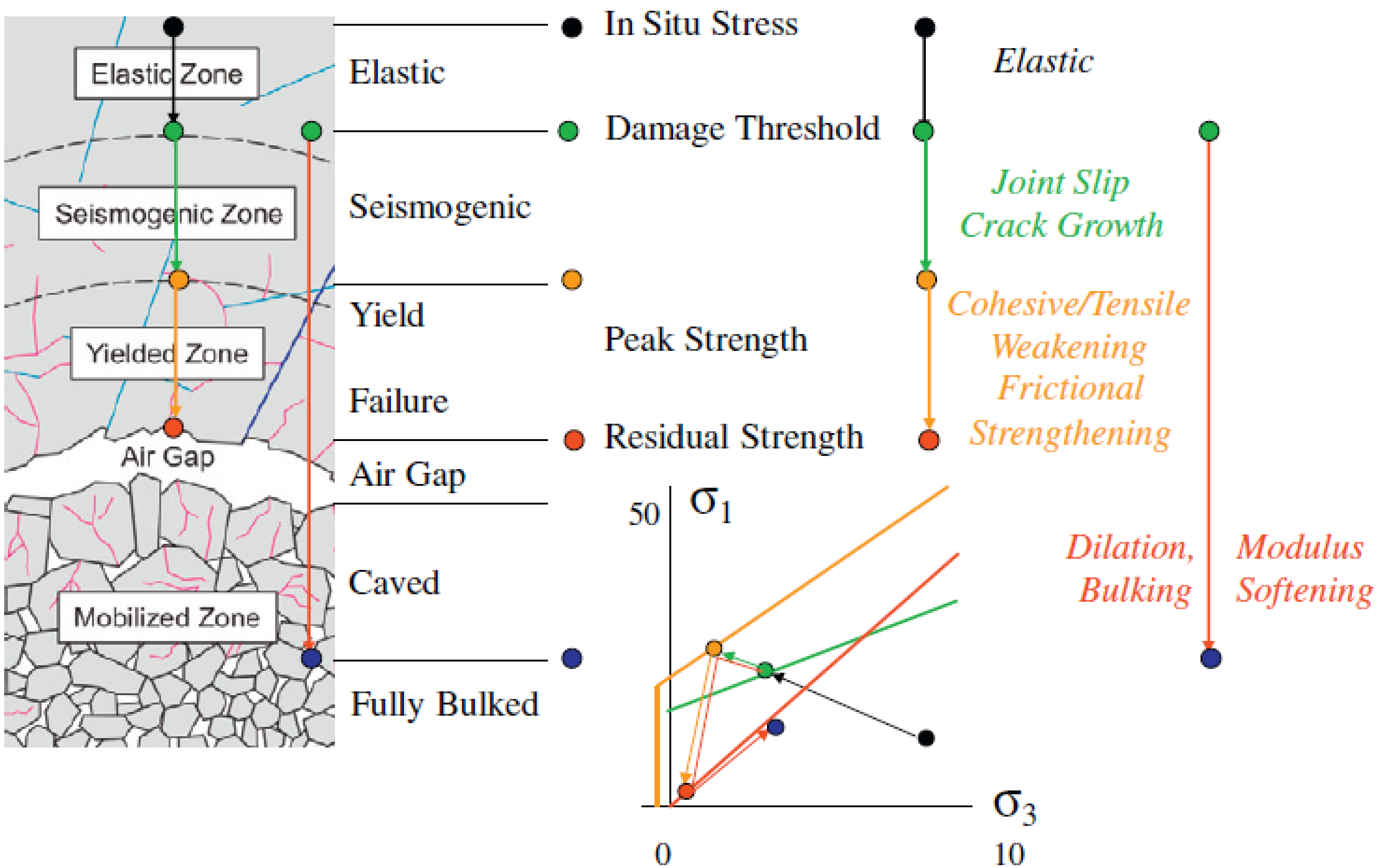
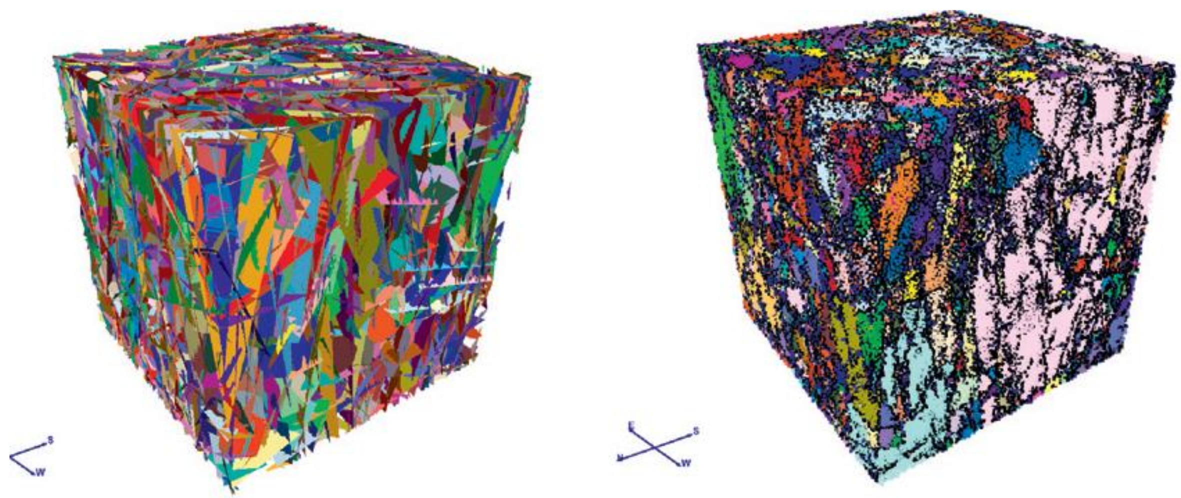
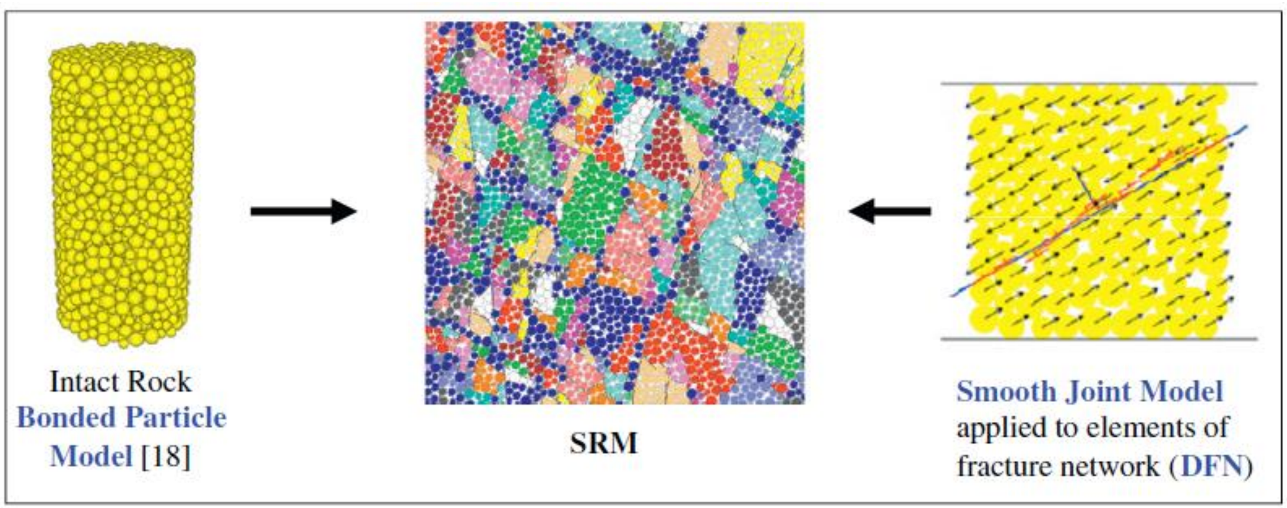


Ivars et al., 2011, IJRMMS

Explicit DEM Examples (6) - Rock Mass Strength
– Uniaxial Strength Test (with a internal fracture)

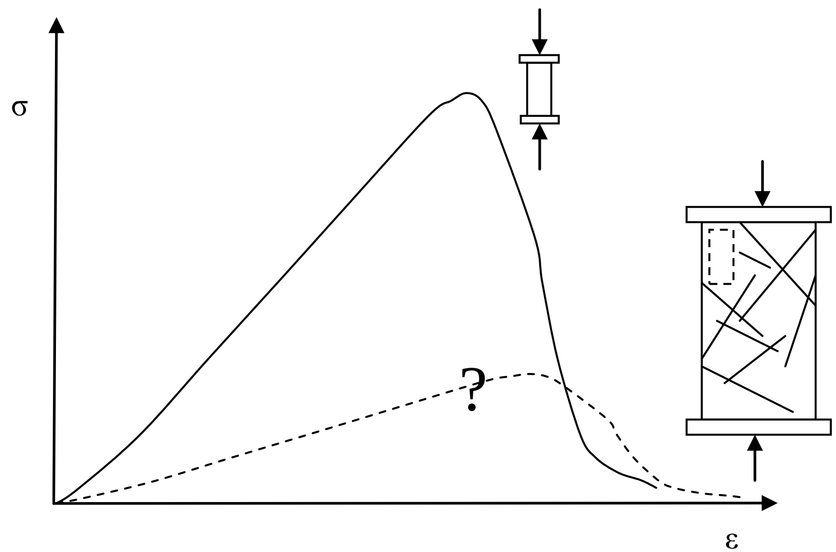


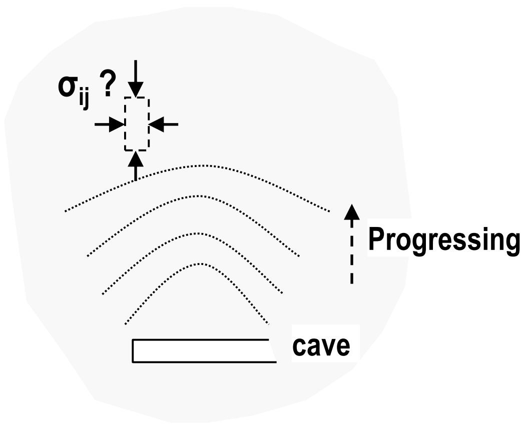


Ivars et al., 2011, IJRMMS

Explicit DEM Examples (6) - Rock Mass Strength

Motivation





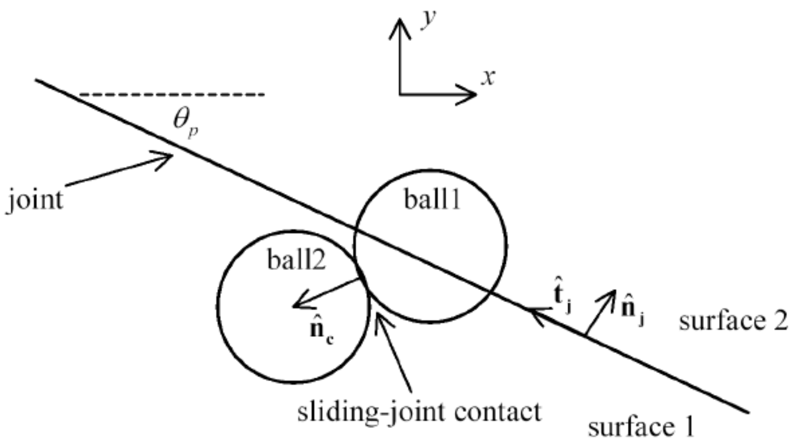
σ_c of intact rock ≠ σ_c of fractured rock mass.

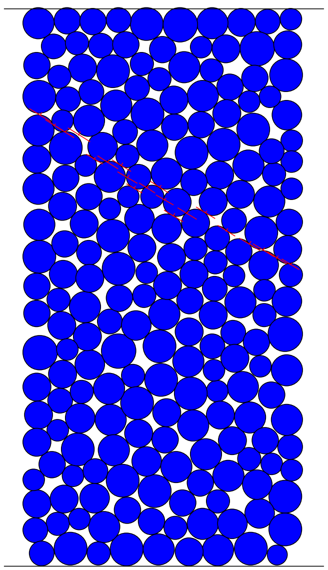
→ <u>Numerical experiment can be an alternative</u>

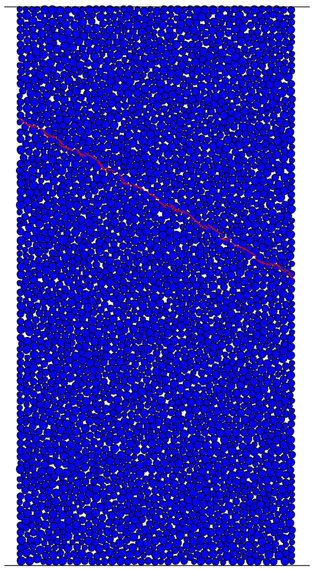
To understand the nature of fracturing around a cave

Use numerically based analysis for caving prediction

Explicit DEM Examples (6) - Rock Mass Strength
Sliding Joint Model







- Identify all contacts between balls that lie upon opposite side of a plane, remove bond
- Kn, Ks defined, Failure - Coulomb failure
- Special command (fishcall in case of PFC) that automatically detect new contact lie in joint

Explicit DEM Examples (6) - Rock Mass Strength – Uniaxial Strength Test (with a internal fracture)

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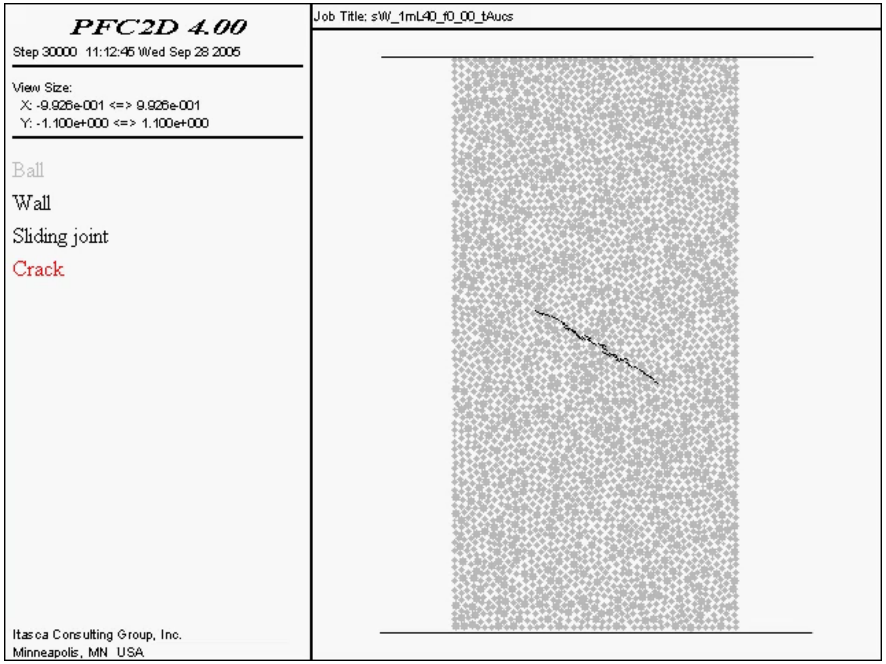
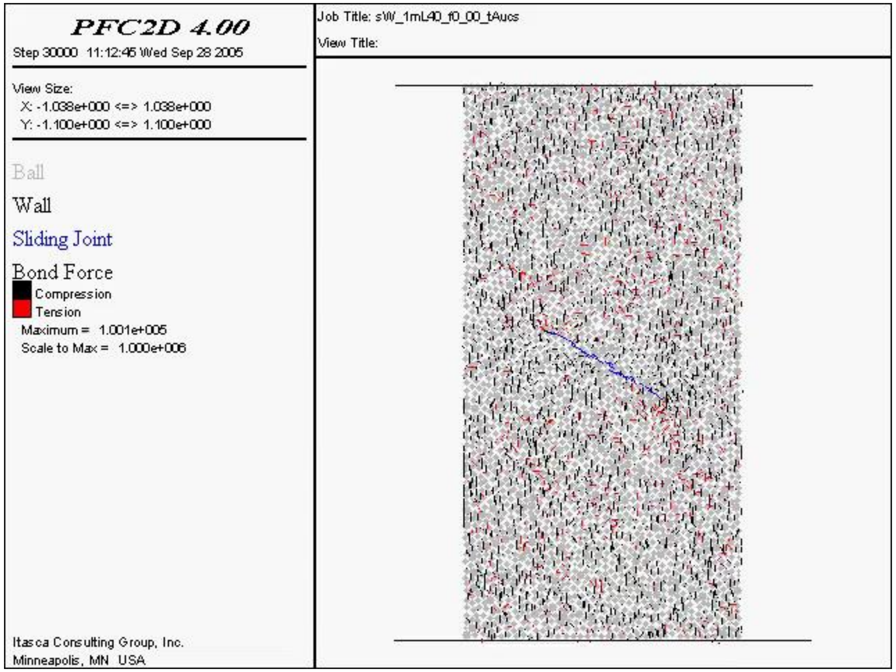


Propagation of crack



Distribution of force

Explicit DEM Examples (7)
Transversely Isotropic Rock

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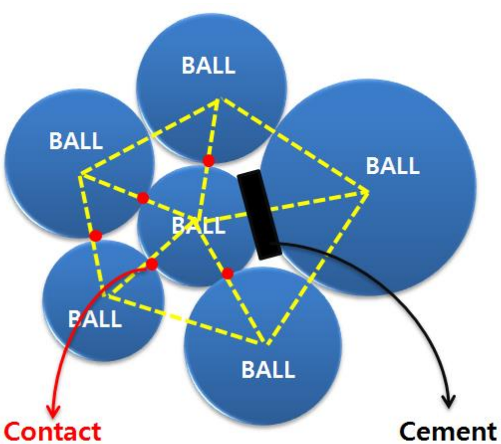
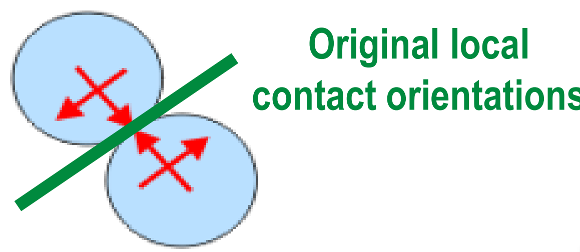
PFC^{2D} with Smooth Joint Model

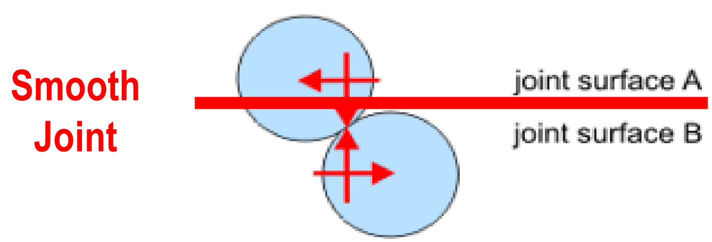
(a) Bonded Particle Model



(b-1) Standard Contact Model



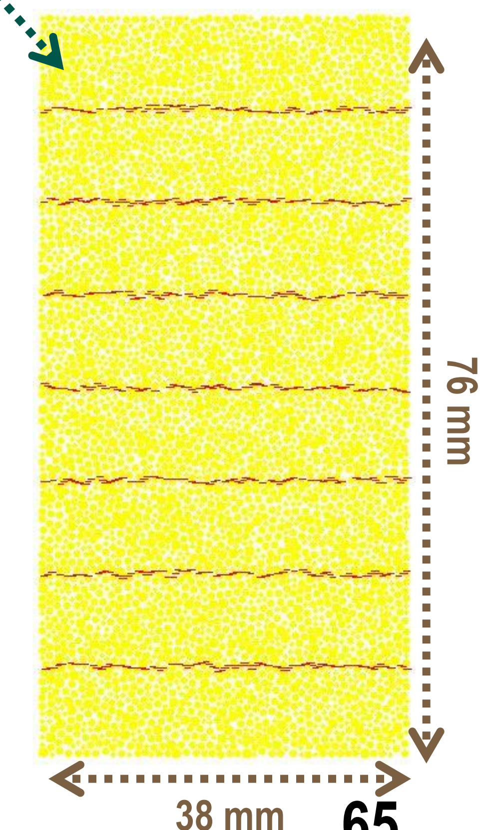
(b-2) Smooth Joint Model



(Mas Ivars *et al.*, 2011)

(c) BPM Embeds Weak Planes

15600 Particles



Explicit DEM Examples (7)
Transversely Isotropic Rock

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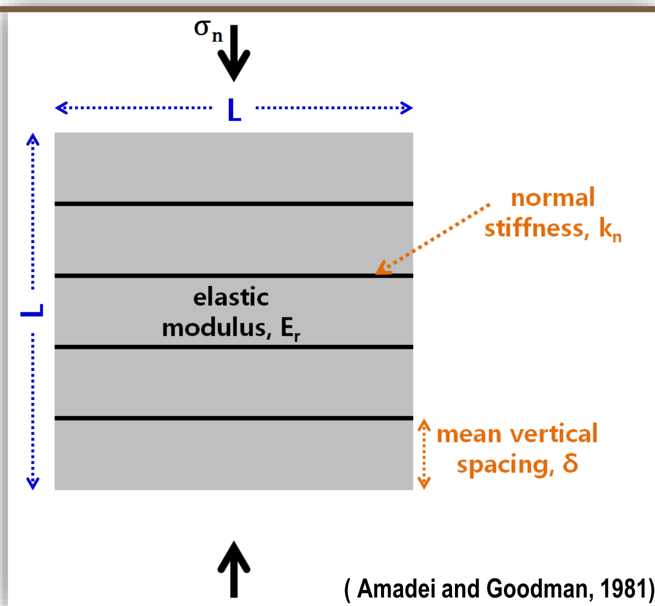
Equivalent Continuum Model



$$\frac{1}{E_e}=\left(\frac{1}{E_r}+\frac{1}{k_n\cdot\delta}\right),\qquad \frac{1}{G_e}=\left(\frac{1}{G_r}+\frac{1}{k_s\cdot\delta}\right)$$

E_e , G_e : equivalent elastic & shear modulus,

E_r, G_r : intact rock elastic & shear modulus,

k_n , k_s : normal & shear stiffness on weak planes,

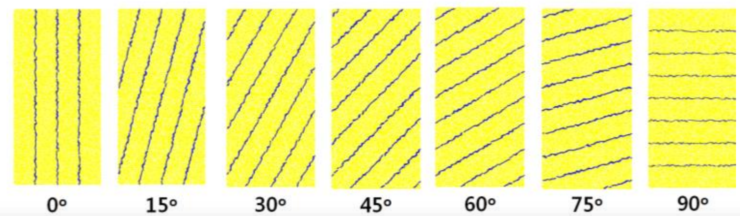
δ : mean vertical spacing

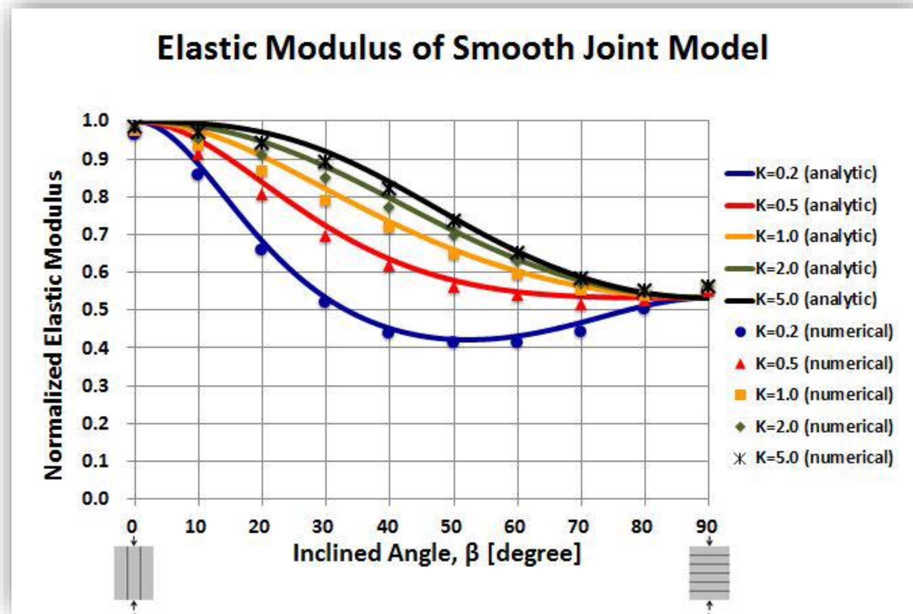
K is stiffness ratio

Tensor Transformation

$$\frac{1}{E_\beta}=\frac{1}{E_r}+\cos^2\beta\cdot\left(\frac{\cos^2\beta}{k_n\cdot\delta}+\frac{\sin^2\beta}{k_s\cdot\delta}\right)$$

(E_β varies with respect to inclination, *β*)





Explicit DEM Examples (7)
Transversely Isotropic Rock

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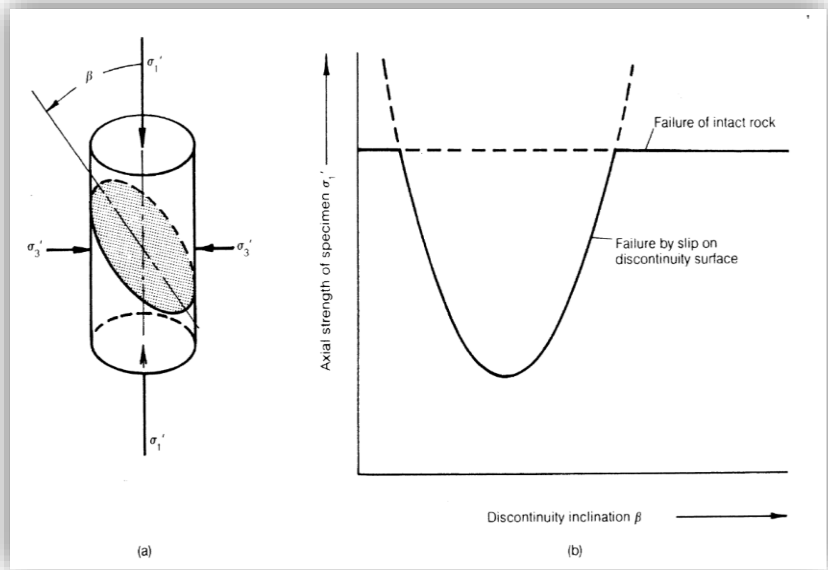
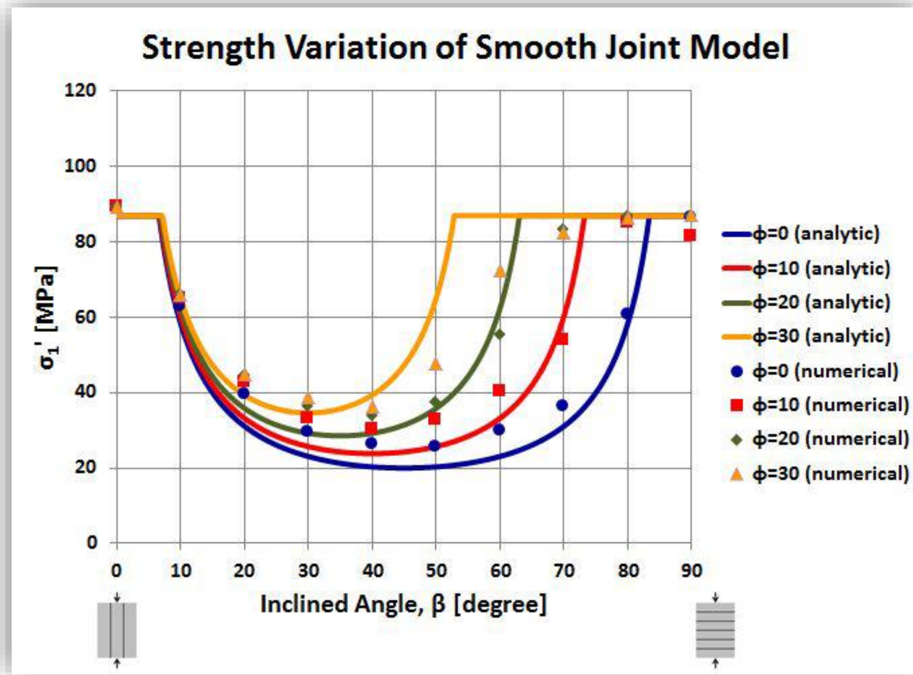
Strength Anisotropy with respect to Weak Planes



(Jaeger and Cook, 2007)



✓ **Strength of Fractured Rock**

$$\sigma_1 = \sigma_3 + \frac{2\cdot(C+\sigma_3\cdot\tan\varphi)}{(1-\tan\varphi\cdot\tan\beta)\cdot\sin 2\beta}$$

σ_1 : axial strength, σ_3 : confining stress,
C : cohesion, φ : friction angle,
β : inclination

✓ Fixed Cohesion (C) : 10 MPa

✓ Various Friction Angle (φ) : 0º – 30º

✓ Mechanical Behaviors -> Smoothly Change

Transversely Isotropic Rock

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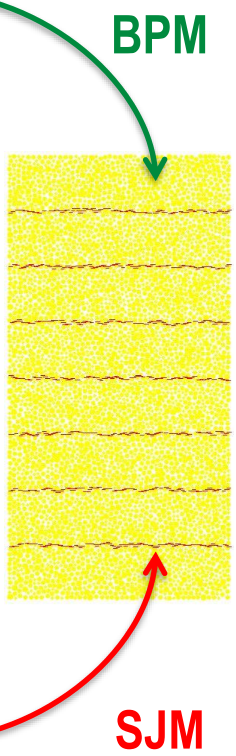
(a) Step 1 (Intact Rock Part)

Microproperties of Bonded Particle Model	
Grain	Cement
Elastic modulus = 38 Gpa	Elastic modulus = 38 Gpa
Stiffness ratio = 3.5	Stiffness Ratio = 3.5
Friction coefficient = 0.839	Tensile stress = 75 Mpa

(b) Step 2 (Weak Plane Part)

Microproperties of Smooth Joint Model	
Normal stiffness = 33700 Gpa/m	Dilation angle = 0°
Shear stiffness = 960 Gpa/m	Tensile strength = 3 Mpa
Friction coefficient = 0.364 (20°)	Cohesion = 15



Explicit DEM Examples (7)
Transversely Isotropic Rock

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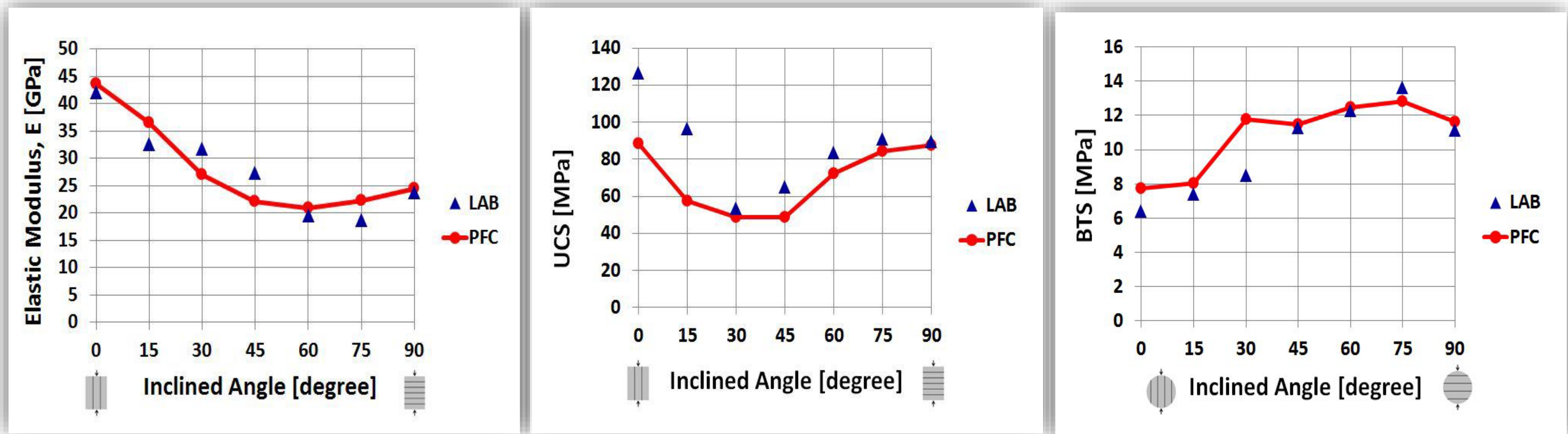
Boryeong Shale vs. Numerical Model

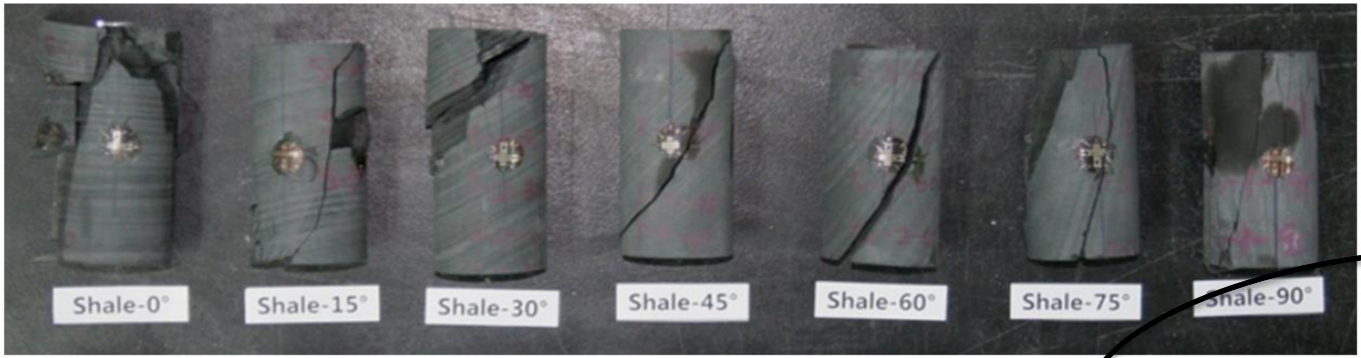
(a) Elastic Modulus (b) UCS (c) BTS



(Cho *et al.*, 2012)

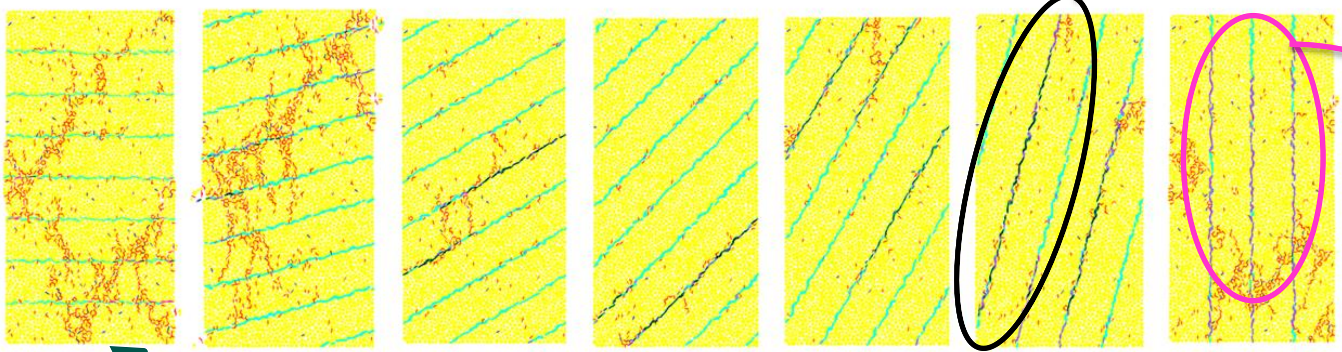
- ✓ ▲ : Lab Experiments / ━●━ : Numerical Results
- ✓ E, UCS and BTS with respect to the smooth joints orientation
- ✓ Capture the overall trend of anisotropic mechanical behaviors

Explicit DEM Examples (7)
Transversely Isotropic Rock



(Cho *et al.*, 2012)

Shear Failure along the Weak Planes



15600 Particles

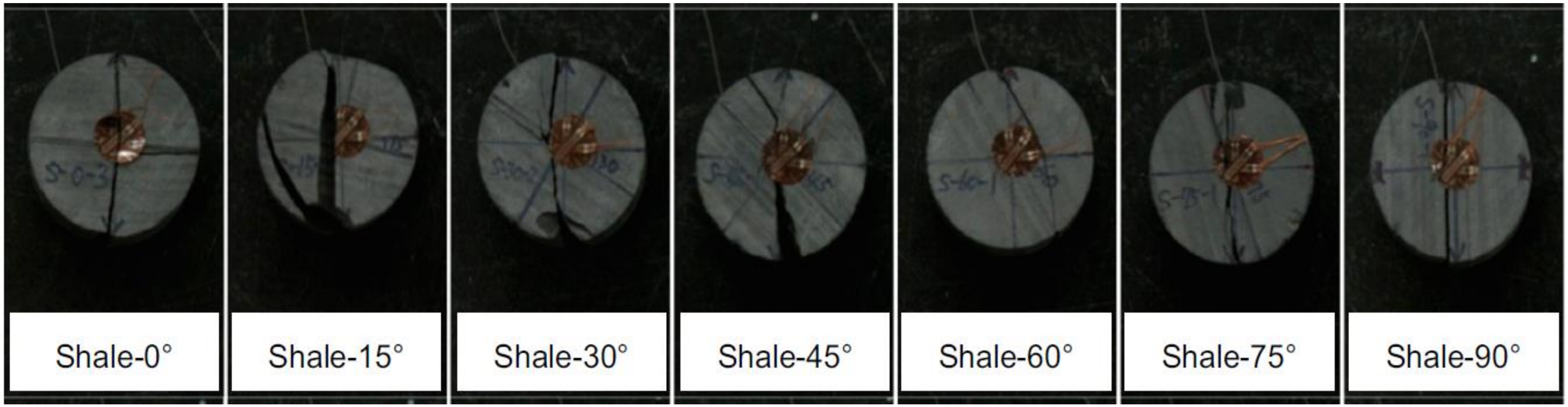
Tensile Failure along the Weak Planes

- ✓ Red: Tensile Cracks on Contact/Parallel Bond
- ✓ Blue: Shear Cracks on Contact/Parallel Bond
- ✓ Magenta: Tensile Cracks on Smooth Joint
- ✓ Black: Shear Cracks on Smooth Joint

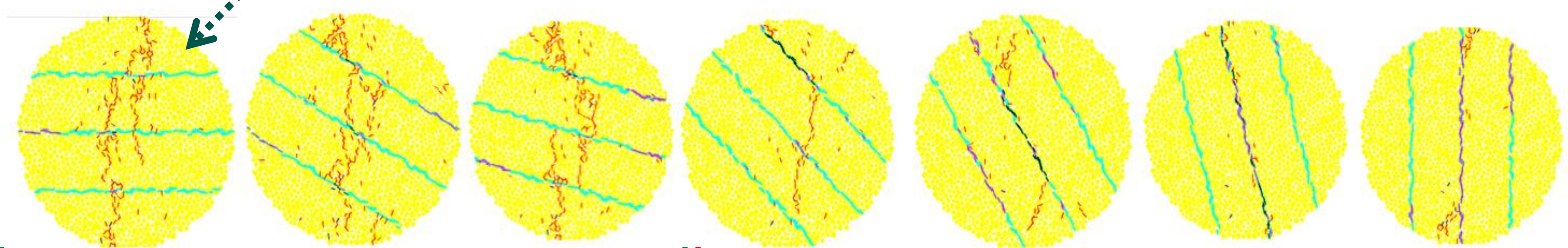
Fracture Propagation & Failure Mechanism

Explicit DEM Examples (7)
Transversely Isotropic Rock

(Lab Results Obtained from Cho *et al.*, 2012)



6158 Particles

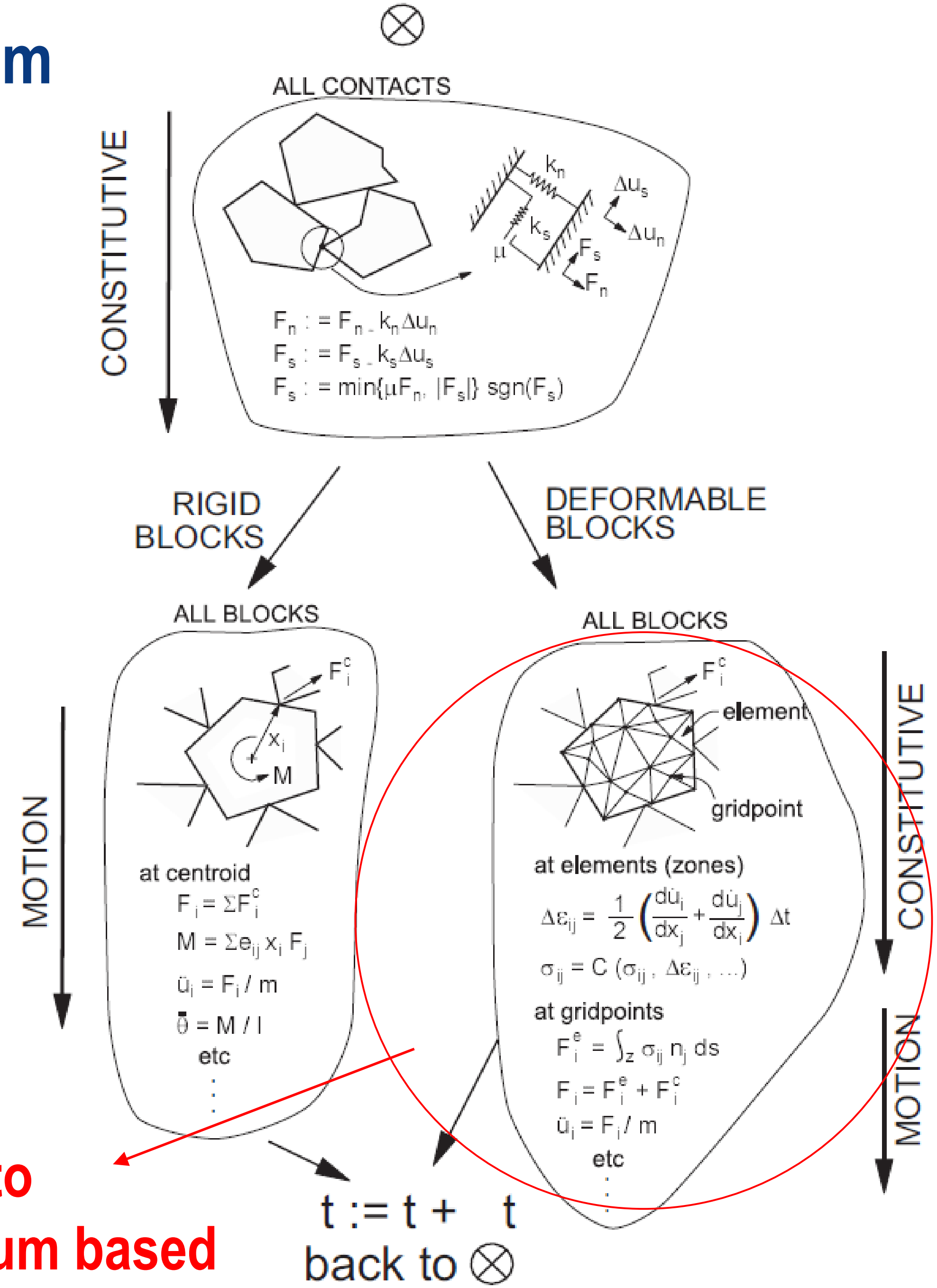


Dominated by Intact Rock (BPM)

Dominated by Layers (Smooth Joint Model)

Explicit DEM in Blocky System
Introduction

- Explicit DEM in blocky system (such as UDEC) essentially have the same calculation scheme – i.e., FDM – for both contact and blocks



Similar to continuum based explicit FDM

UDEC manual, 2011

Explicit DEM in Blocky System
Contact

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Identification of contact character between 2 blocks

We need to know:

1) type of contact (e.g. corner-to-corner, corner-to-edge, etc.)
2) direction of normal to sliding direction
3) gap between blocks, or contact overlap

Explicit DEM in Blocky System
Contact

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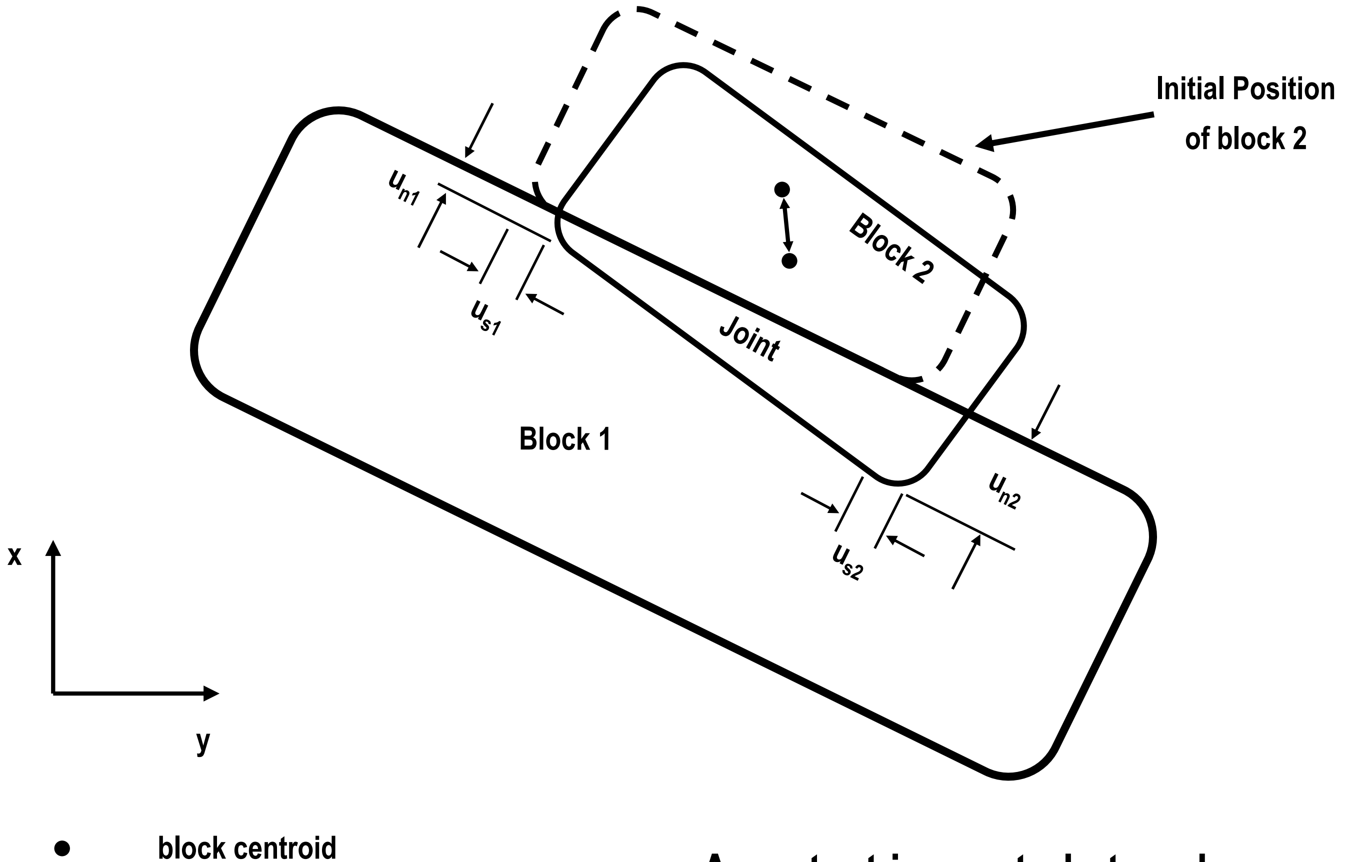
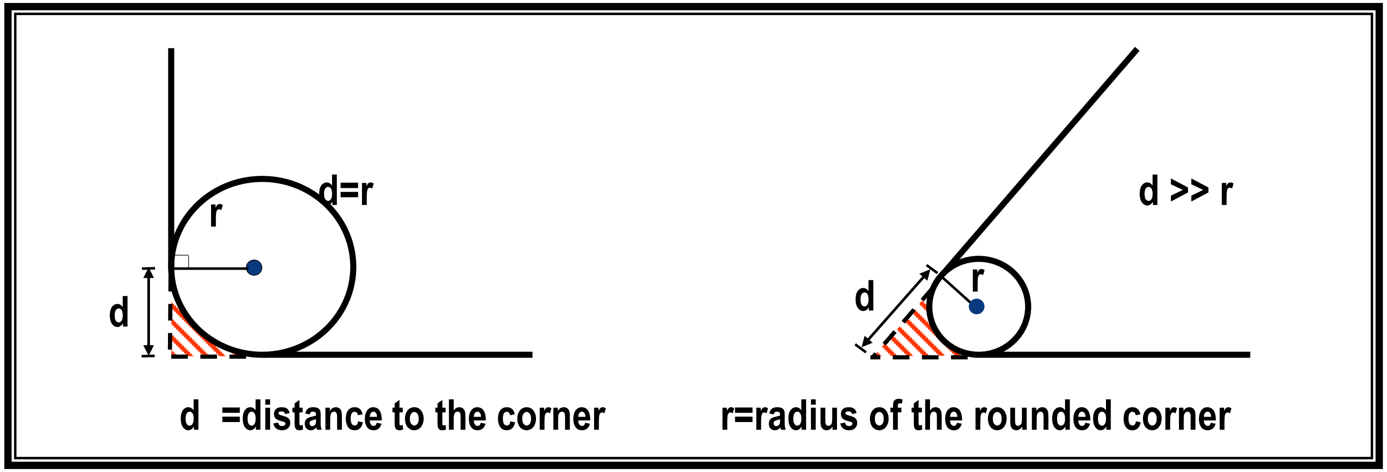
Contact Between two Rigid Blocks

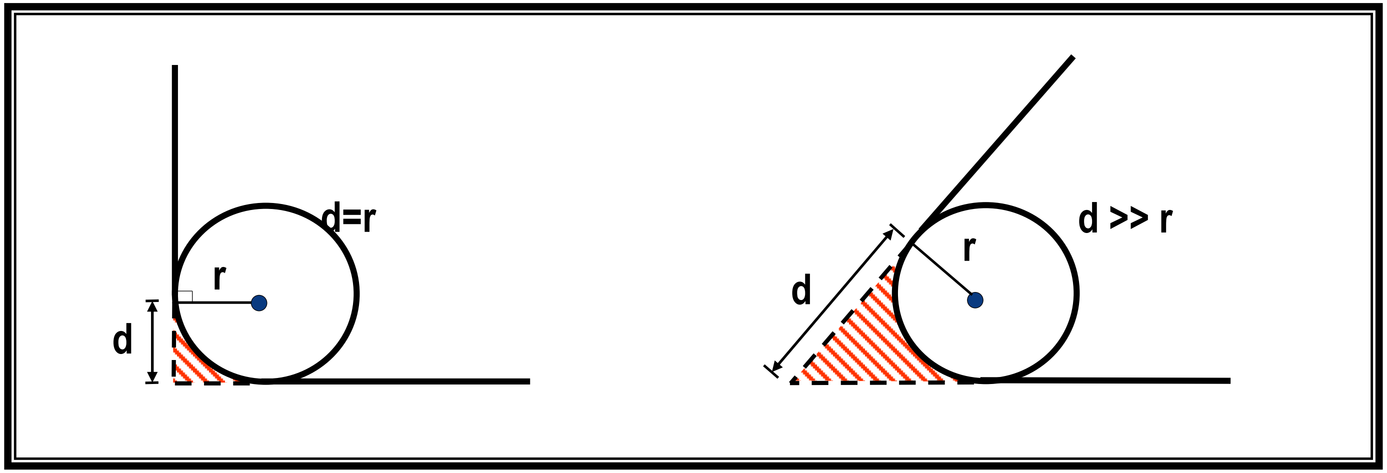


A contact is created at each corner interacting with a corner or edge of an opposing block.

Explicit DEM in Blocky System
Corner handling

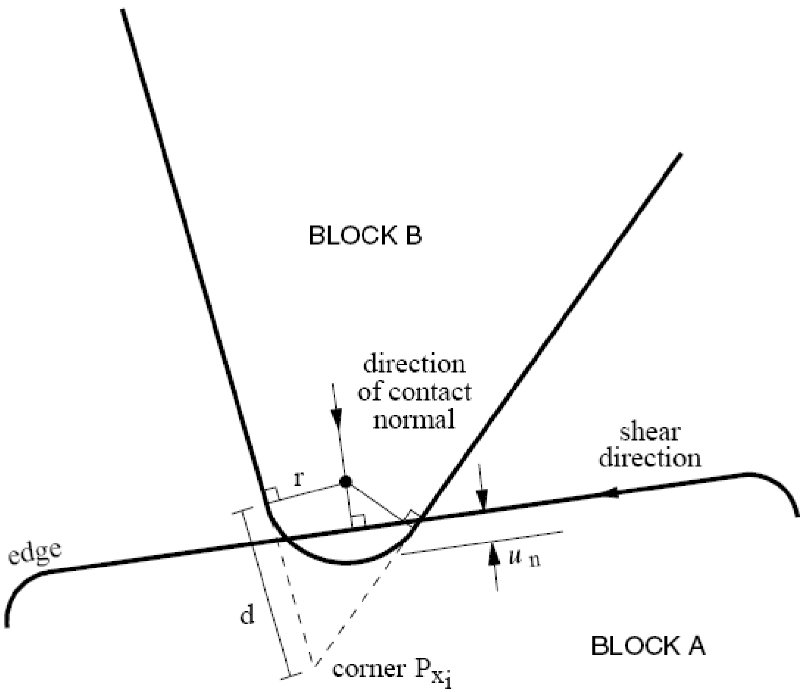


Corner rounding scheme with constant length d

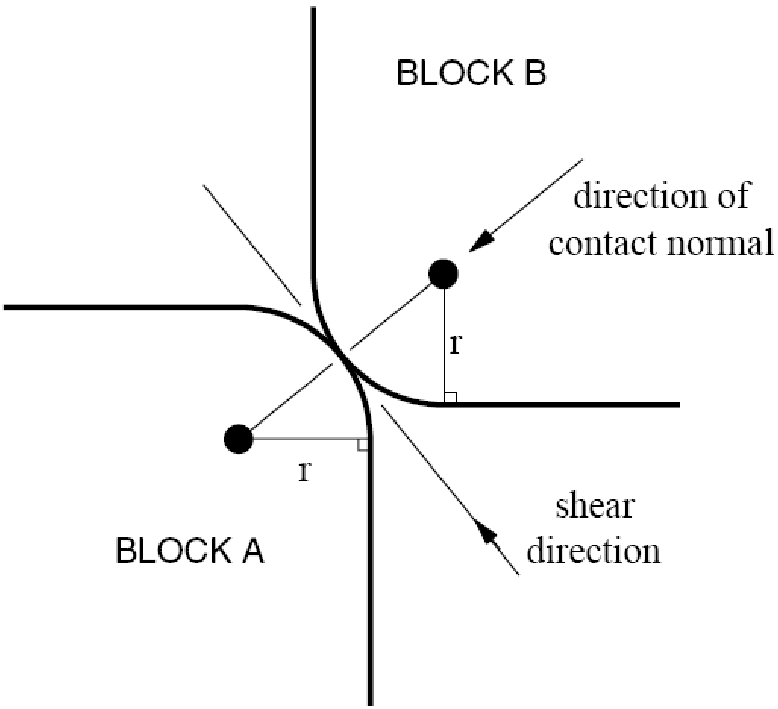


Corner rounding scheme with constant radius r, showing that small angles in the corner leads to large distances d

Explicit DEM in blocky system
Contact



Rounded corner-to-edge contact

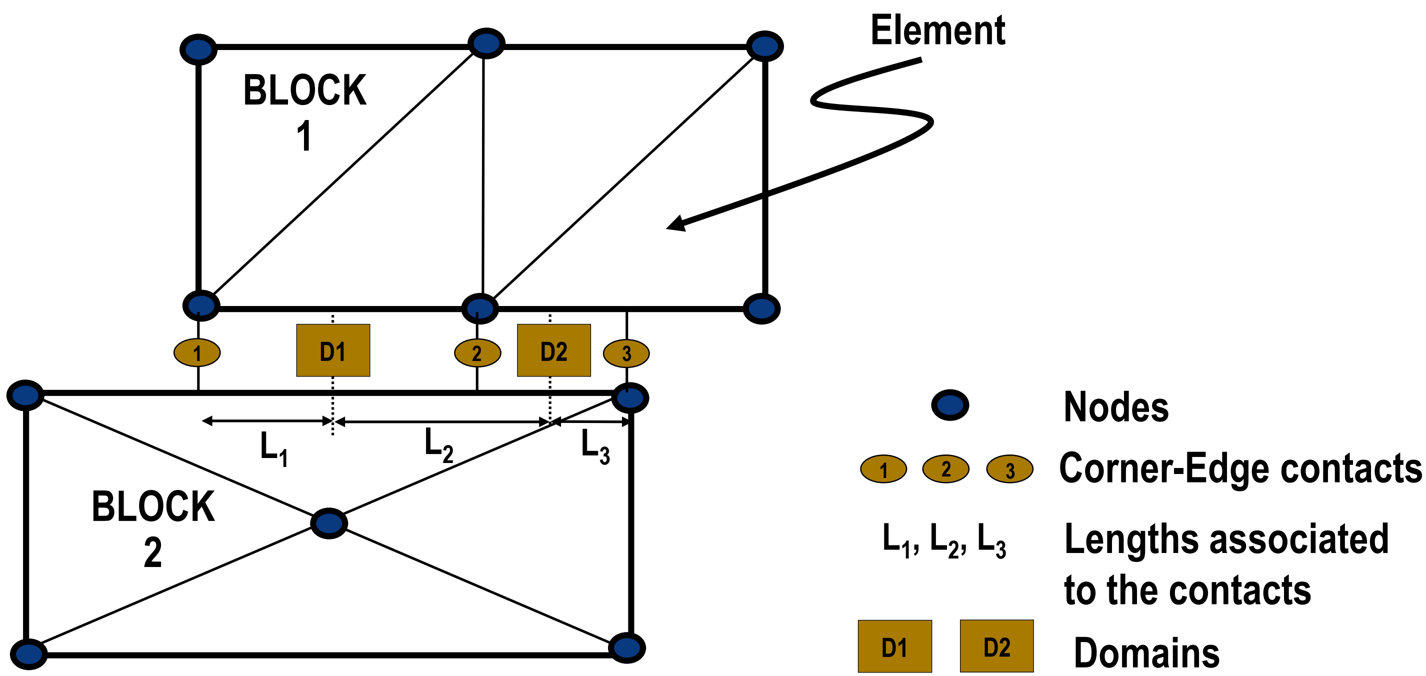


Rounded corner-to-corner contact

Definition of contact normal

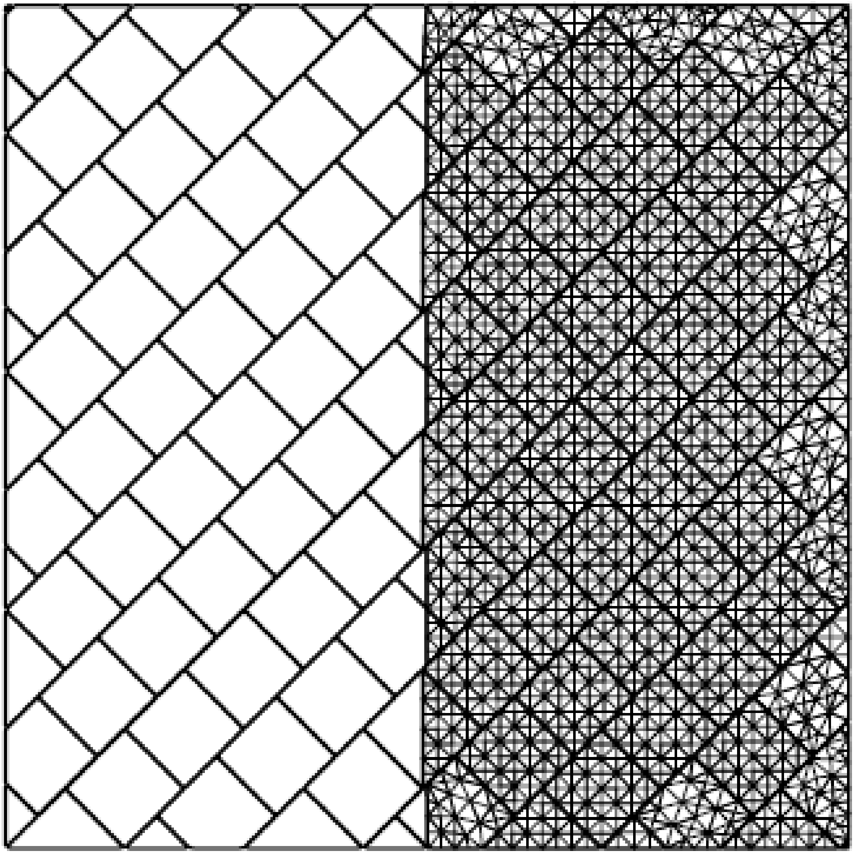
Explicit DEM in blocky system
Contacts and Domains

Contacts and Domains between Two Deformable Blocks



Explicit DEM in blocky system
Blocks

- **Two formulations for Solid body mechanics**
  - **- Rigid body translation and rotation**
  - **- Deformable body mechanics**



(a) distinct element blocks (b) zoning within blocks

Explicit DEM in blocky system
Joint Model

- Geometry
  - joint set assigned with orientation/length/spacing…
  - complete Discrete Fracture Network (DFN) not implemented but easily combined via FISH or separate program
  - dead-end joint cannot be generated
  - continuum model → use of fictitious joint with high stiffness values (UDEC) or glue (3DEC)

- Constitutive model
  - Linear model $F = K_n \delta_n$ $F = K_s \delta_s$
  - Nonlinear model
    - step-wise nonlinear
    - continuously yielding model
    - Barton-Bandis model

Explicit DEM in blocky system
Joint Model (Barton-Bandis model)

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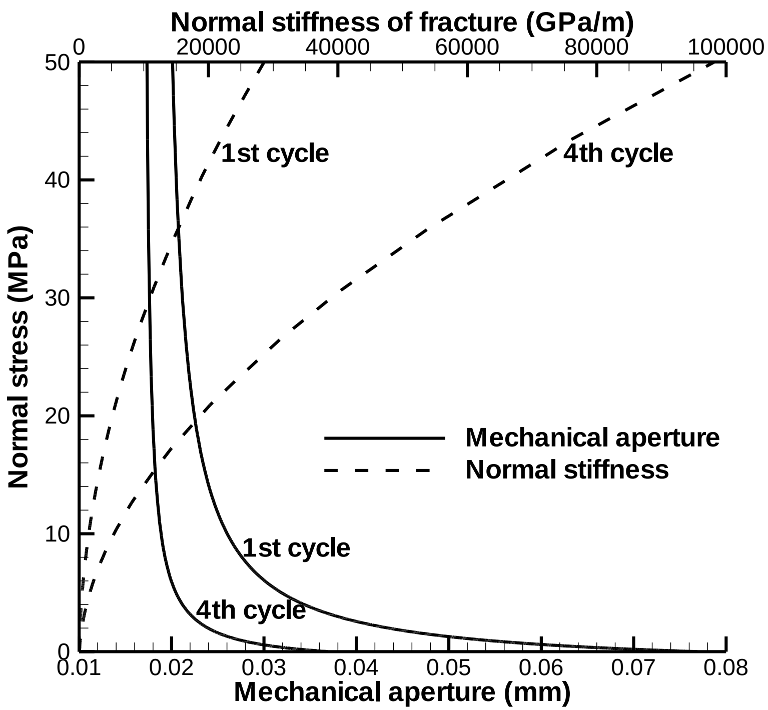

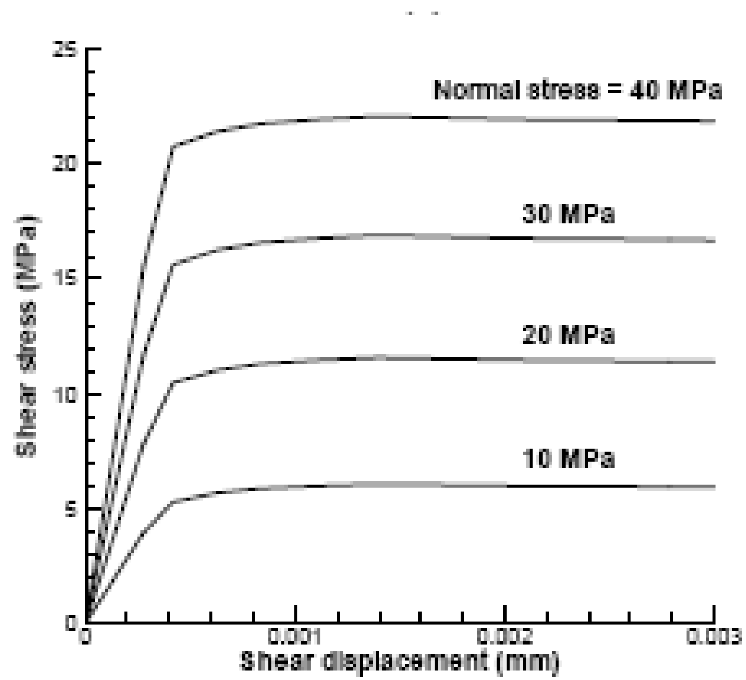

- Normal - nonlinear
- Shear - Affected by the magnitude of normal stress and JRC

(Min & Jing, 2004)

Explicit DEM in blocky system
Joint Model (Barton-Bandis model)

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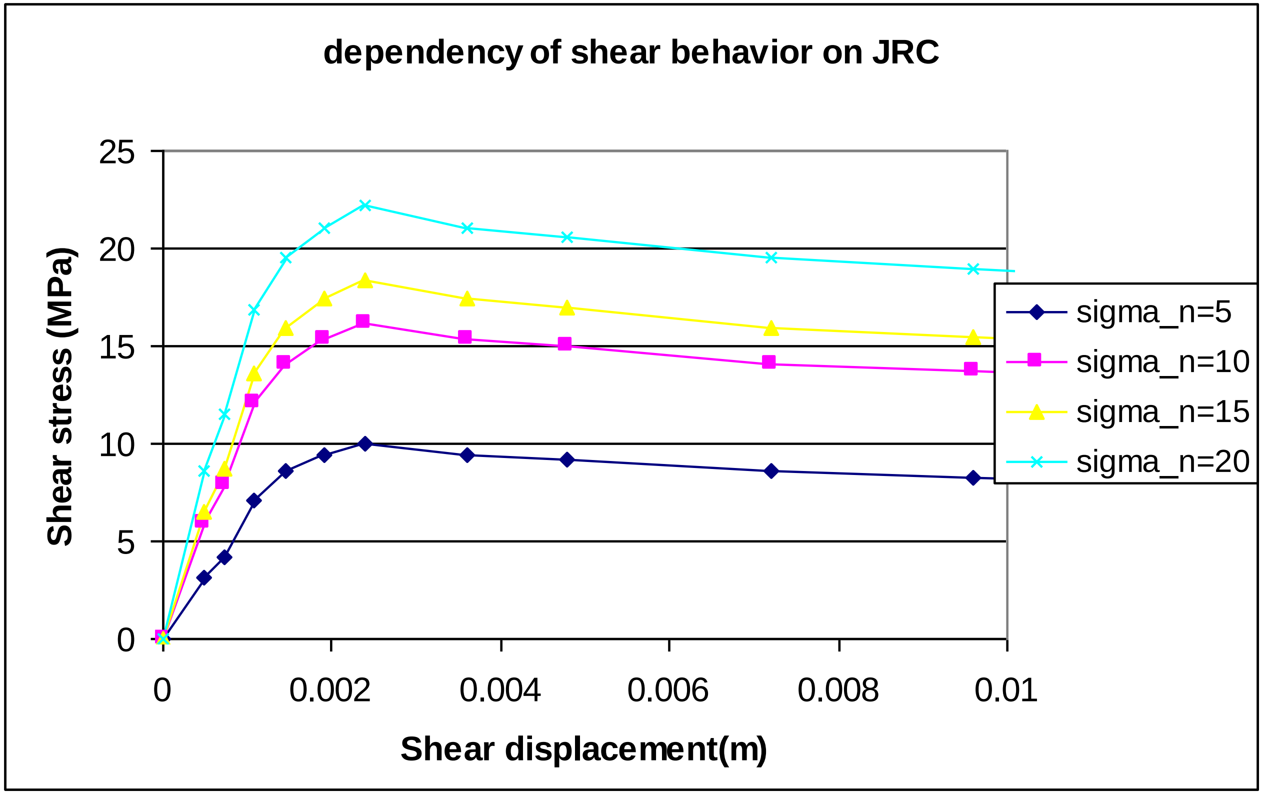
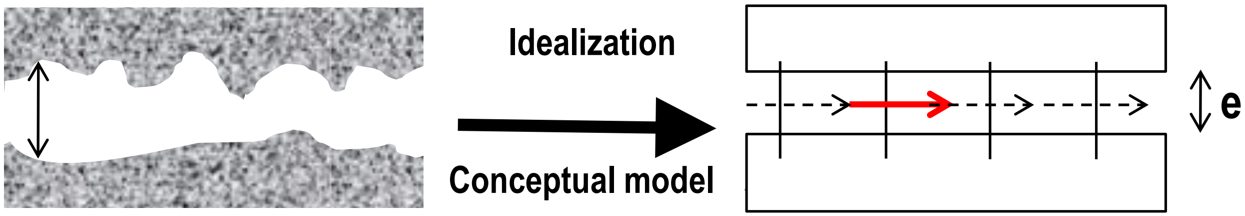


- More prominent peak with the increase of JRC

Explicit DEM in blocky system
Fluid flow



Real rock fracture

Idealized rock fracture

$$p = p_0 + K_w Q \frac{\Delta t}{V} - K_w \frac{\Delta V}{V_m}$$

$$Q = -\frac{e^3}{12\mu}\frac{\partial p}{\partial x}$$

p_0 :domain pressure in the preceding timestep

Q :sum of flowrate into the domain

K_w :bulk modulus of fluid

$\Delta V = V - V_0$

$V_m = (V + V_0)/2$

$$\Delta t_f = \min\left[\frac{V}{K_w \sum Q_i / \Delta p_i}\right]$$

For edge-to-edge contact

Explicit DEM in blocky system
Fluid flow and coupled hydromechanical calculation

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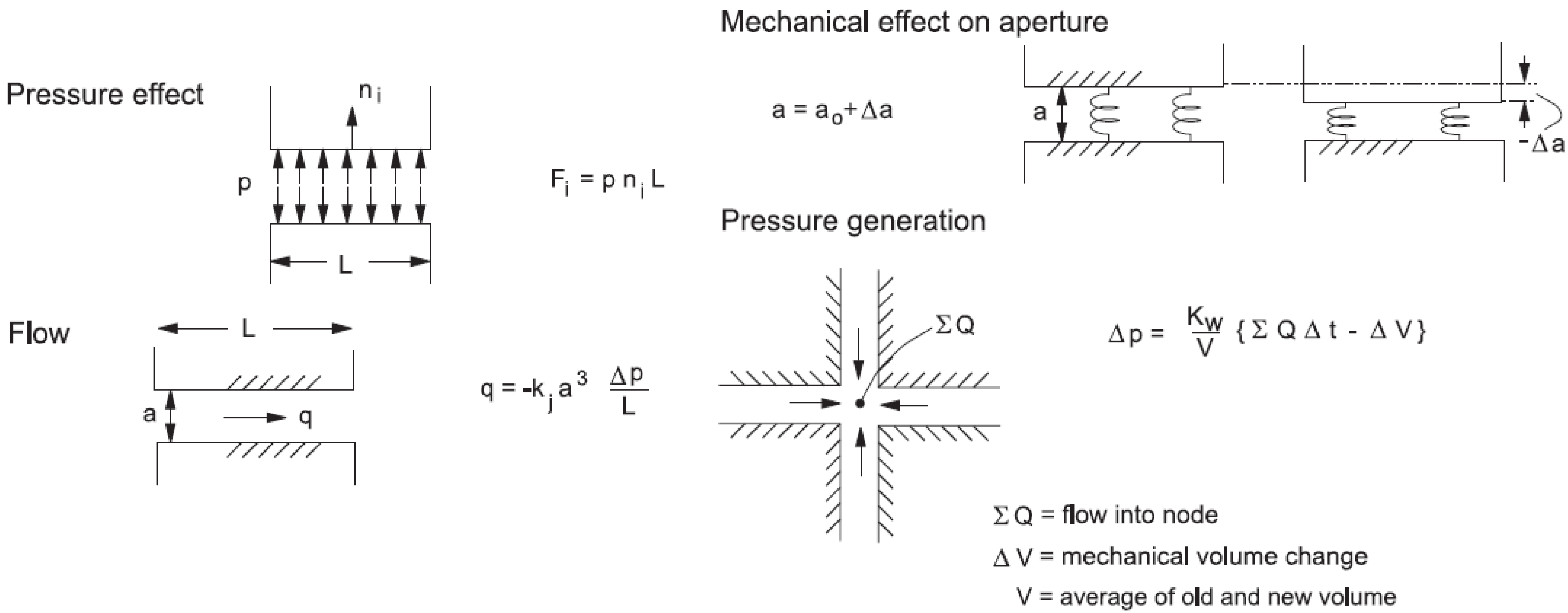

From UDEC manual

Explicit DEM in blocky system
Fluid flow and coupled hydromechanical calculation

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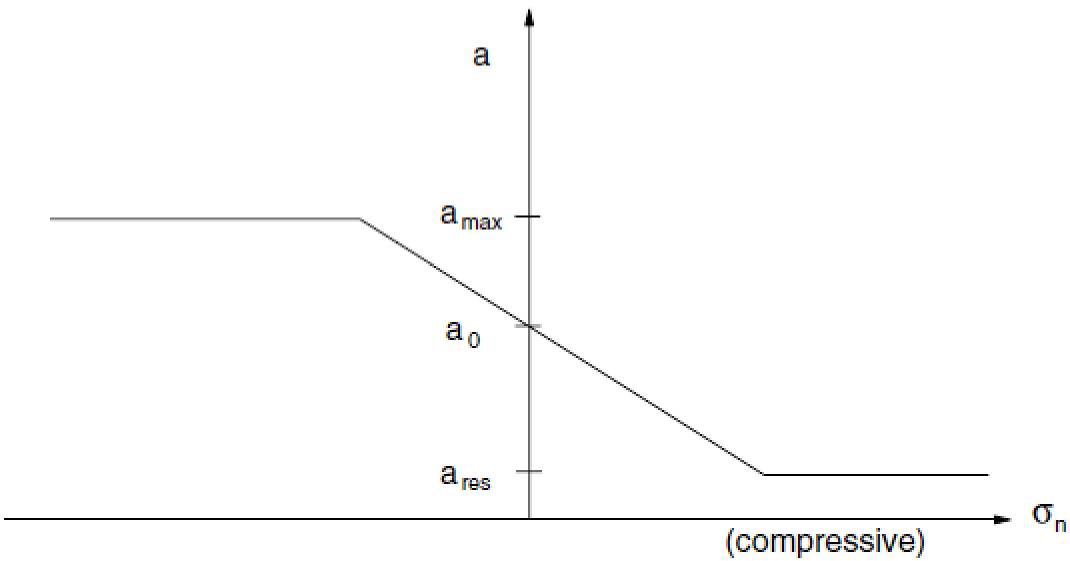
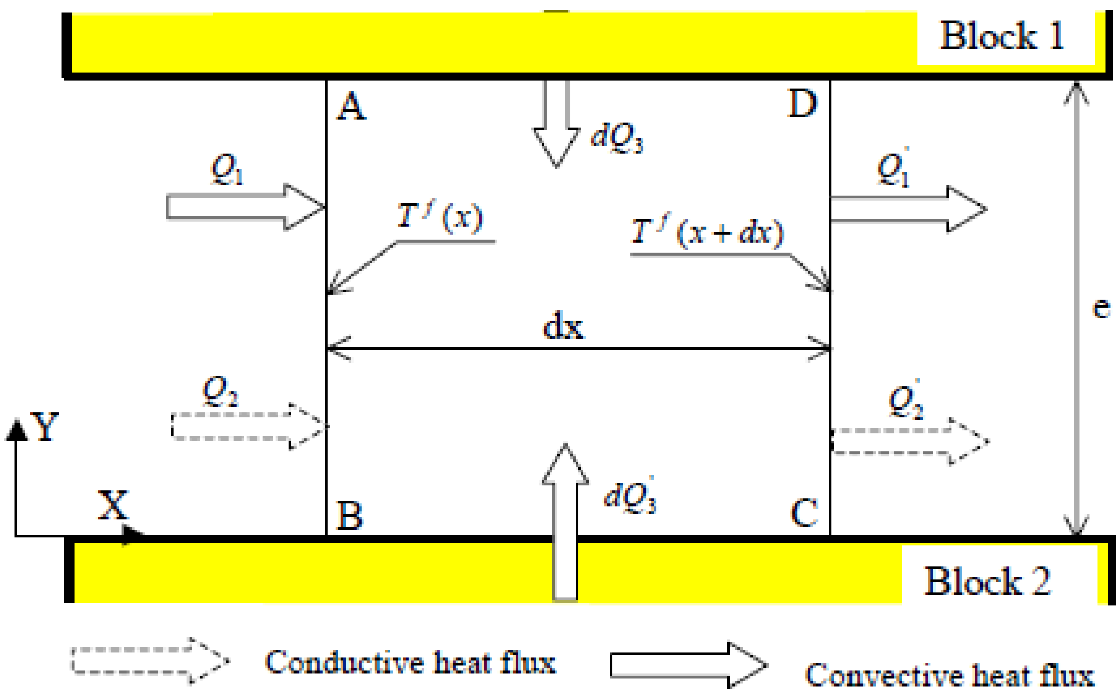
- Relation between hydraulic aperture, a, and joint normal stress in UDEC (from UDEC manual)



$$a = a_0 + u_n$$

Explicit DEM in blocky system
Convective Heat Transfer



Jing and Stephansson, 2007

Explicit DEM in Blocky System
Examples

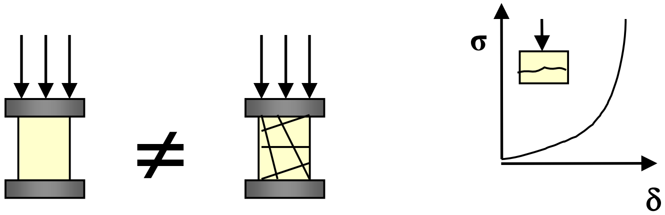
- 2D Explicit DEM in blocky system
  - Stability analysis of fractured rock for low and intermediate level nuclear waste underground repository
  - determination of rock mass elastic properties (E, v)
  (Min & Jing, 2003), (Min et al., 2005)

- 3D Explicit DEM in blocky system
  - effect of fracture zones on stress distribution (Min, 2009)

- Coupled Hydromechanical Analysis
  - effect of stress on permeability (Min et al., 2004)

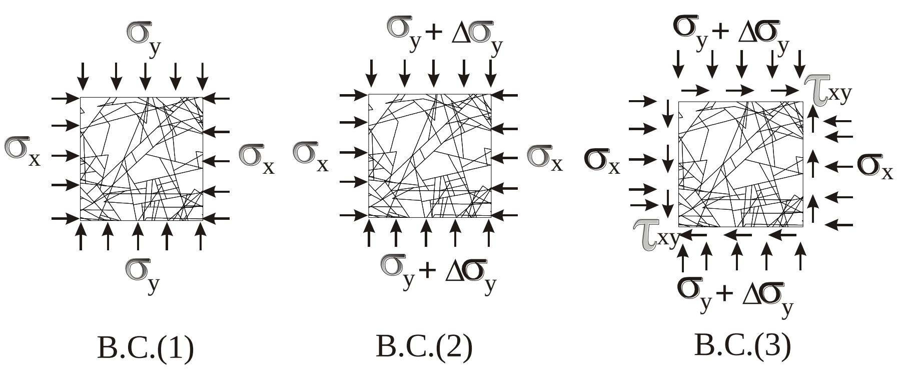
Explicit DEM in Blocky System

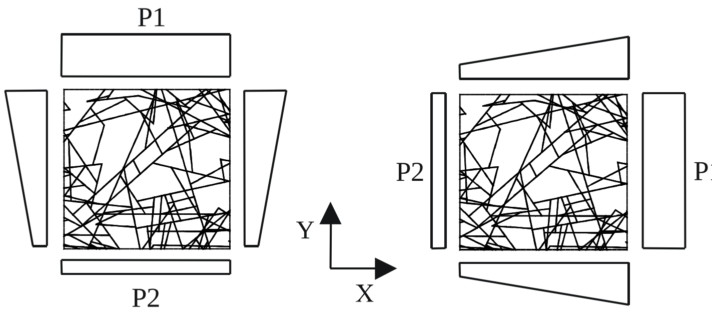
Example - Rock mass E and ν (Min and Jing, 2003)



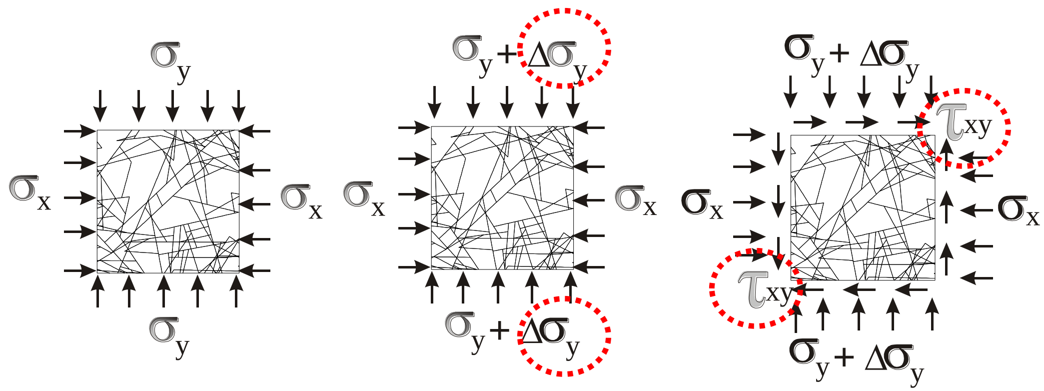
- In situ experiments is difficult – scale, boundary condition, cost
- <u>*Numerical Experiments*</u> can be alternative and effective to handle all these difficult questions as long as……
- DFN-DEM approach

{ Discrete Fracture Network (DFN) – geometry of fractured rock mass
  Distinct Element Method (DEM) – solution technique





Example - Rock Mass Determination
Methodology



B.C.(1) B.C.(2) B.C.(3)

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl}$$

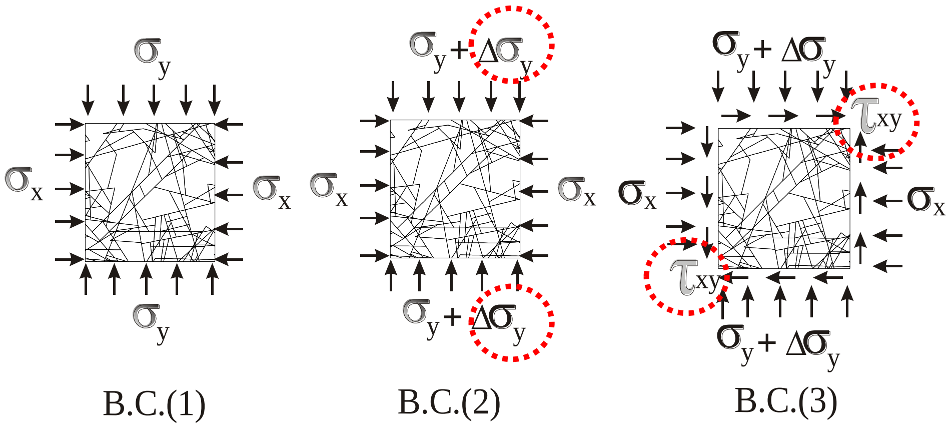
$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \lambda_{yz} \\ \lambda_{xz} \\ \lambda_{xy} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{pmatrix}$$

- Three linearly independent B.C.s and consider full anisotropy
- This overcomes the difficulty in Stietel at al (1996)

Example - Rock Mass Determination
Methodology

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl}$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{pmatrix}$$



6 Boundary conditions

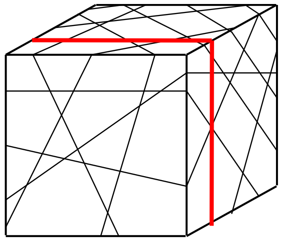
$$\begin{pmatrix} \varepsilon_{xx}^{(1)} & \varepsilon_{xx}^{(2)} & \varepsilon_{xx}^{(3)} & \varepsilon_{xx}^{(4)} & \varepsilon_{xx}^{(5)} & \varepsilon_{xx}^{(6)} \\ \varepsilon_{yy}^{(1)} & \varepsilon_{yy}^{(2)} & \varepsilon_{yy}^{(3)} & \varepsilon_{yy}^{(4)} & \varepsilon_{yy}^{(5)} & \varepsilon_{yy}^{(6)} \\ \varepsilon_{zz}^{(1)} & \varepsilon_{zz}^{(2)} & \varepsilon_{zz}^{(3)} & \varepsilon_{zz}^{(4)} & \varepsilon_{zz}^{(5)} & \varepsilon_{zz}^{(6)} \\ \gamma_{yz}^{(1)} & \gamma_{yz}^{(2)} & \gamma_{yz}^{(3)} & \gamma_{yz}^{(4)} & \gamma_{yz}^{(5)} & \gamma_{yz}^{(6)} \\ \gamma_{zx}^{(1)} & \gamma_{zx}^{(2)} & \gamma_{zx}^{(3)} & \gamma_{zx}^{(4)} & \gamma_{zx}^{(5)} & \gamma_{zx}^{(6)} \\ \gamma_{xy}^{(1)} & \gamma_{xy}^{(2)} & \gamma_{xy}^{(3)} & \gamma_{xy}^{(4)} & \gamma_{xy}^{(5)} & \gamma_{xy}^{(6)} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{pmatrix} \begin{pmatrix} \sigma_{xx}^{(1)} & \sigma_{xx}^{(2)} & \sigma_{xx}^{(3)} & \sigma_{xx}^{(4)} & \sigma_{xx}^{(5)} & \sigma_{xx}^{(6)} \\ \sigma_{yy}^{(1)} & \sigma_{yy}^{(2)} & \sigma_{yy}^{(3)} & \sigma_{yy}^{(4)} & \sigma_{yy}^{(5)} & \sigma_{yy}^{(6)} \\ \sigma_{zz}^{(1)} & \sigma_{zz}^{(2)} & \sigma_{zz}^{(3)} & \sigma_{zz}^{(4)} & \sigma_{zz}^{(5)} & \sigma_{zz}^{(6)} \\ \sigma_{yz}^{(1)} & \sigma_{yz}^{(2)} & \sigma_{yz}^{(3)} & \sigma_{yz}^{(4)} & \sigma_{yz}^{(5)} & \sigma_{yz}^{(6)} \\ \sigma_{zx}^{(1)} & \sigma_{zx}^{(2)} & \sigma_{zx}^{(3)} & \sigma_{zx}^{(4)} & \sigma_{zx}^{(5)} & \sigma_{zx}^{(6)} \\ \sigma_{xy}^{(1)} & \sigma_{xy}^{(2)} & \sigma_{xy}^{(3)} & \sigma_{xy}^{(4)} & \sigma_{xy}^{(5)} & \sigma_{xy}^{(6)} \end{pmatrix}$$

- 6 linearly independent B.C. – 3D
- 3 linearly independent B.C. – 2D

Example - Rock Mass Determination
Methodology

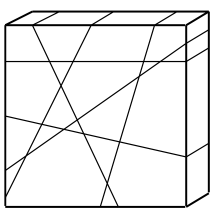
$$\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}S_{11} & S_{12} & S_{13} & S_{16}\\ S_{21} & S_{22} & S_{23} & S_{26}\\ S_{31} & S_{32} & S_{33} & S_{36}\\ S_{61} & S_{62} & S_{63} & S_{66}\end{pmatrix}\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\end{pmatrix}$$

$$\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}S_{11} & S_{12} & S_{16}\\ S_{21} & S_{22} & S_{26}\\ S_{61} & S_{62} & S_{66}\end{pmatrix}\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{pmatrix}+\begin{pmatrix}S_{13}\\ S_{23}\\ S_{63}\end{pmatrix}\left(\sigma_{zz}\right)$$



3 Boundary conditions

2D DFN



$$\begin{pmatrix}\varepsilon_{xx}^{(1)} & \varepsilon_{xx}^{(2)} & \varepsilon_{xx}^{(3)}\\ \varepsilon_{yy}^{(1)} & \varepsilon_{yy}^{(2)} & \varepsilon_{yy}^{(3)}\\ \gamma_{xy}^{(1)} & \gamma_{xy}^{(2)} & \gamma_{xy}^{(3)}\end{pmatrix}-\begin{pmatrix}S_{13}\\ S_{23}\\ S_{63}\end{pmatrix}\begin{pmatrix}\sigma_{zz}^{(1)} & \sigma_{zz}^{(2)} & \sigma_{zz}^{(2)}\end{pmatrix}=\begin{pmatrix}S_{11} & S_{12} & S_{16}\\ S_{21} & S_{22} & S_{26}\\ S_{61} & S_{62} & S_{66}\end{pmatrix}\begin{pmatrix}\sigma_{xx}^{(1)} & \sigma_{xx}^{(2)} & \sigma_{xx}^{(3)}\\ \sigma_{yy}^{(1)} & \sigma_{yy}^{(2)} & \sigma_{yy}^{(3)}\\ \sigma_{xy}^{(1)} & \sigma_{xy}^{(2)} & \sigma_{xy}^{(3)}\end{pmatrix}$$

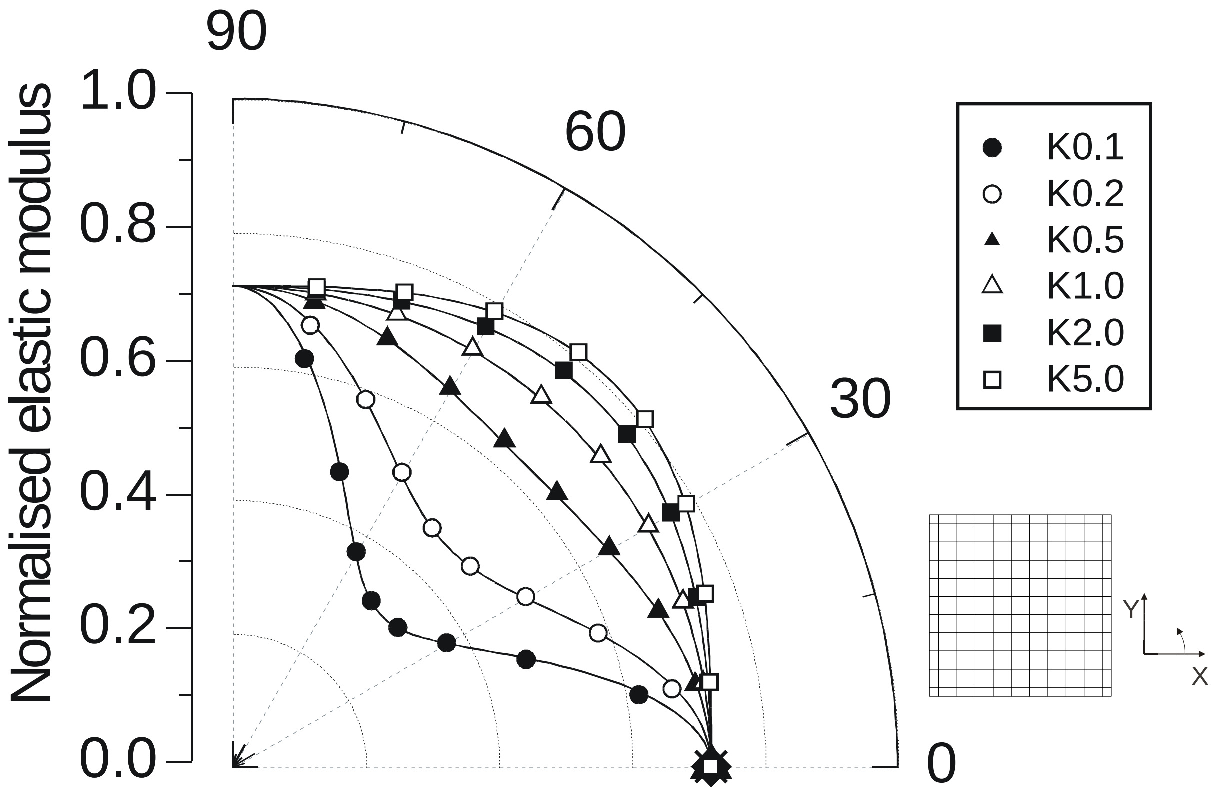
$$[\varepsilon]-[S_z][\sigma_z]=[S][\sigma]$$

$$[\varepsilon][\sigma]^{-1}-[S_z][\sigma_z][\sigma]^{-1}=[S]$$

σ_{ij} : BC
ε_{ij} : measured
S_{ij} : pre-determined
S_{ij} : ?

- In 2D plane strain condition, 6 elastic constants are determined.

Example - Rock Mass Determination
Methodology - Verification



Example - Rock Mass Determination
Data from Forsmark, Sweden

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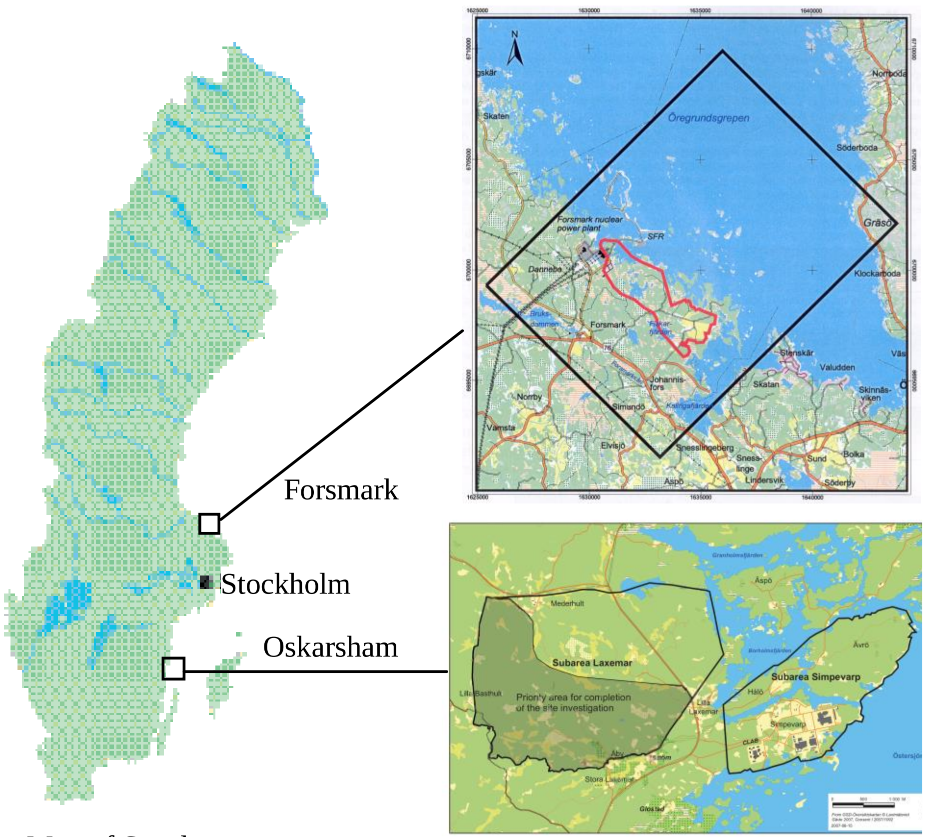
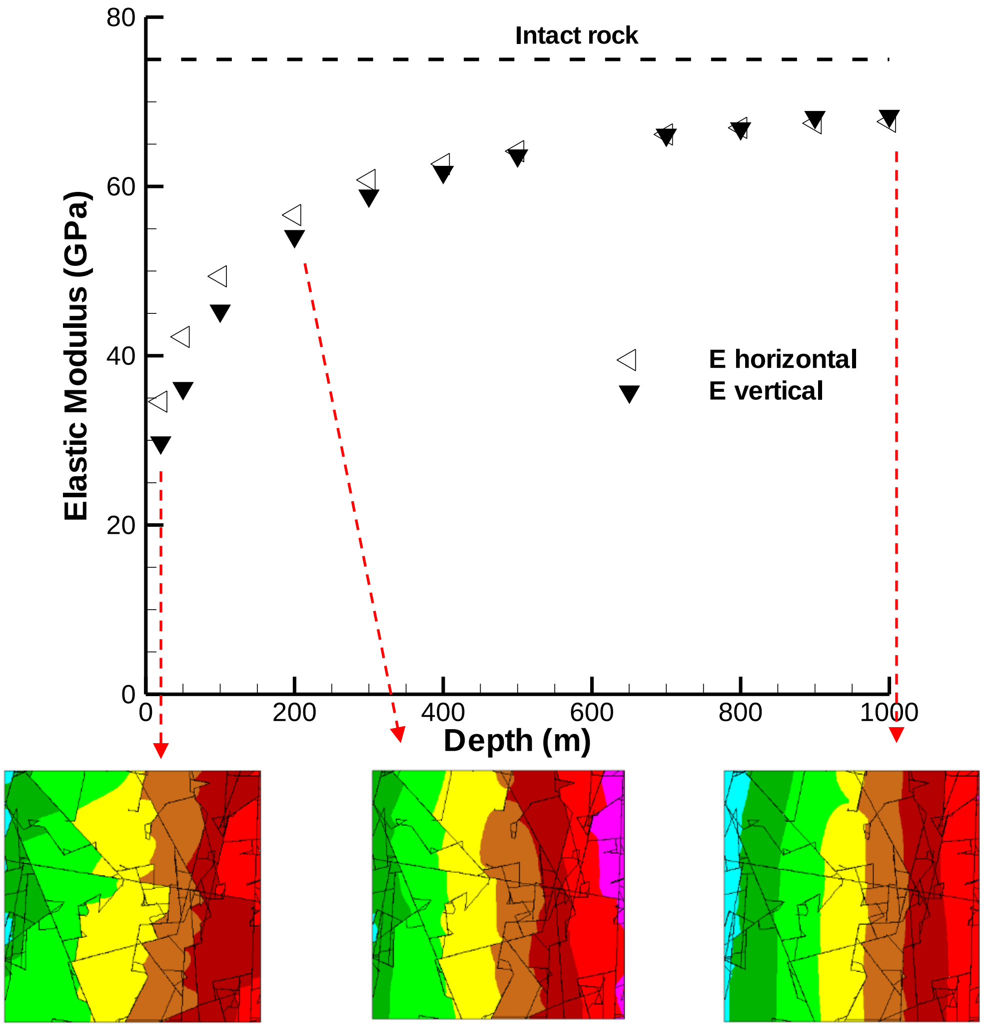


Map of Sweden

- Forsmark and Oskarshamn, two candidate sites for Swedish Program.
- 2002-2009: site investigation.
- 2009: Forsmark, as the final site
- 2011: License application
- 2014: decision (?)
- 2025: Operation (?)

Example - Rock Mass Determination
E mass results (elastic modulus from Forsmark)



Displacement distribution

Elastic moduli ↑ with stress ↑
- highly stress-dependent

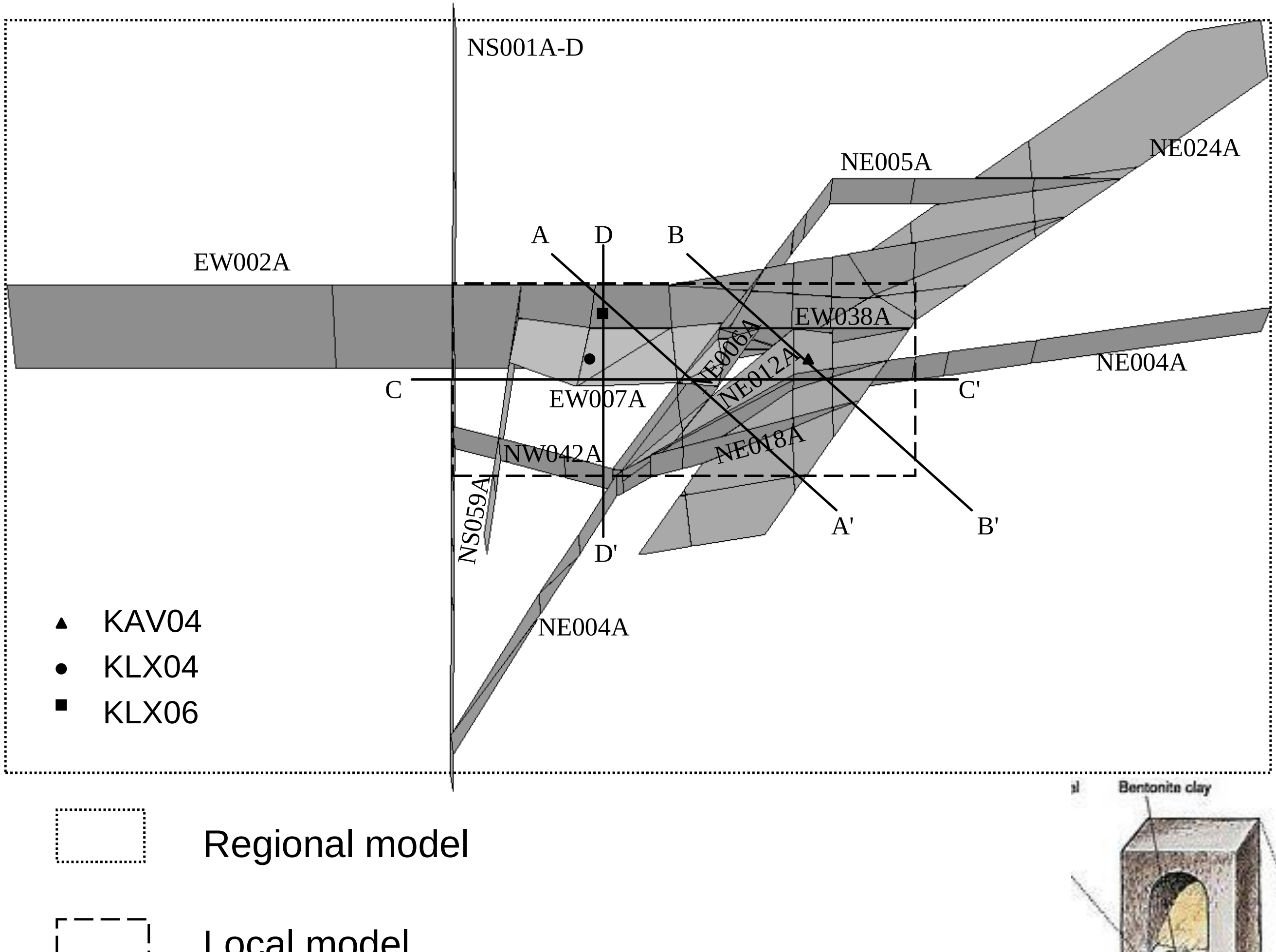
Stress induced anisotropy
- E_h 20% higher than E_v in shallow depth

Effect of stress is more evident in low stress condition.

(Min, Jing & Stephansson, 2005)

Explicit DEM in Blocky System

Example – Stress state modeling (Min, 2009)



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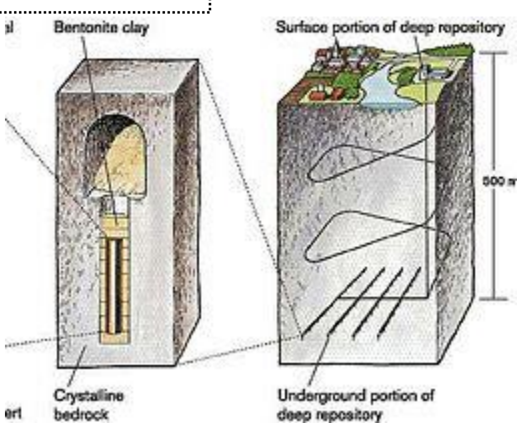
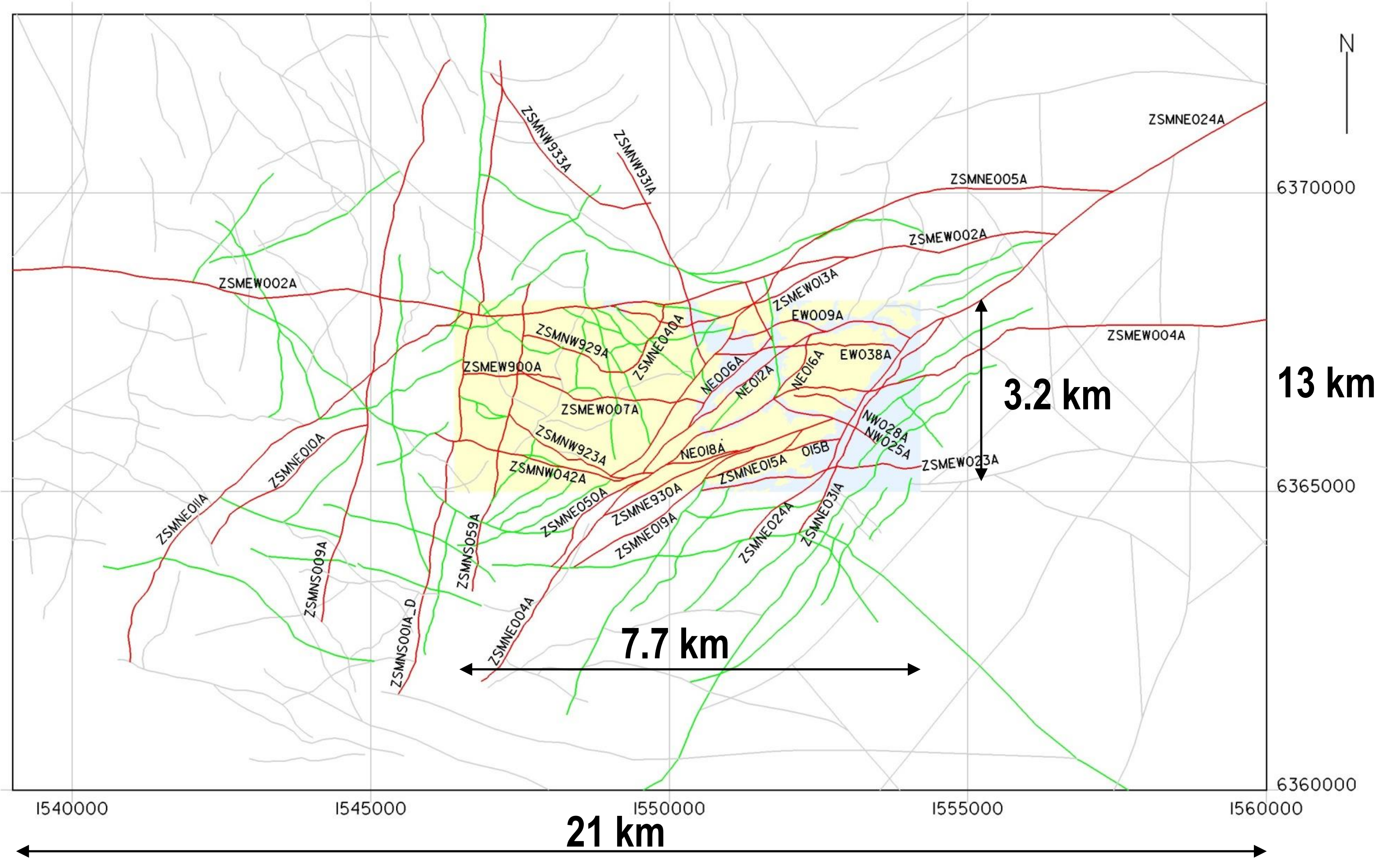


- A study at Laxemar(Oskarshamn), Sweden which was one of the two candidate sites for Swedish nuclear waste disposal

Example – Stress state modeling
Geometry



Example – Stress state modeling
Boundary Condition

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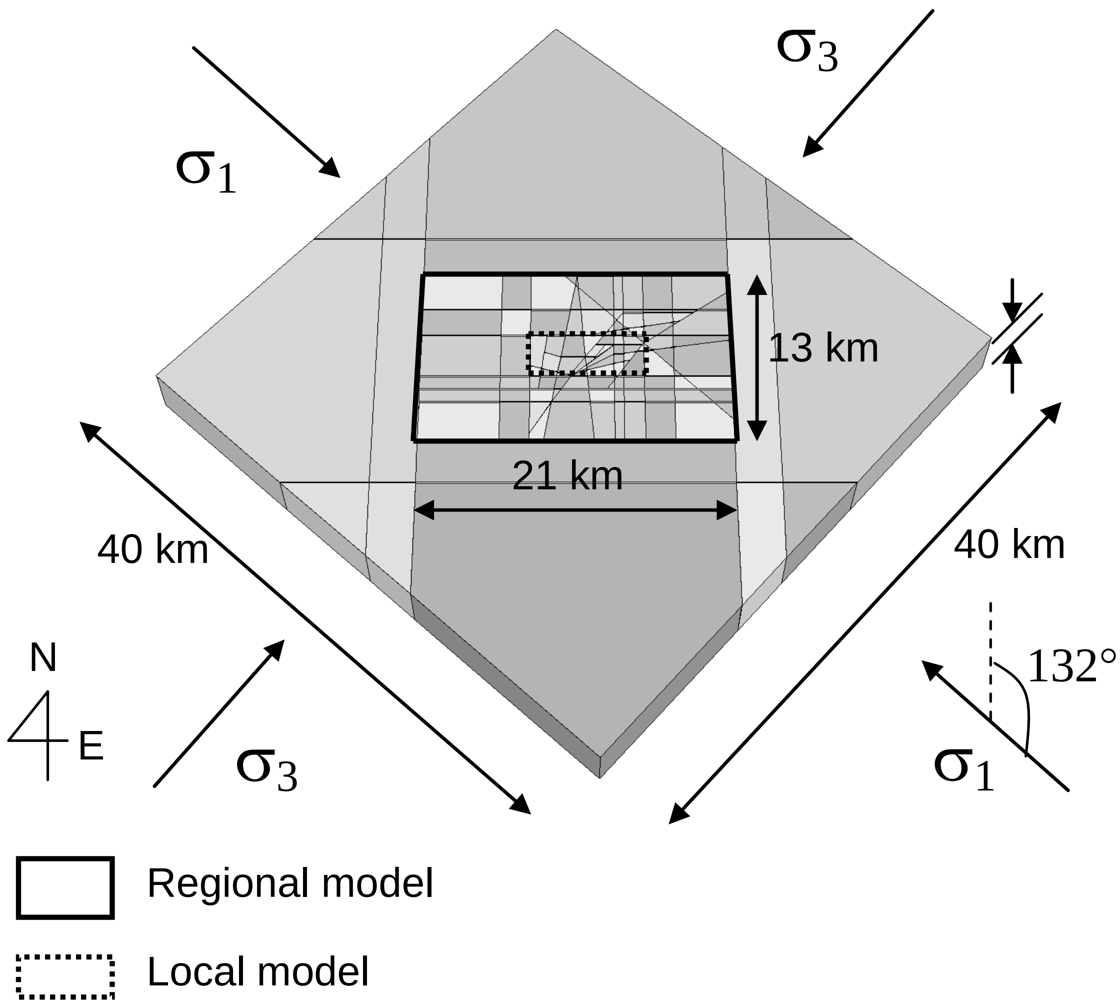

Example – Stress state modeling
Mesh generation

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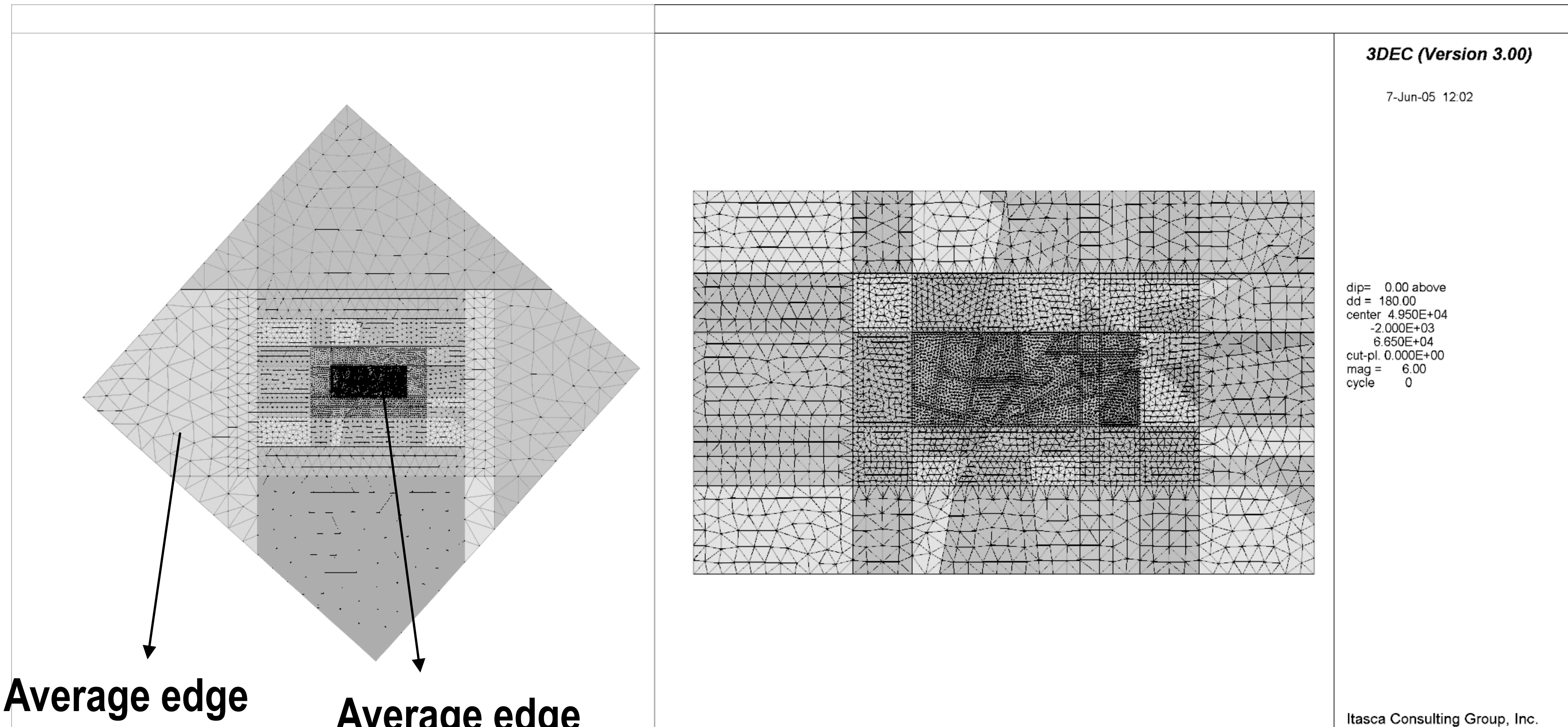
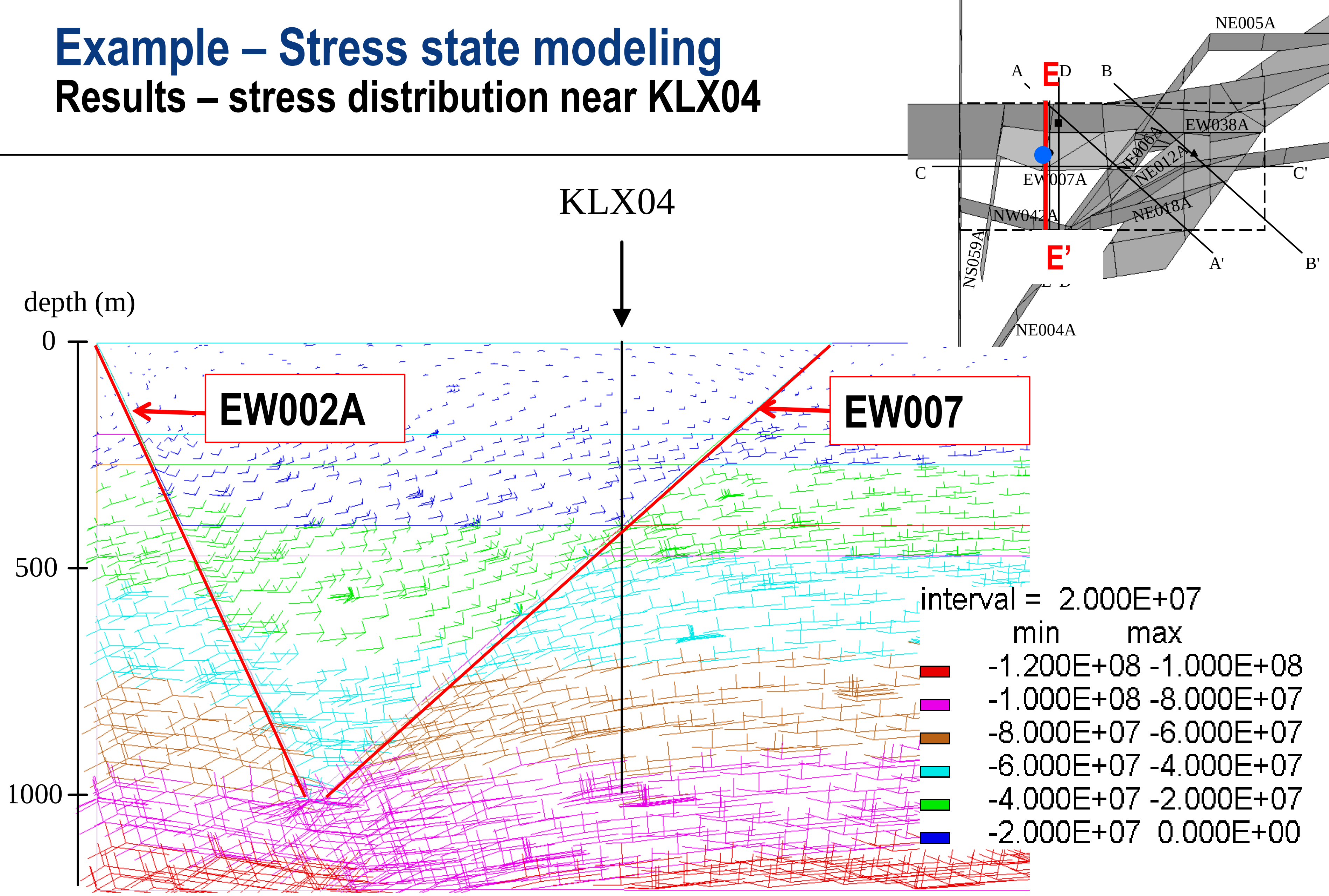


Average edge length 2,000 m

Average edge length 100 m

Example – Stress state modeling

Results – stress distribution near KLX04



(Min, 2009)

Example – Stress state modeling

Results - Predicted vs. Measured stress (KLX04)

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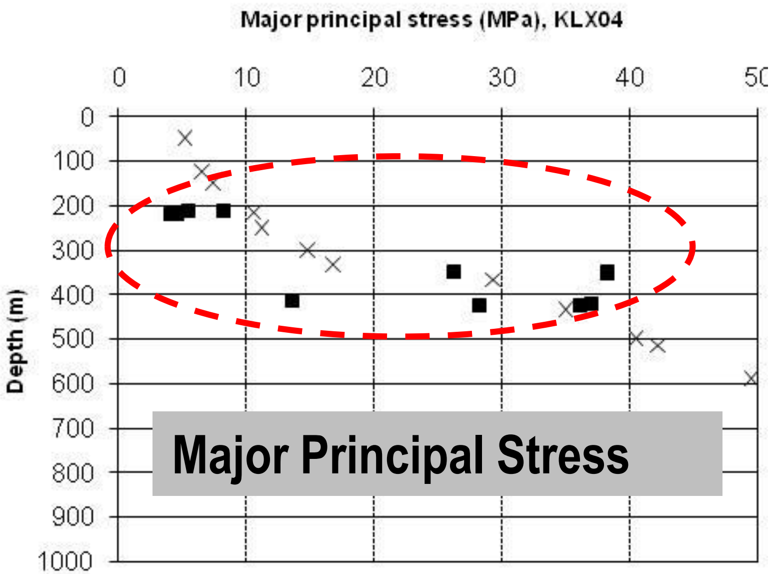
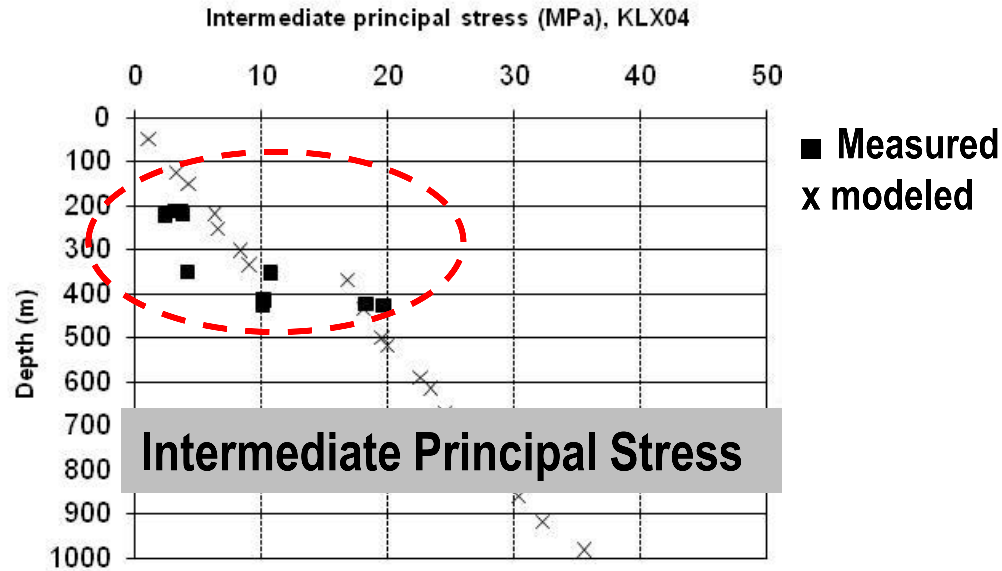
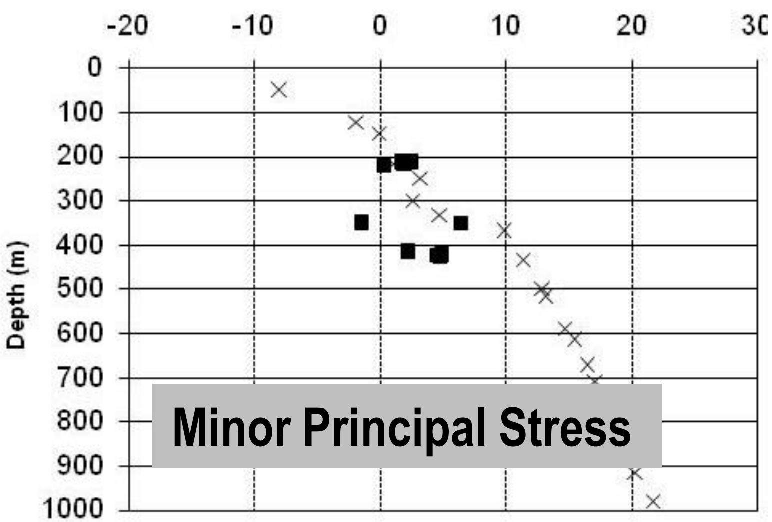




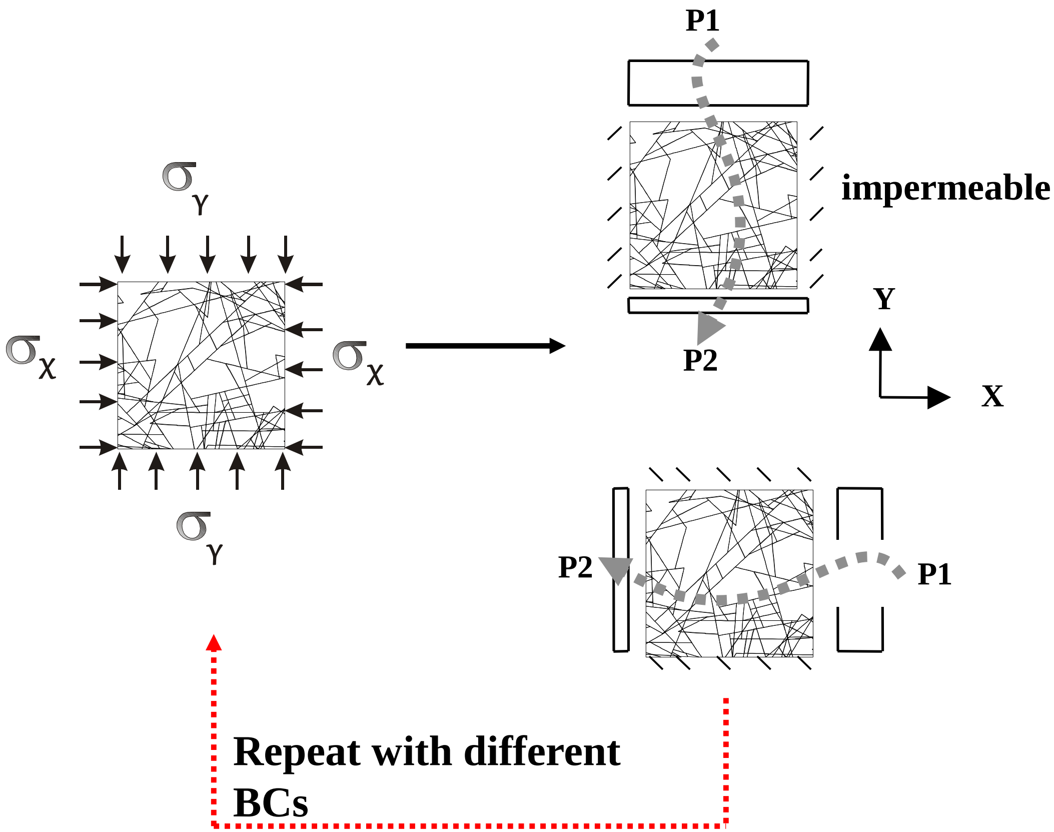


Numerically predictions capture the dramatic change of in situ stress

(Min, 2009)

Explicit DEM in Blocky System

Example - Stress-dependent permeability (Min et al., 2004)



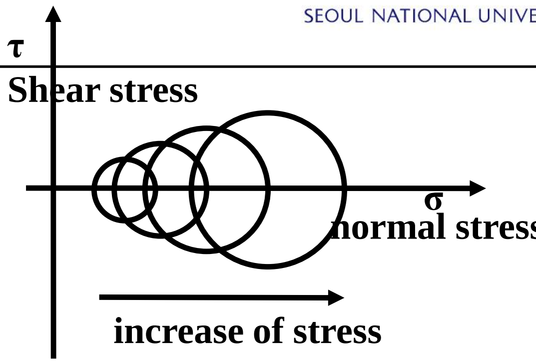
(1)

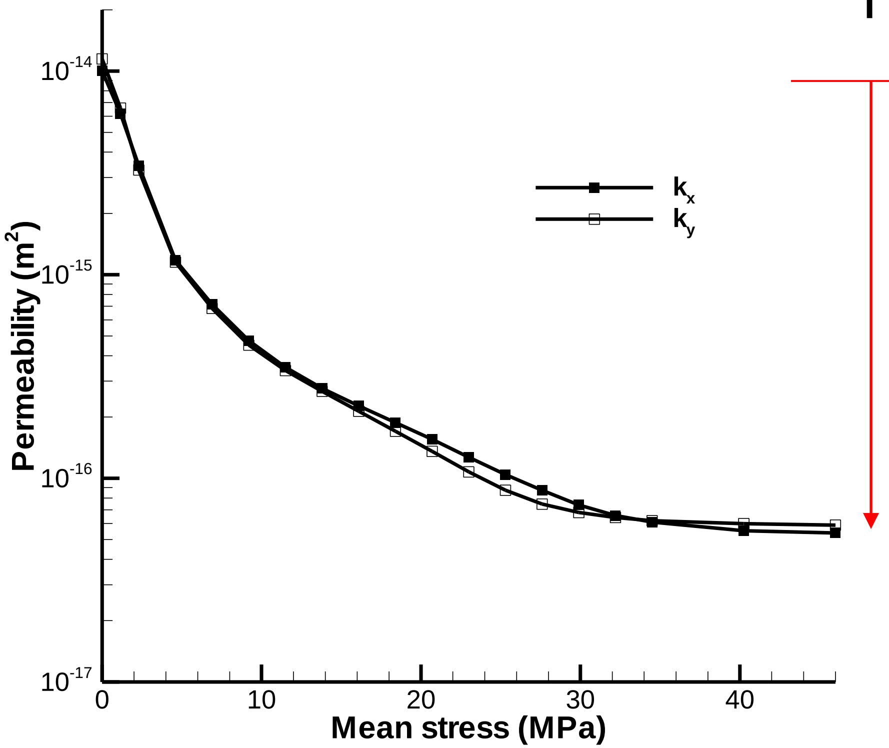
Apply Stress

(2)

Measure permeability

Example - Stress-dependent permeability (Min et al., 2004) - Results



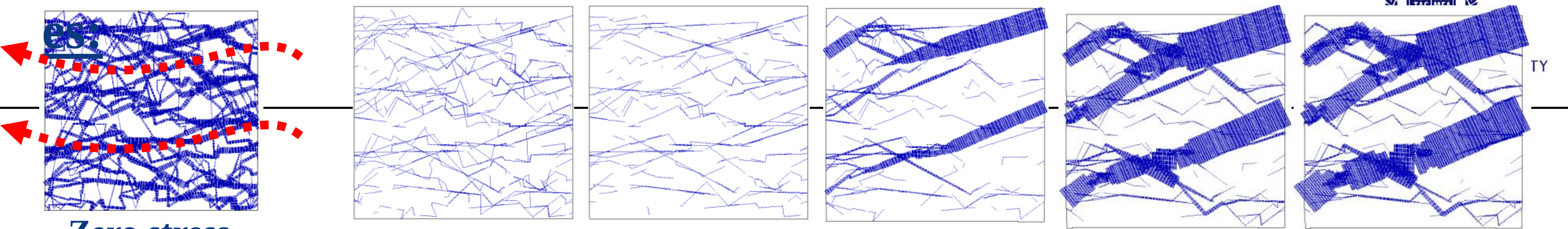


2 order magnitude reduction

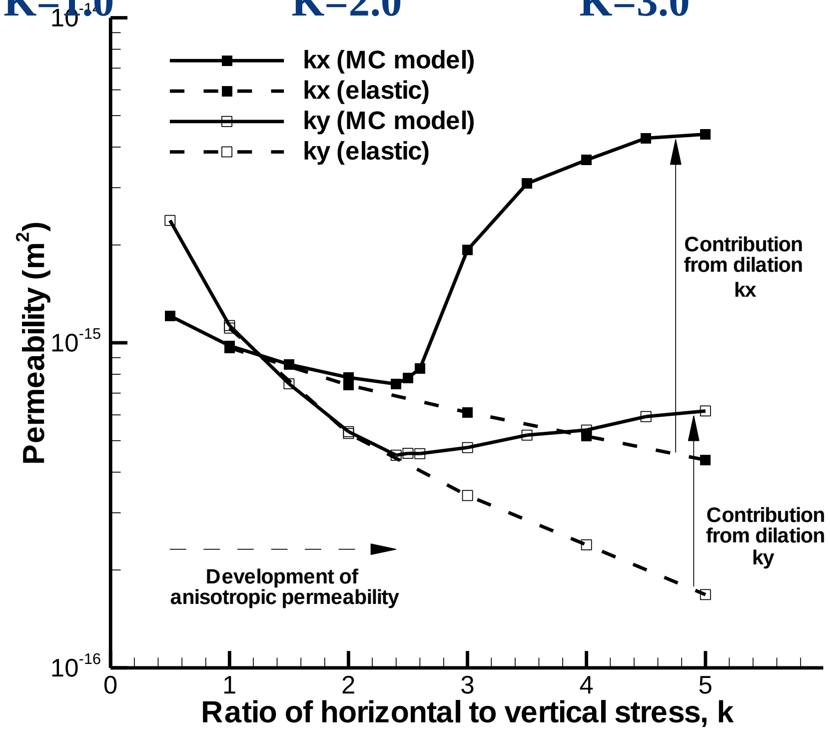
More sensitive at low stress

- Deformation of aperture occur uniformly
- Normal closure is dominating the k_x,k_y change

Flowrates:

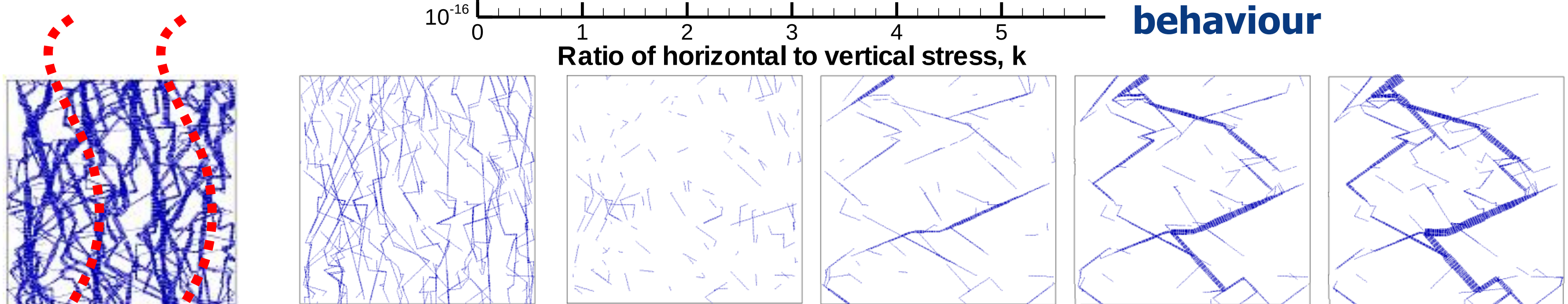


Zero stress K=1.0 K=2.0 K=3.0 K=4.0 K=5.0



- stress-induced channelling is reproduced.

- This partly explains why fluid flow in a few fractures are dominating the fluid behaviour



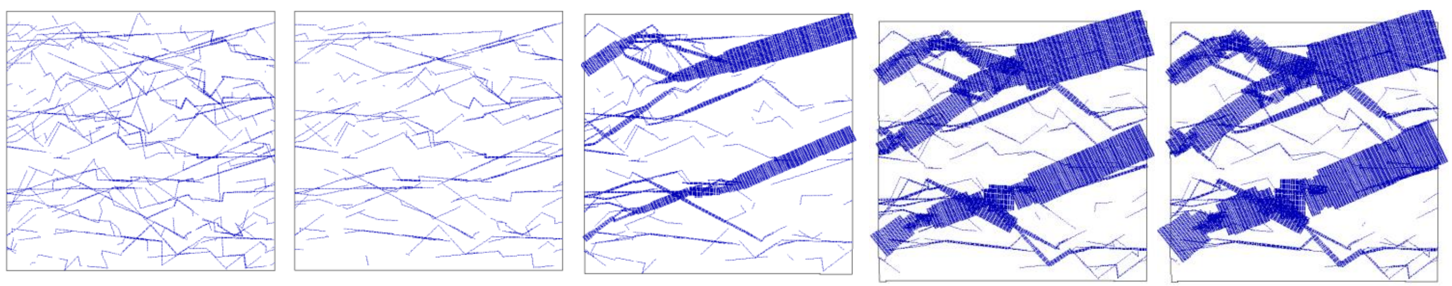
K=1.0 K=2.0 K=3.0 K=4.0 K=5.0

Example - Stress-dependent permeability (Min et al., 2004)
Results - Implications

CO2 Geosequestration → Increase of injected CO2 pressure

Nuclear waste repository → Generation of thermal stress

THERMOSHEARING



K=1.0 K=2.0 K=3.0 K=4.0 K=5.0

Enhanced Geothermal System
Hydrofrac for shale gas reservoir → Increased hydraulic pressure → hydraulic stimulation

HYDROSHEARING

A few words for application of DEM

- Start from very very simple model when you do complicated modeling – try to gain insight into the implemented physics & constitutive equation of code
- Modeler should be able to explain every bit of observation – numerical code is not a black box!
- There can be many many interesting applications - you are encouraged to apply DEM to novel applications
- There are much rooms for improvement in DEM development

Explicit DEM
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Explicit DEM
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