Announcement

- To be updated

Chap. 5 Stresses in Beams

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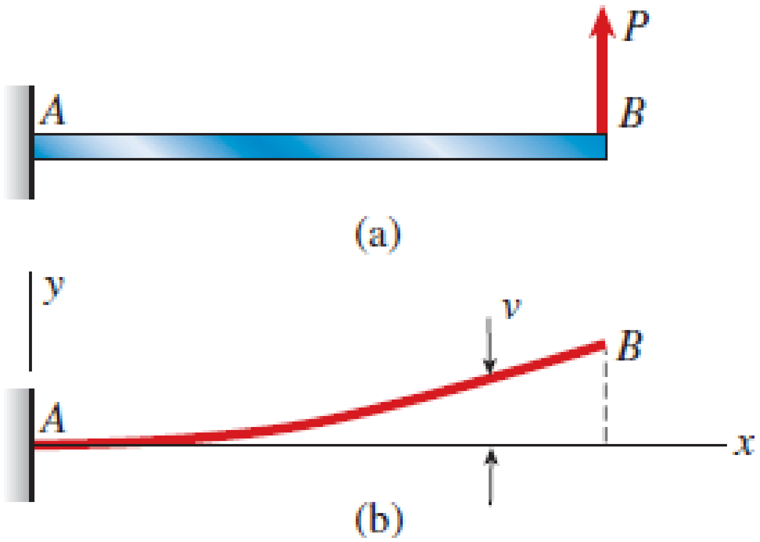
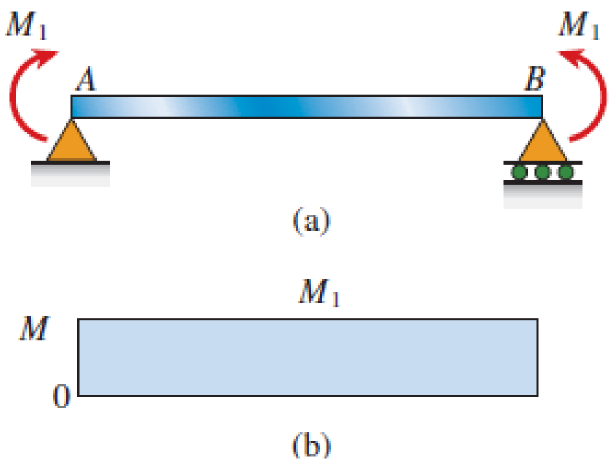
Introduction

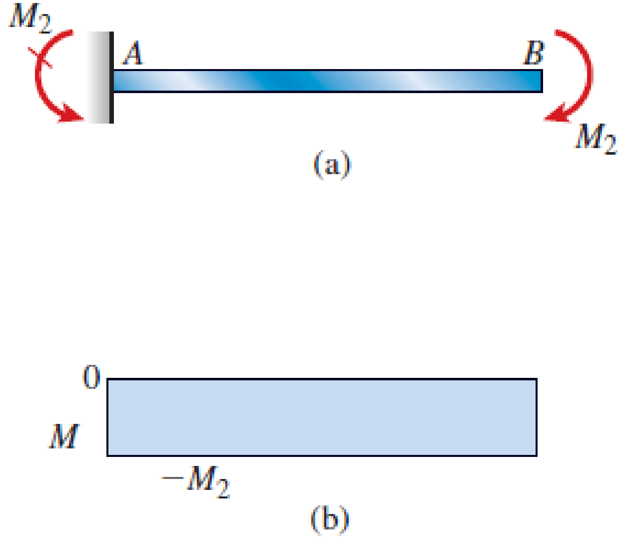
Bending of a cantilever beam: beam with load and resulting deflection curve



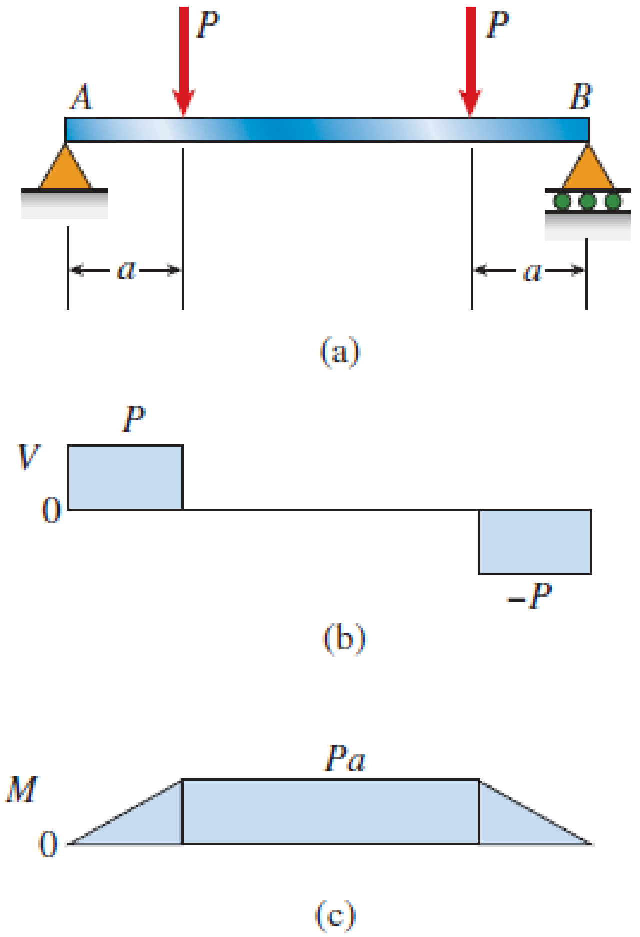
Pure Bending and Non-uniform Bending



<Simple beam in pure bending>

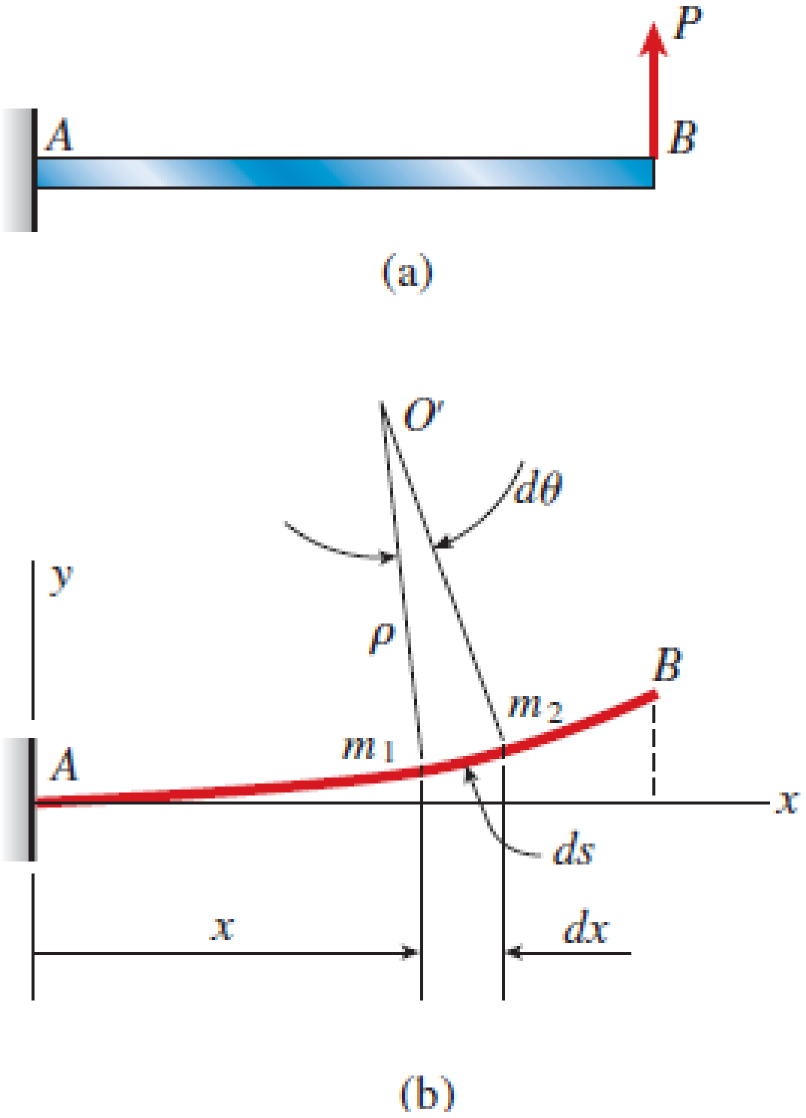


<Cantilever beam in pure bending>



<Simple beam in non-uniform bending>

Curvature of a beam



Radius of curvature = $\rho = m_1 O' = m_2 O'$

Reciprocal of the radius of curvature = κ

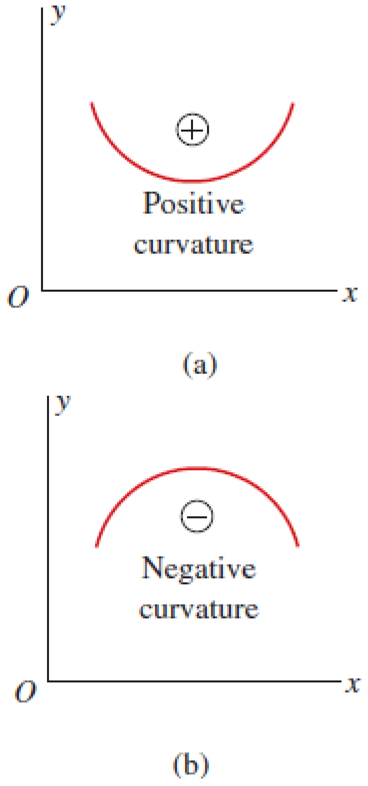
$$\kappa = \frac{1}{\rho}$$

$$\rho \, d\theta = ds$$

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds}$$

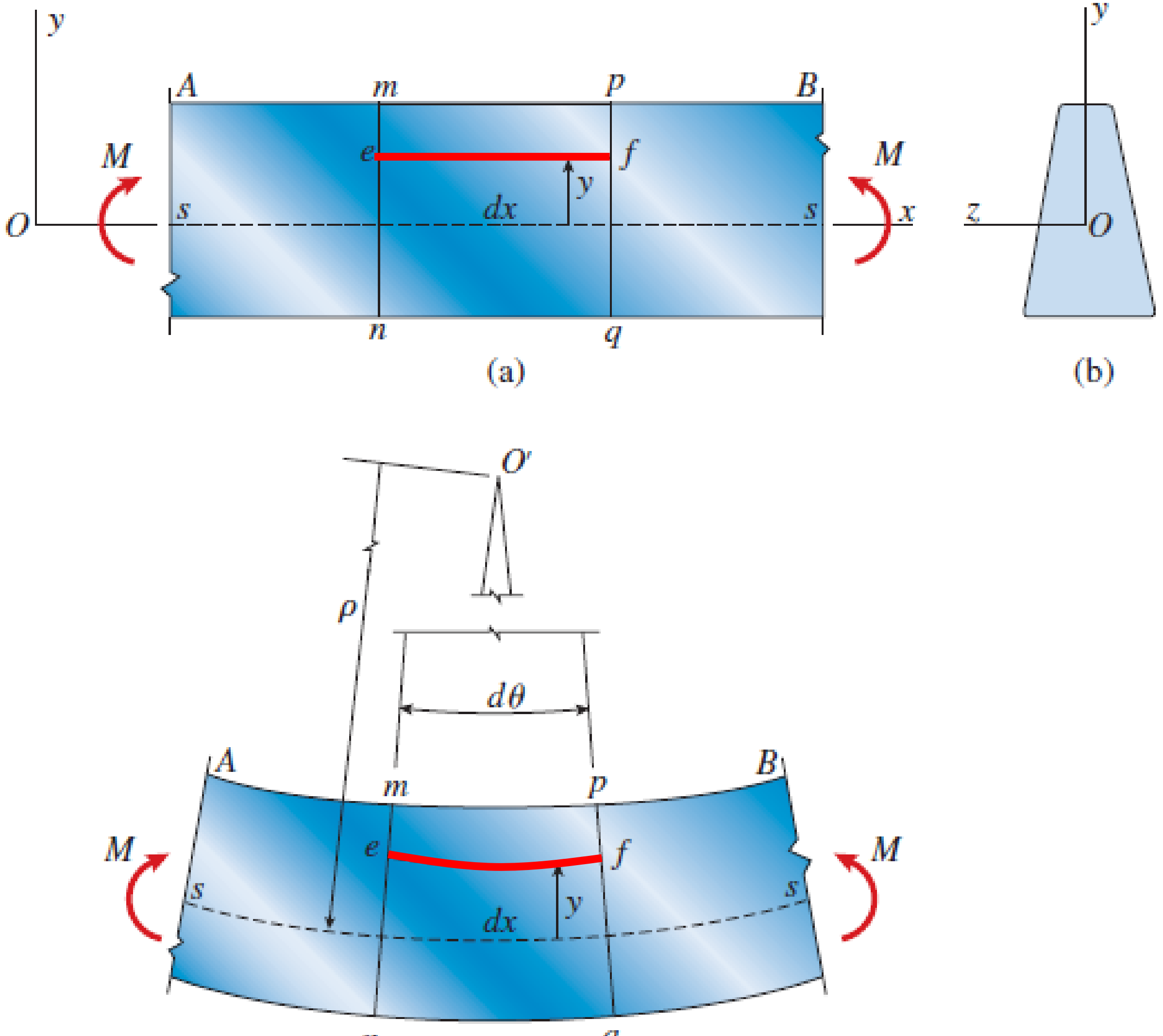
Assumption: $ds = dx$

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{dx}$$



Sign convention for curvature

Longitudinal Strains in Beams



(a) (b) (c)

original length of line ef is dx.

length L_1 of line ef after bending

$$L_1 = (\rho - y)\, d\theta = dx - \frac{y}{\rho} dx$$

elongation

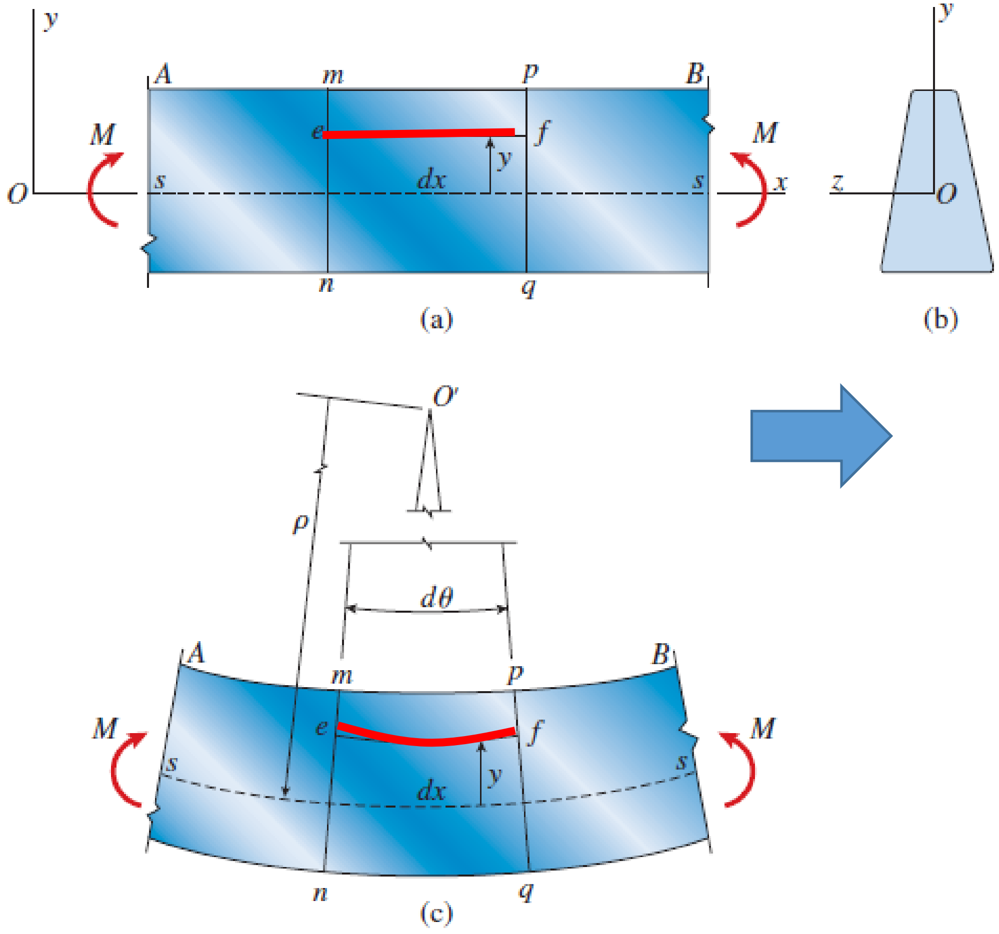
$$L_1 - dx, \text{ or } -y\, dx/\rho$$

Strain (=elongation / original length of dx)

$$\epsilon_x = -\frac{y}{\rho} = -\kappa y$$

Strain-curvature relation

Longitudinal Strains in Beams



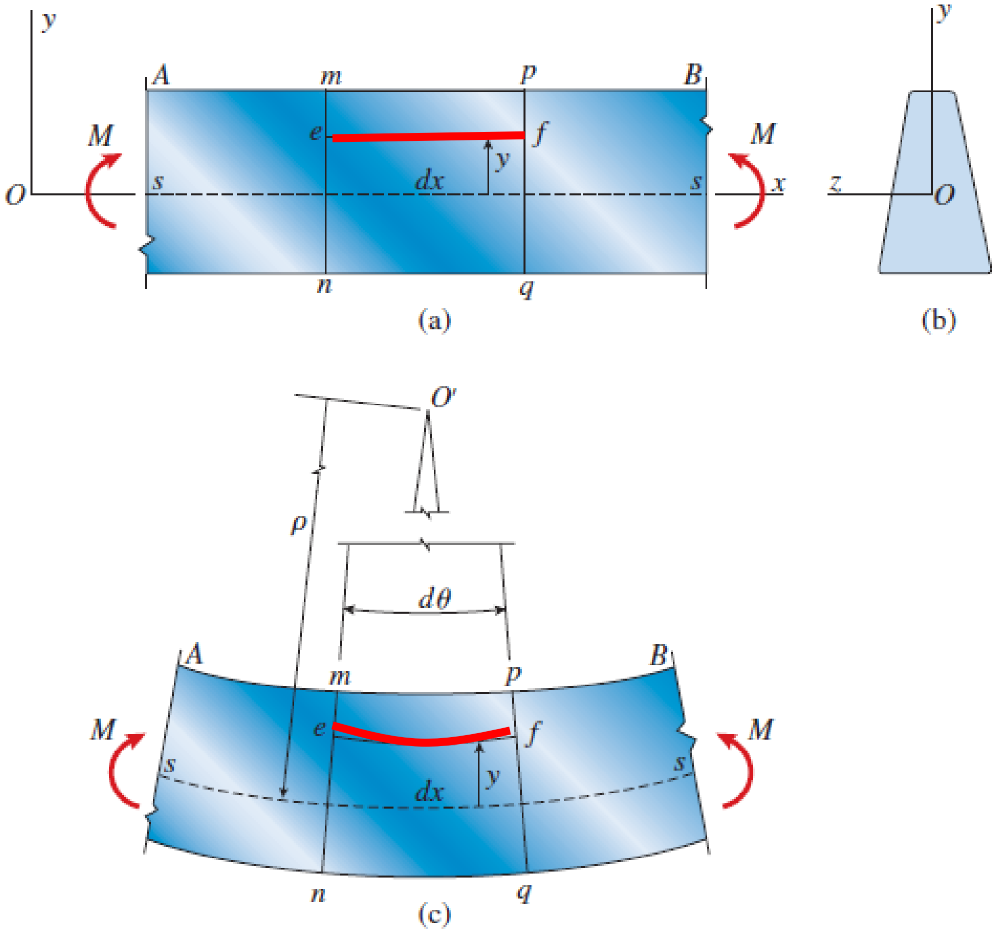
Strain

$$\epsilon_x = -\frac{y}{\rho} = -\kappa y$$

Strain-curvature relation

- Normal strains in a beam was derived solely from the geometry of the deformed beam.
- Properties of the material did NOT enter into the derivation.
- The strains in a beam in pure bending vary linearly with distance from the neutral surface regardless of the shape of the stress-strain curve of the material.

Normal Stresses in Beams (Linearly Elastic Materials)



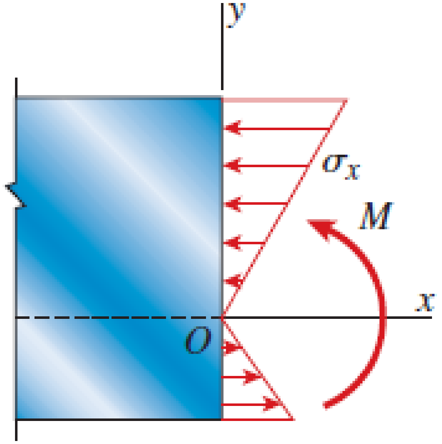
Strain in bending

$$\epsilon_x = -\frac{y}{\rho} = -\kappa y$$

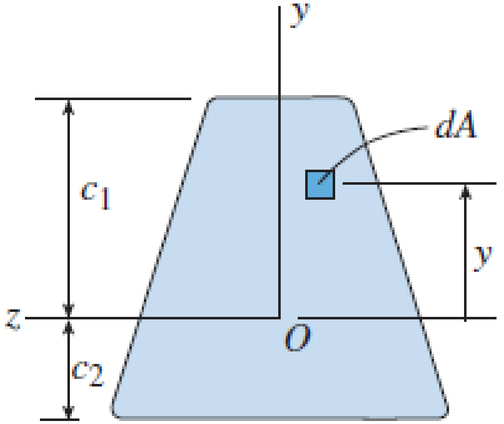
Linearly Elastic Materials

$$\sigma_x = E\epsilon_x$$

Normal Stresses in Beams (Linearly Elastic Materials)



(a)



(b)

Strain in bending

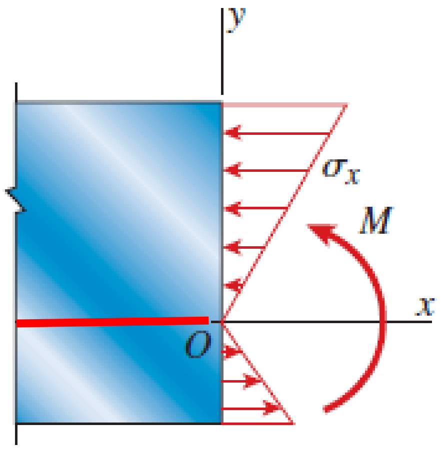
$$\epsilon_x = -\frac{y}{\rho} = -\kappa y$$

Linearly Elastic Materials

$$\sigma_x = E\epsilon_x$$

$$\sigma_x = E\epsilon_x = -\frac{Ey}{\rho} = -E\kappa y$$

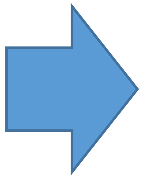
Normal Stresses in Beams (Linearly Elastic Materials)



(a)

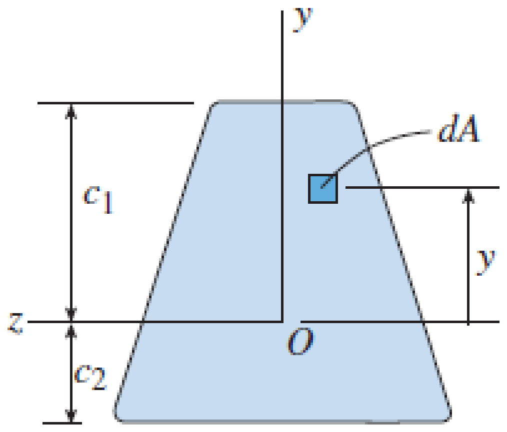
But…Where is origin axis?

$$\int_A \sigma_x dA = -\int_A E\kappa y dA = 0$$



$$\int_A y\, dA = 0$$

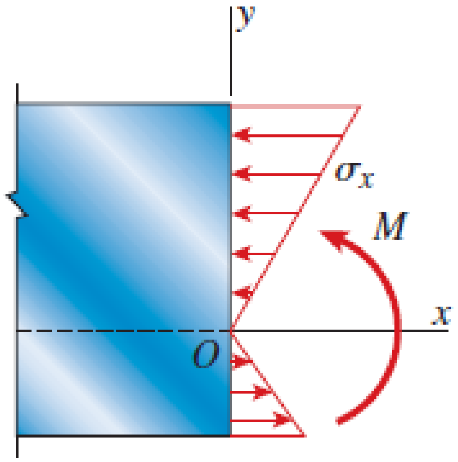
Neutral axis



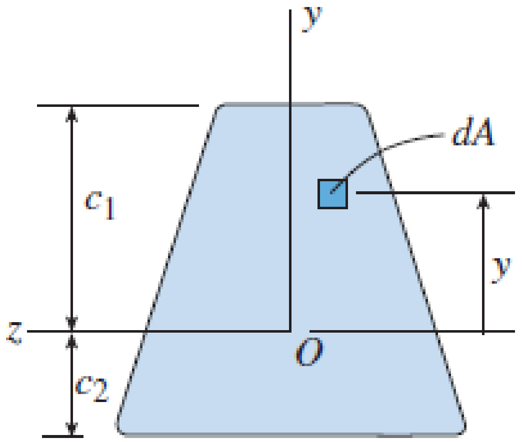
(b)

- Neutral axis passes through the centroid of the cross-sectional area when the material follows Hooke's law and there is no axial force acting on the cross section.

Moment-Curvature Relationship



(a)



(b)

$$dM = -\sigma_x y \, dA$$

$$M = -\int_A \sigma_x y \, dA$$

$$M = \int_A \kappa E y^2 \, dA = \kappa E \int_A y^2 \, dA$$

$$\sigma_x = -E\kappa y$$

$$I = \int_A y^2 \, dA$$

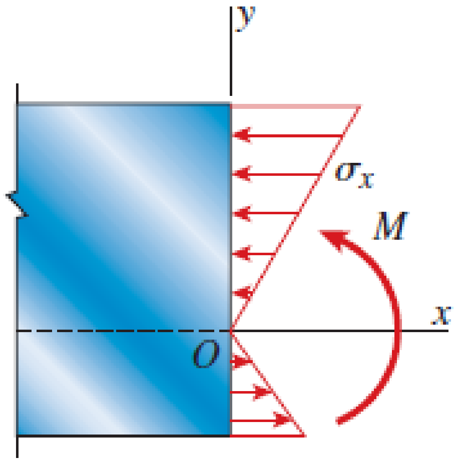
2nd moment of inertia

$$M = \kappa EI$$

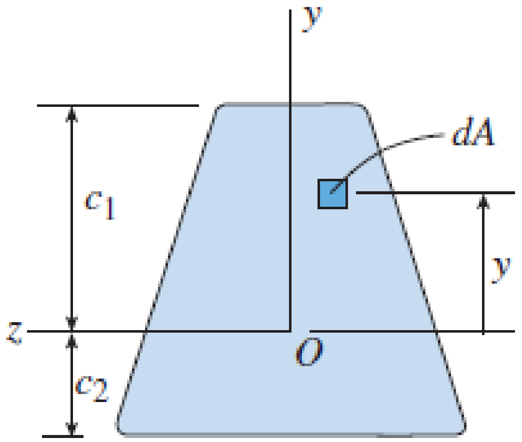
$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

Curvature-Moment

Flexure Formula



(a)



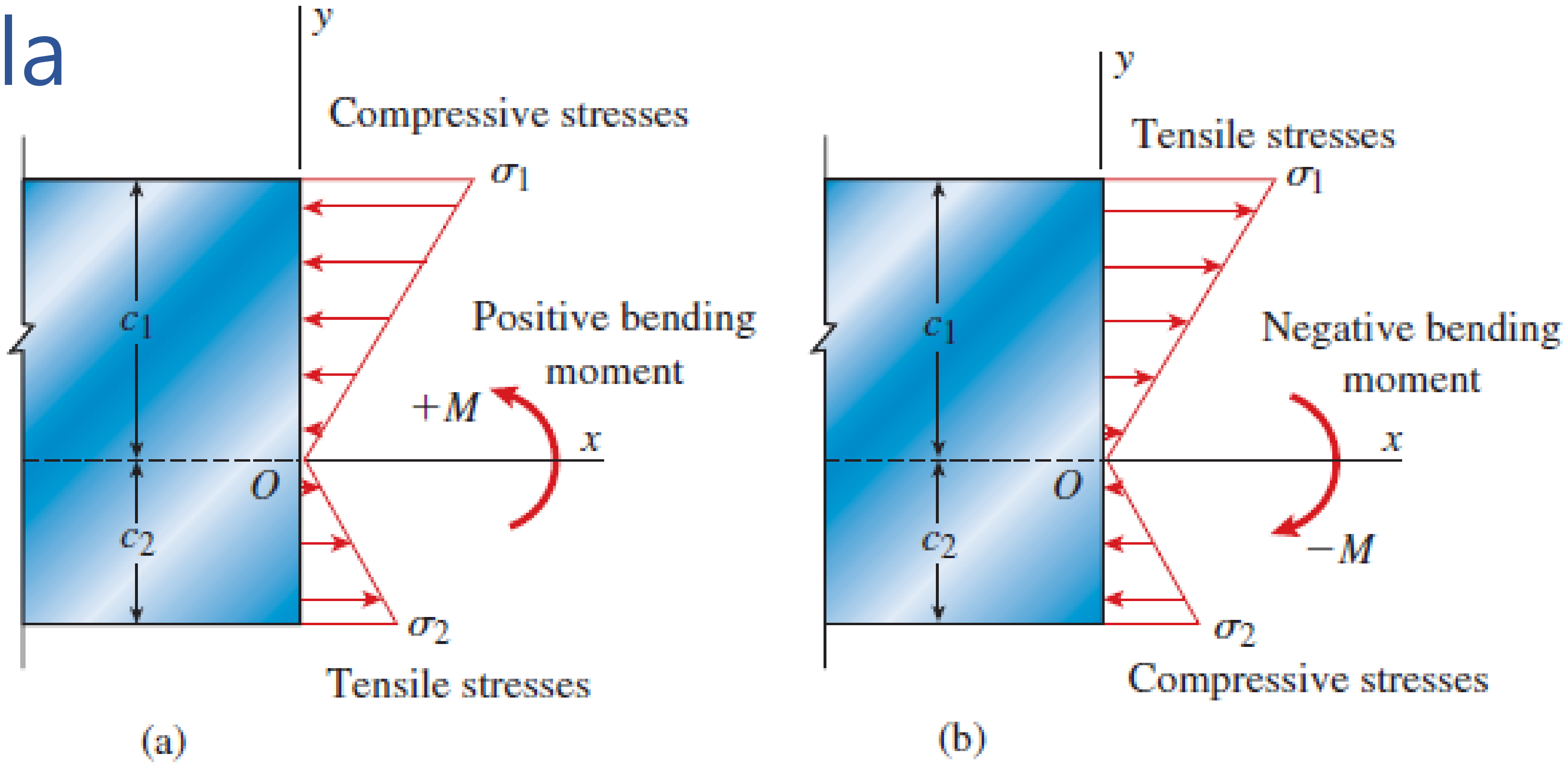
(b)

$$\sigma_x = E\epsilon_x = -\frac{Ey}{\rho} = -E\kappa y$$

$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

$$\sigma_x = -\frac{My}{I}$$

Flexure Formula



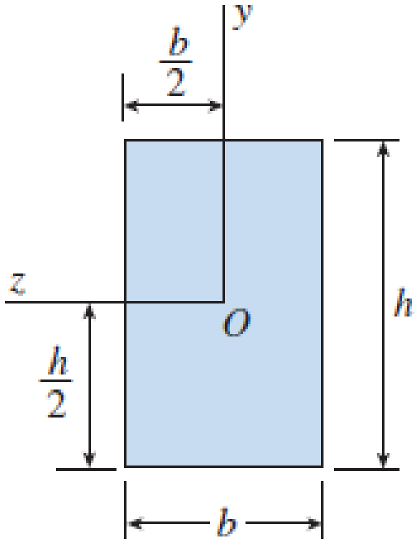
$$\sigma_x = -\frac{My}{I}$$

$$\sigma_1 = -\frac{Mc_1}{I} = -\frac{M}{S_1} \qquad \sigma_2 = \frac{Mc_2}{I} = \frac{M}{S_2}$$

where $S_1 = \frac{I}{c_1}$ $\quad S_2 = \frac{I}{c_2}$ Section moduli of the cross-sectional area

Moment of Inertia (I) and Section Modulus (S)

<Rectangular cross section>



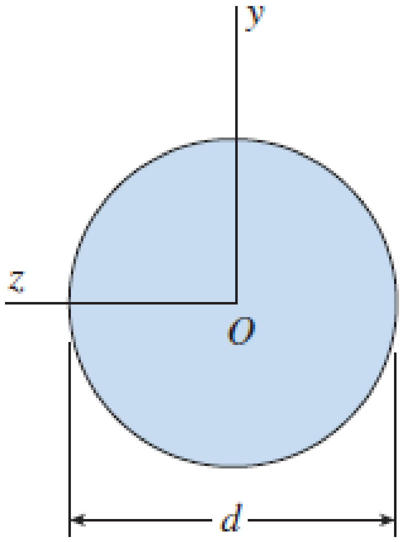
$$I = \int_A y^2\, dA$$

$$S = \frac{I}{c}$$

$$I = \frac{bh^3}{12}$$

$$S = \frac{bh^2}{6}$$

<Circular cross section>



$$I = \frac{\pi d^4}{64}$$

$$S = \frac{\pi d^3}{32}$$

Polar moment of inertia

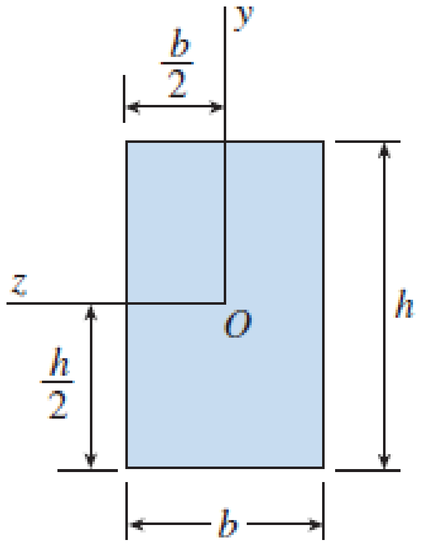
$$I_P = \int_A \rho^2 dA$$

$$I_P = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

Moment of Inertia (I) derivation

$$I = \int_A y^2 \, dA$$

<Rectangular cross section>



$$I_x = \int_{-b/2}^{b/2} \int_{-h/2}^{h/2} y^2 \mathrm{d}y \mathrm{d}x$$

$$I_x = \int_{-b/2}^{b/2} \frac{1}{3} y^3 \Big|_{-h/2}^{h/2} \mathrm{d}y \mathrm{d}x$$

$$I_x = \int_{-b/2}^{b/2} \frac{1}{3} \frac{h^3}{4} \mathrm{d}x$$

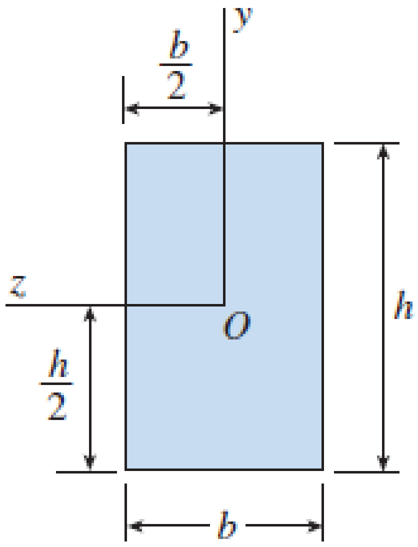
$$I_x = \frac{1}{3} \frac{h^3}{4} x \Big|_{-b/2}^{b/2}$$

$$I_x = \frac{bh^3}{12}$$

$$I = \frac{bh^3}{12}$$

Moment of Inertia (I) derivation

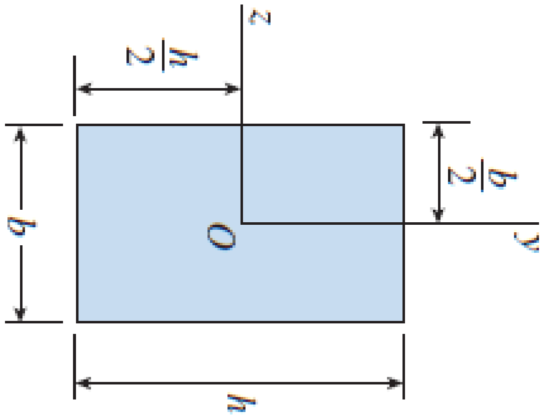
<Rectangular cross section>



$$I = \frac{bh^3}{12}$$

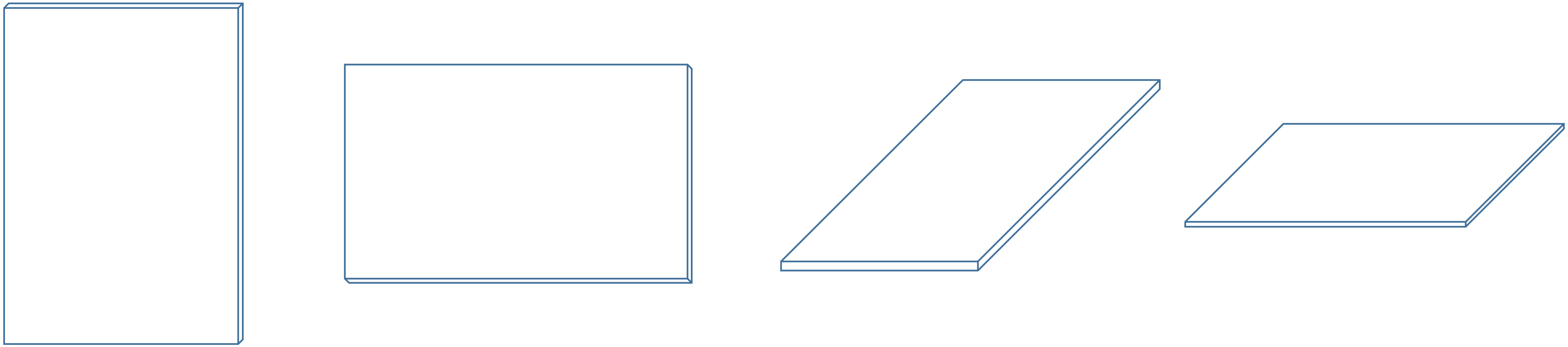
90 degree rotation

<Rectangular cross section>



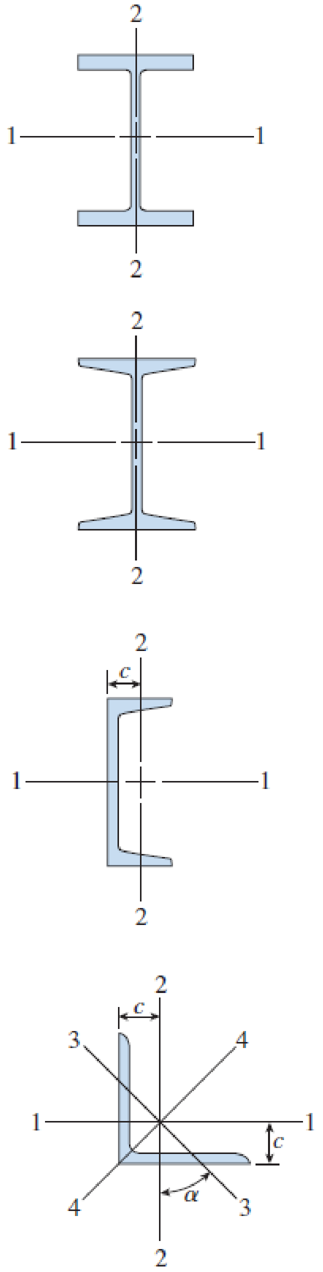
$$I = \frac{hb^3}{12}$$

Paper folding

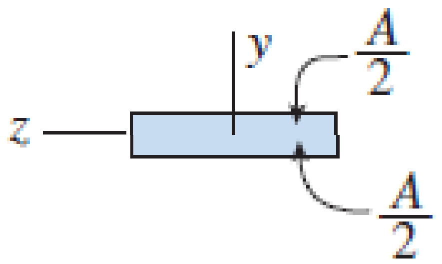


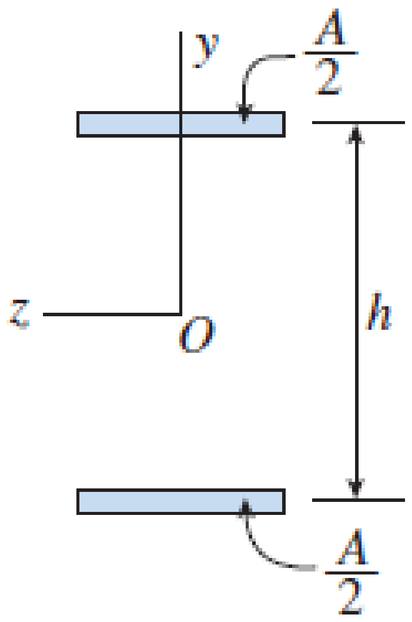
Engineering

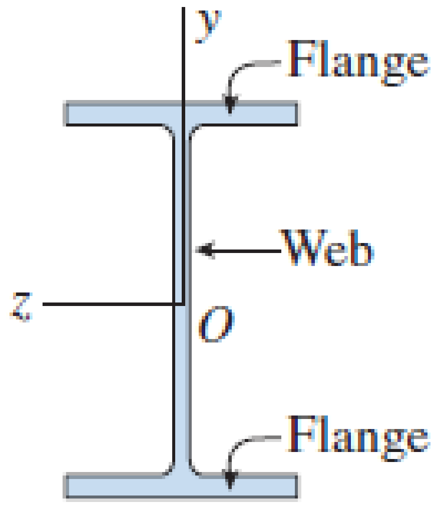
APPENDIX E



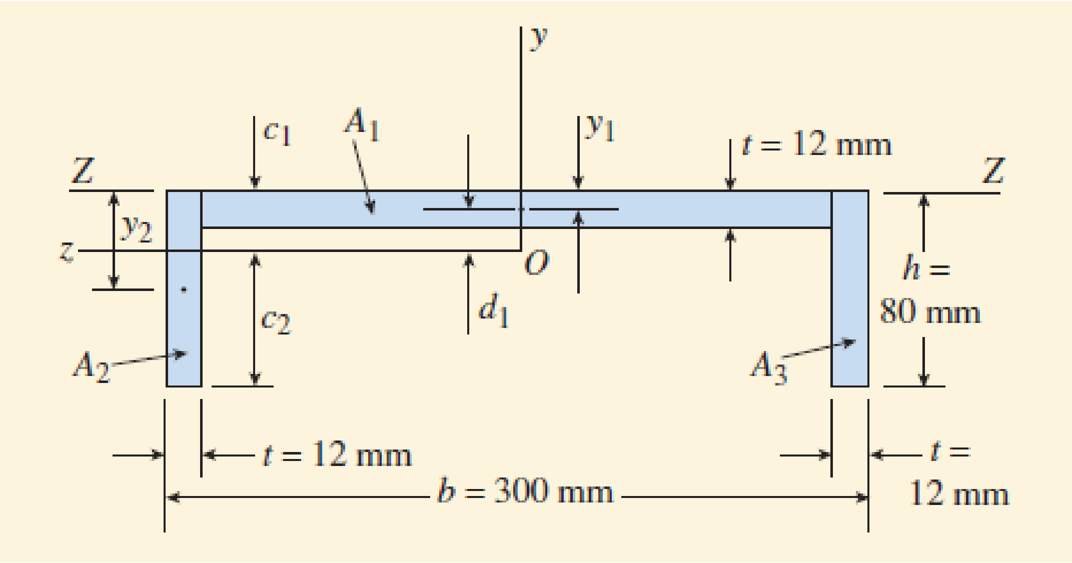
Customized section







Customized section



Where is the neutral axis?

Area 1: $y_1 = t/2 = 6\text{ mm}$

$A_1 = (b - 2t)(t) = (276\text{ mm})(12\text{ mm}) = 3312\text{ mm}^2$

Area 2: $y_2 = h/2 = 40\text{ mm}$

$A_2 = ht = (80\text{ mm})(12\text{ mm}) = 960\text{ mm}^2$

Area 3: $y_3 = y_2 \qquad A_3 = A_2$

$$c_1 = \frac{\sum y_i A_i}{\sum A_i} = \frac{y_1 A_1 + 2y_2 A_2}{A_1 + 2A_2}$$

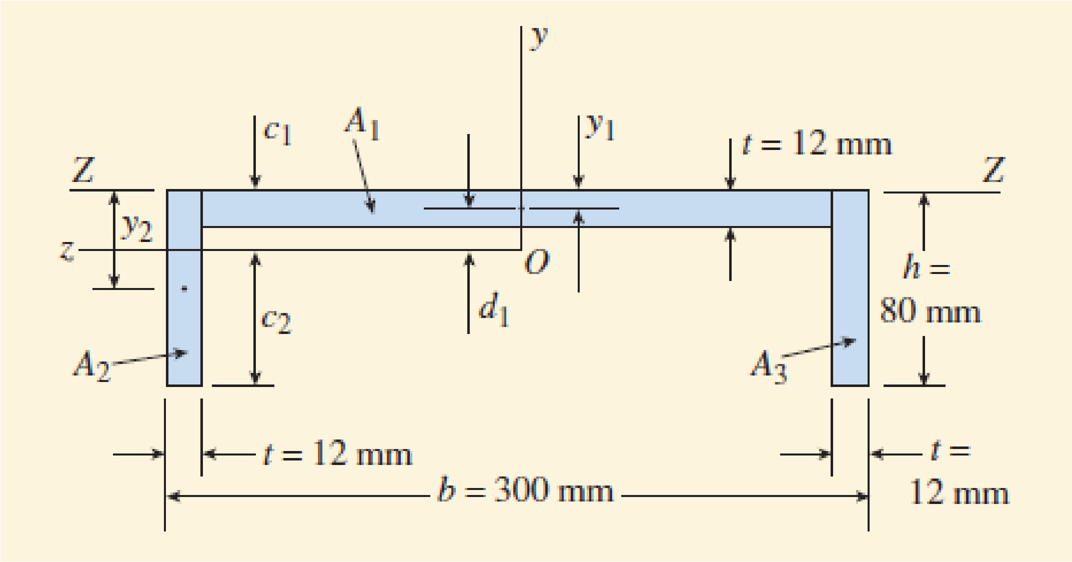
$$= \frac{(6\text{ mm})(3312\text{ mm}^2) + 2(40\text{ mm})(960\text{ mm}^2)}{3312\text{ mm}^2 + 2(960\text{ mm}^2)} = 18.48\text{ mm}$$

$$c_2 = h - c_1 = 80\text{ mm} - 18.48\text{ mm} = 61.52\text{ mm}$$

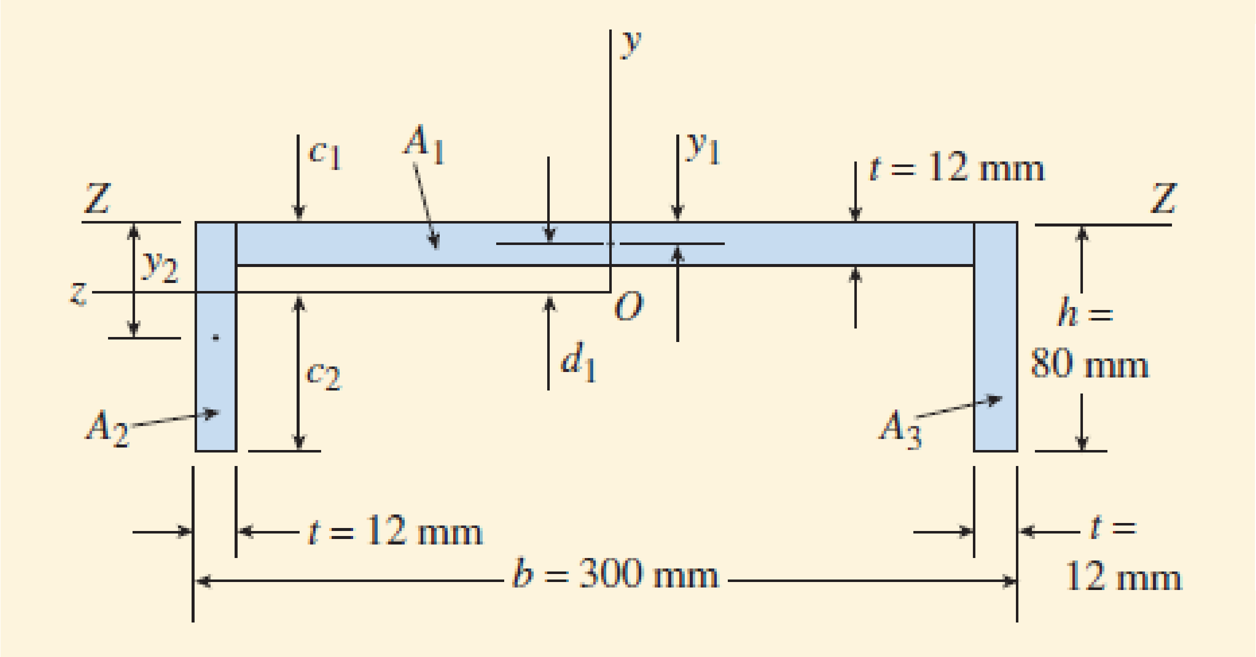
Customized section

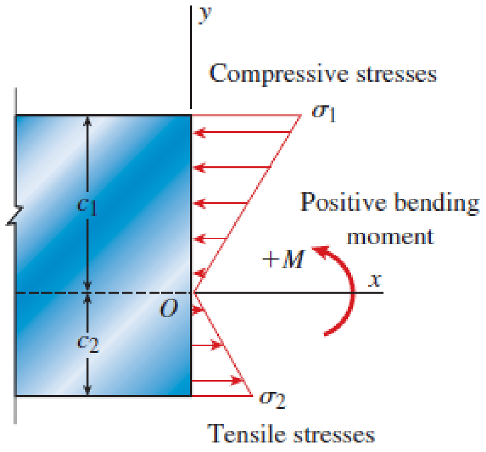
Moment of Inertia $(I_z)_1 = (I_c)_1 + A_1 d_1^2$

(if neutral axis is different from the centroid of each components)



Customized section





(a)

$$\sigma_1 = -\frac{Mc_1}{I}$$

$$\sigma_2 = \frac{Mc_2}{I}$$