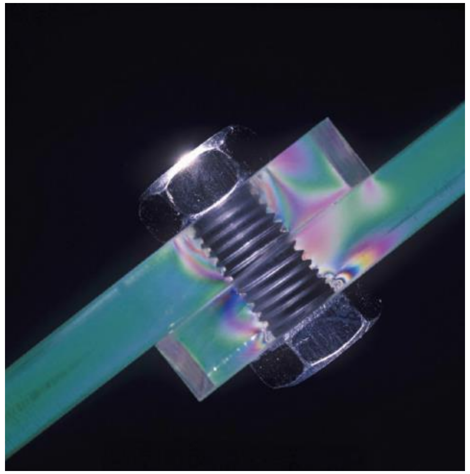
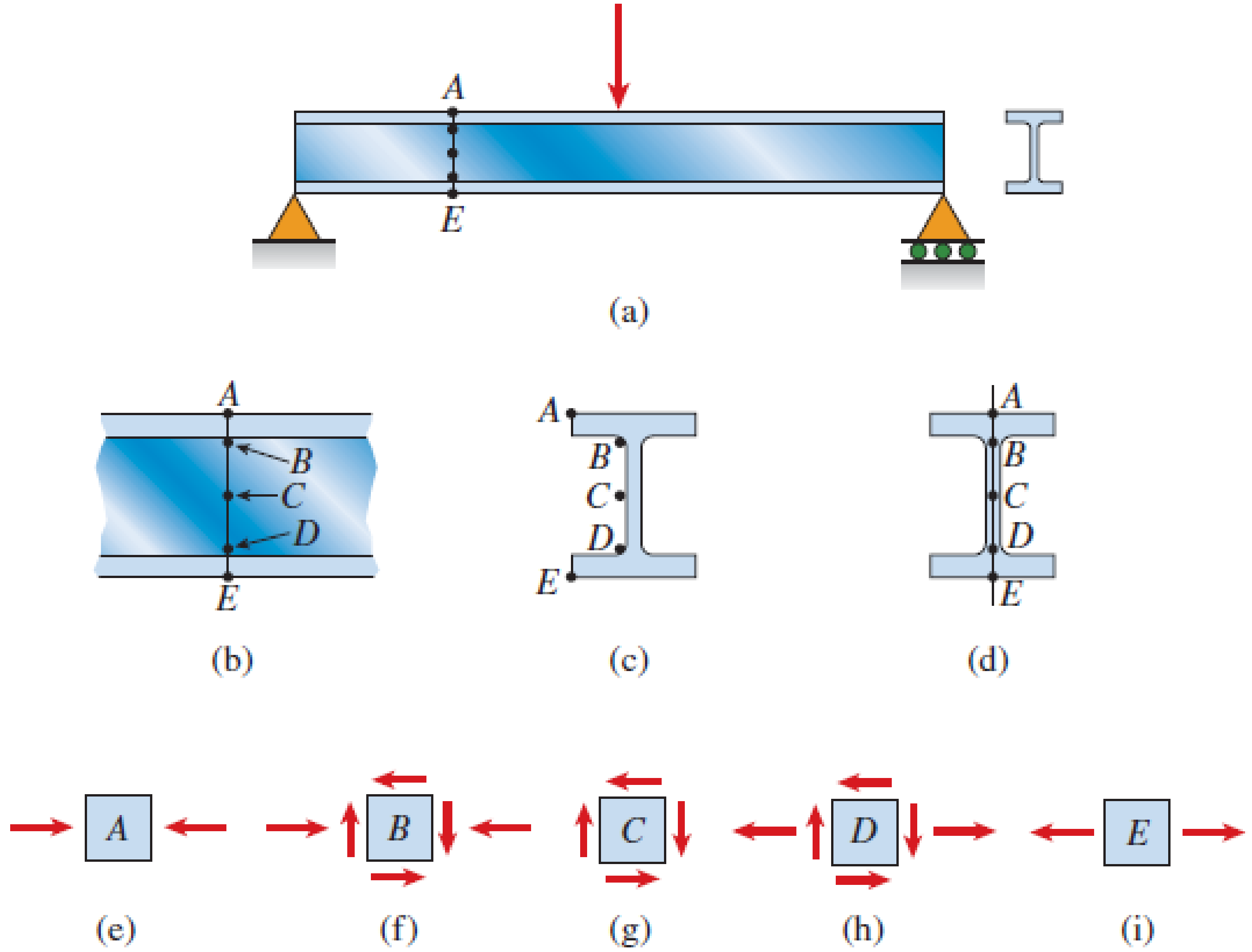
Announcement

- To be updated

Chap. 5 Stresses in Beams

Shear in bending?

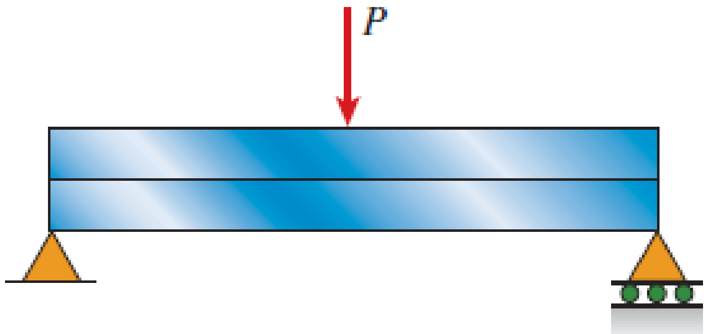




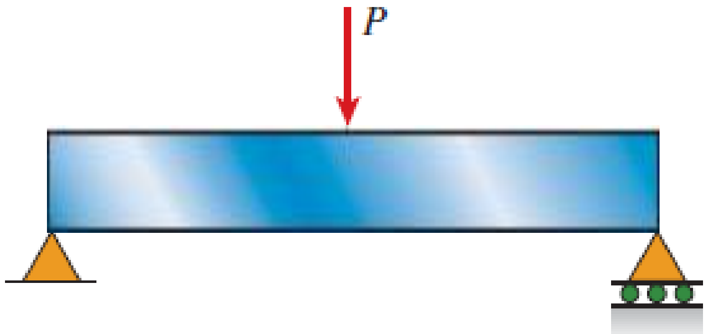
SFD

BMD

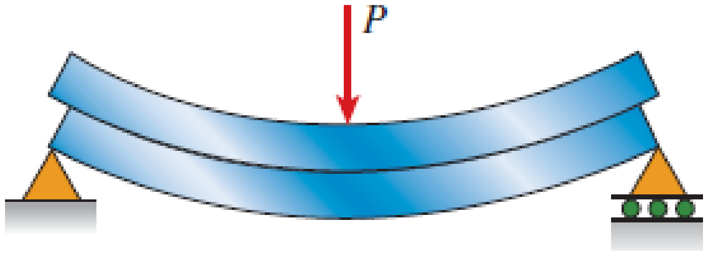
Shear in bending?



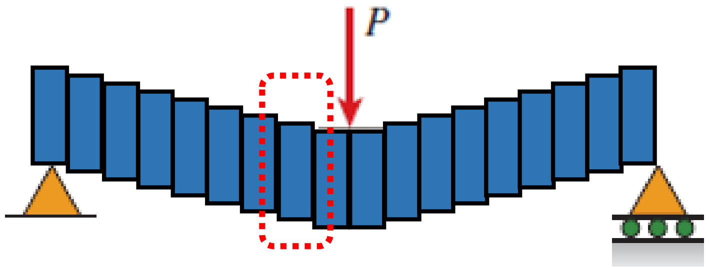
(a)

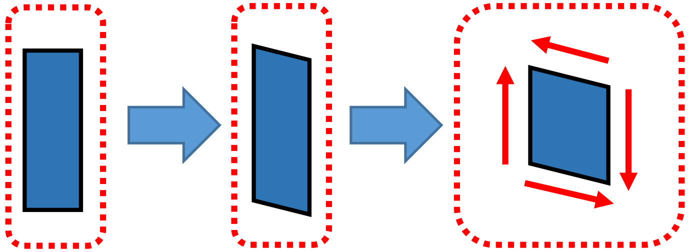


(a)

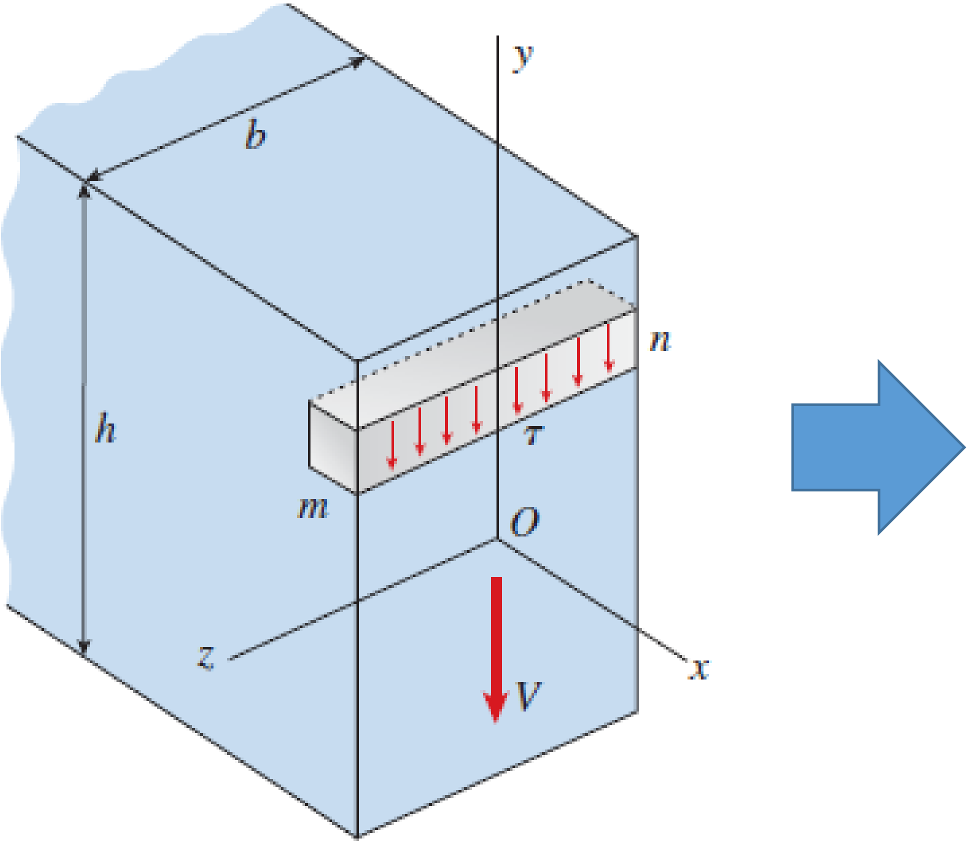


(b)





Shear stress in beams of rectangular section



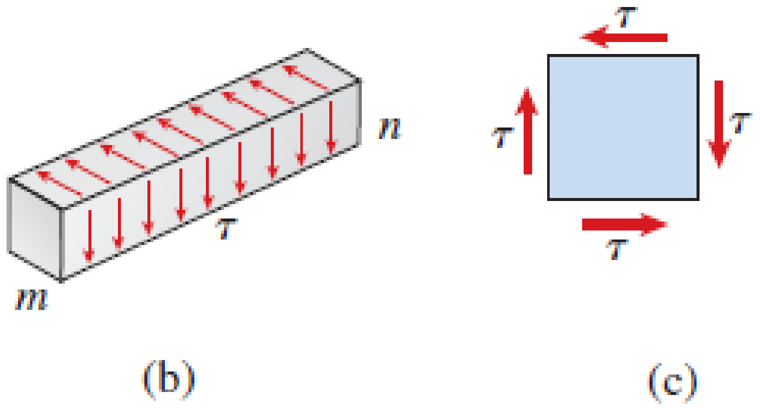
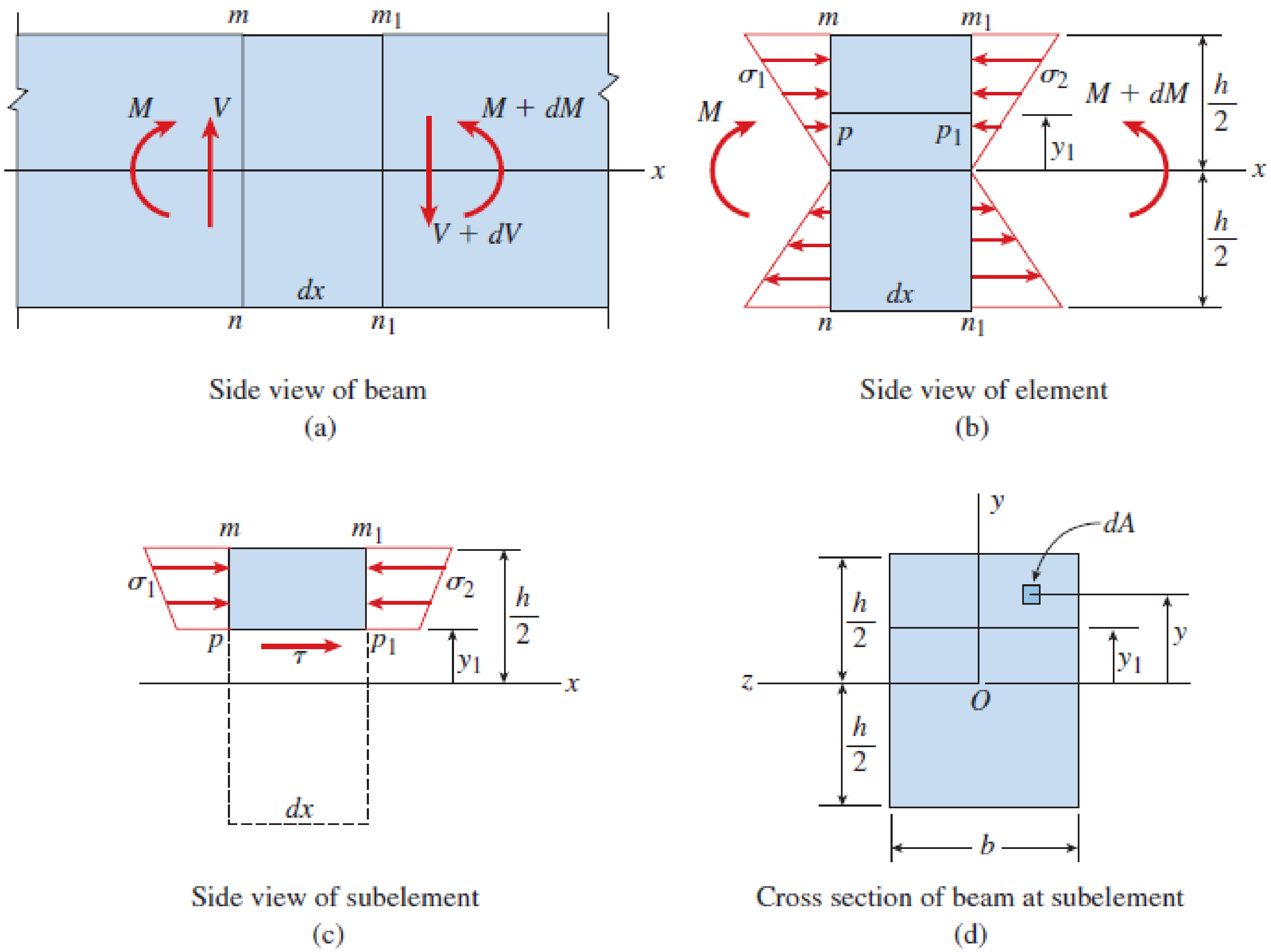


FIG. 5-26 Shear stresses in a beam of rectangular cross section

Shear stress in beams of rectangular section



Shear stress in beams of rectangular section

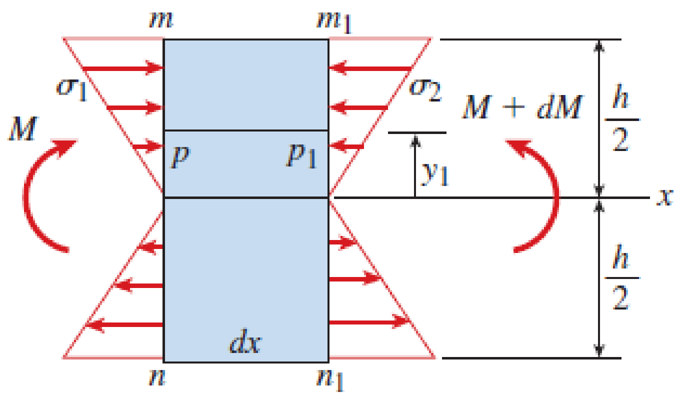
$$\sigma_1 = -\frac{My}{I}$$

$$\sigma_2 = -\frac{(M+dM)y}{I}$$

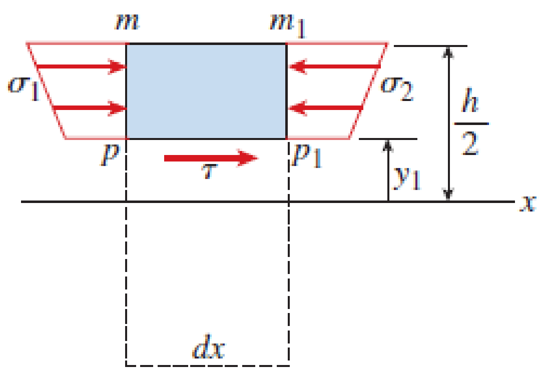
$$\sigma_1 dA = \frac{My}{I} dA$$

$$F_1 = \int \sigma_1 \, dA = \int \frac{My}{I} dA$$

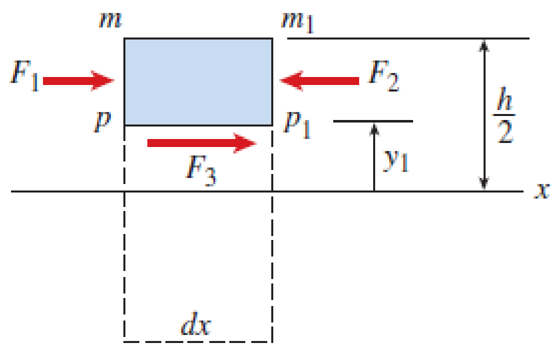
$$F_2 = \int \sigma_2 dA = \int \frac{(M+dM)y}{I} dA$$



Side view of element



Side view of subelement



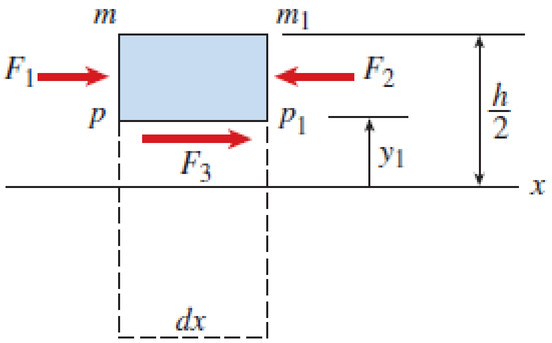
Side view of subelement

$$F_3 = F_2 - F_1$$

$$F_3 = \int \frac{(M + dM)y}{I} dA - \int \frac{My}{I} dA = \int \frac{(dM)y}{I} dA$$

$$\Rightarrow \quad F_3 = \frac{dM}{I} \int y dA$$

Shear stress in beams of rectangular section



$$F_3 = \frac{dM}{I}\int y dA$$

$$F_3 = \tau b \, dx$$

(Second moment of inertia)

$$I = \int_A y^2 \, dA$$

(Neutral axis finding)

$$\int_A y \, dA = 0$$

$$\tau = \frac{dM}{dx}\left(\frac{1}{Ib}\right)\int y \, dA$$

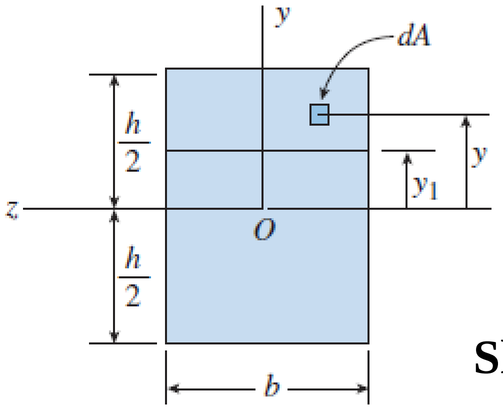
$$V = \frac{dM}{dx}$$

$$\tau = \frac{V}{Ib}\int y \, dA$$

$$Q = \int y \, dA$$

(First moment of inertia)

Why not zero?

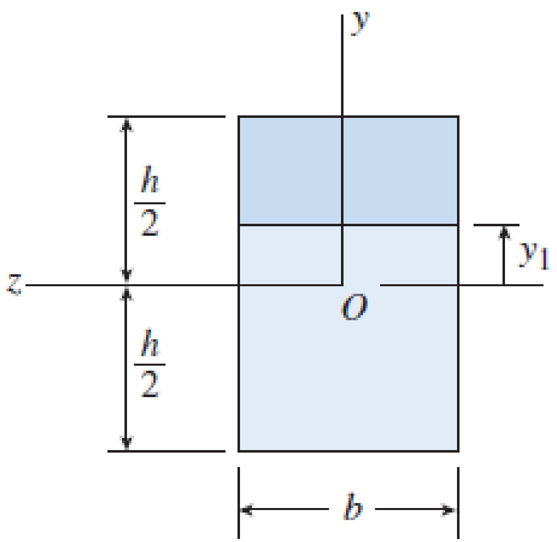


Cross section of beam at subelement

Shear Formula

$$\tau = \frac{VQ}{Ib}$$

Shear stress in beams of rectangular section



$$Q = \int y\,dA$$ (First moment of inertia)

$$Q = b\left(\frac{h}{2} - y_1\right)\left(y_1 + \frac{h/2 - y_1}{2}\right) = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right)$$

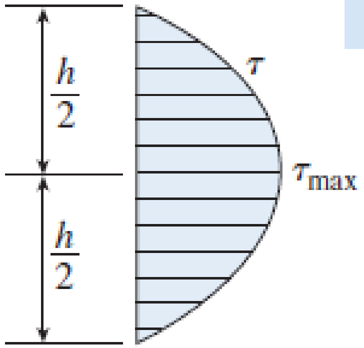
$$Q = \int y\,dA = \int_{y_1}^{h/2} yb\,dy = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right)$$

$$\tau_{max} = \frac{Vh^2}{8I} = \frac{3V}{2A}$$

$$\tau = \frac{VQ}{Ib}$$

Shear Formula

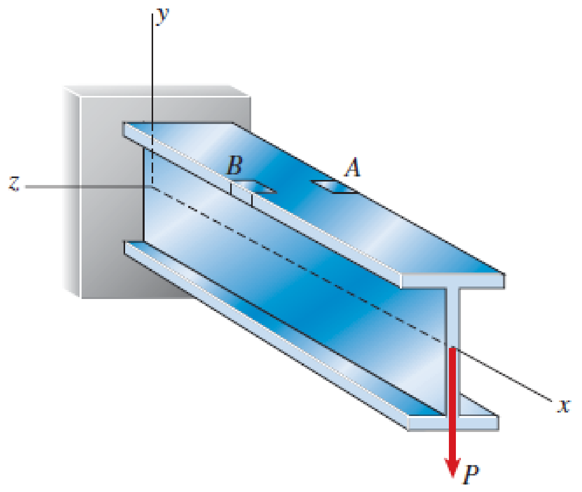
$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

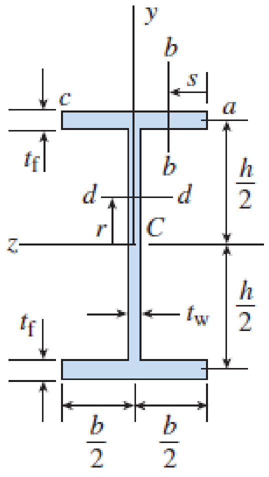


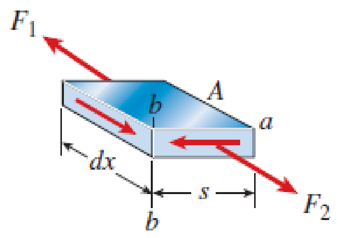
In 3D

Other examples

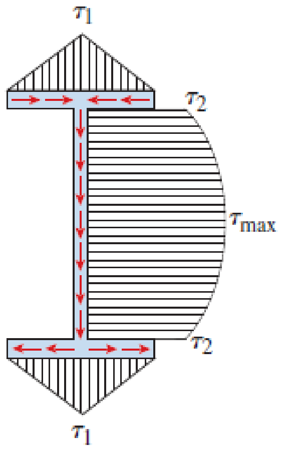
I section





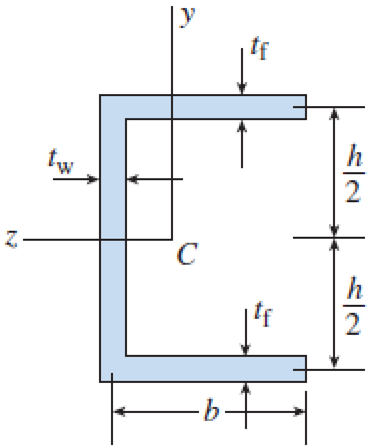


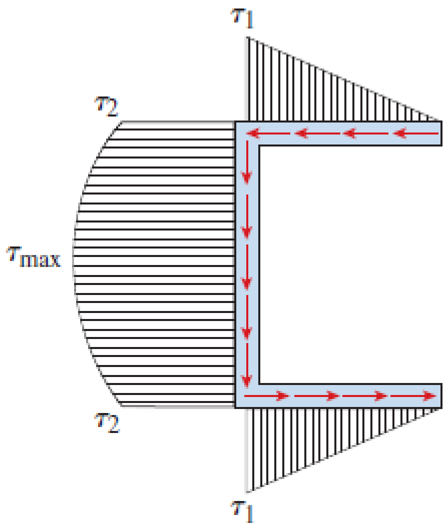
(c)

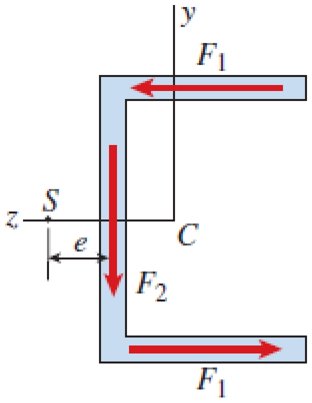


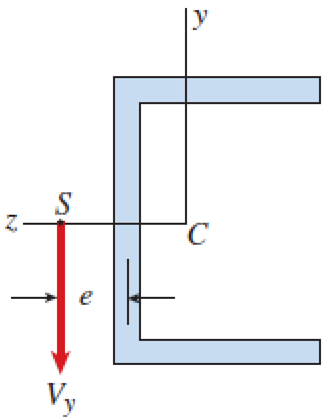
(d)

C channel





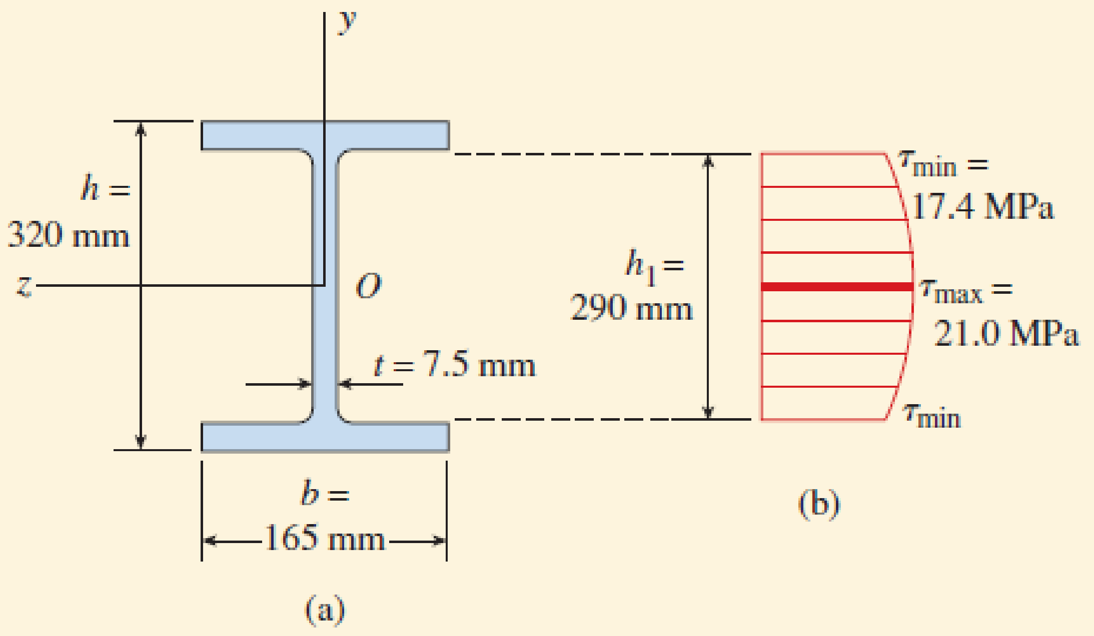




Example #1

A beam of wide-flange shape (Fig. 5-39a) is subjected to a vertical shear force $V = 45$ kN. The cross-sectional dimensions of the beam are $b = 165$ mm, $t = 7.5$ mm, $h = 320$ mm, and $h_1 = 290$ mm.

Determine the maximum shear stress, minimum shear stress, and total shear force in the web. (Disregard the areas of the fillets when making calculations.)



Example #2

A beam having a T-shaped cross section (Fig. 5-40a) is subjected to a vertical shear force $V = 10{,}000$ lb. The cross-sectional dimensions are $b = 4$ in., $t = 1.0$ in., $h = 8.0$ in., and $h_1 = 7.0$ in.

Determine the shear stress τ_1 at the top of the web (level nn) and the maximum shear stress τ_{max}. (Disregard the areas of the fillets.)