Seoul National University
Dept. of Civil and Environmental Engineering

Instructor: Junho Song
junhosong@snu.ac.kr

457.646 Topics in Structural Reliability
In-Class Material: Class 01

I. Introduction

◎ Uncertainties in Engineering

① **( ) :** Inherent randomness (or physical fluctuation)

e.g. earthquake intensity (PGA, PGV, …), wind velocity, maximum flow rate

⇒ ( ) be reduced

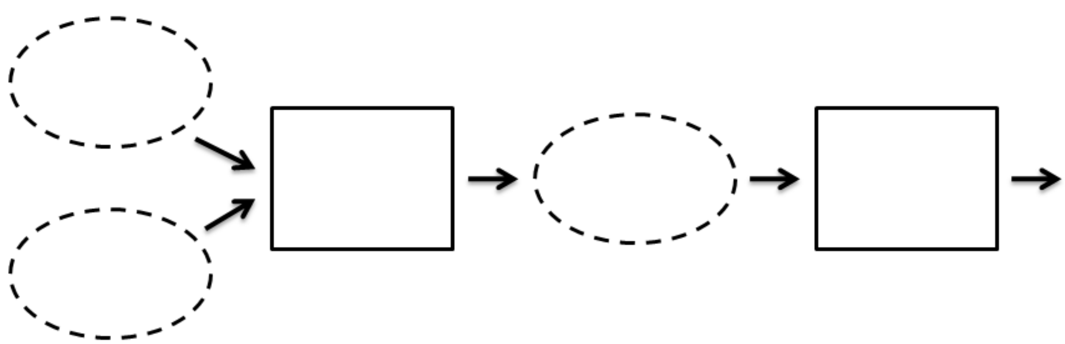
② **( ) :** uncertainty due to insufficient ( )

- ( ) uncertainty: imperfect or simplified model (e.g. 3D→2D)
  missing variables or effects
- ( ) uncertainty: insufficient data
  e.g. "sample mean is not the true mean"

⇒ ( ) be reduced by investing more in knowledge and data

Der Kiureghian, A., and O. Ditlevsen (2009). Aleatory or epistemic? Does it matter? *Structural Safety*, **31**: 105-112

◎ Uncertainty, Risk and Decisions



Decision making under ( ) leads to ( )
Need to quantify ( ) caused by ( )

◎ 457.646 Topics in Structural Reliability (Theory)

- Focus: methods for quantifying risk & applications
- Provide overview and applications of " " reliability methods

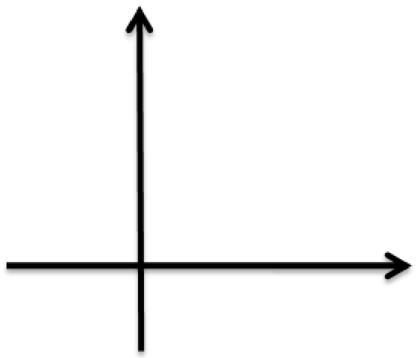
  ⇒ The word " " does not refer to physical structures (buildings and bridges, …)

  ⇒ in an ( ) & ( ) manner

Instructor: Junho Song
junhosong@snu.ac.kr

◎ Part 2: Basic theory of probability & statistics (≤ 3 weeks) (ref. A&T textbook)

◎ Part 3: Structural Reliability Analysis (SRA) - Component

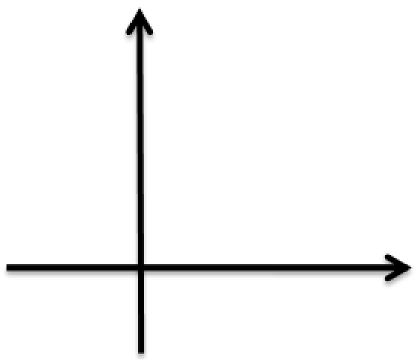


$P_f =$

- Reliability index: $\beta_{MVFOSM}, \beta_{HL}$

- Reliability methods: FORM, SORM, etc. (how to integrate ↖)

◎ Part 4: Structural Reliability Analysis (SRA) - System



$P_f =$

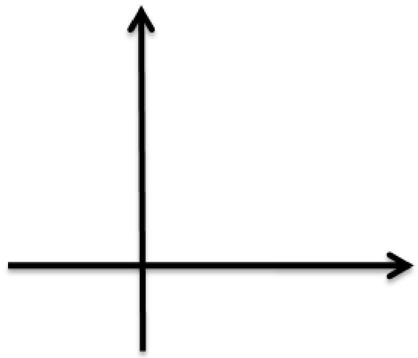
- Reliability methods developed to handle system failure domains

 : "System" reliability methods

◎ Part 5: Structural Reliability under Epistemic Uncertainty

$$P_f = \int_{g(\mathbf{x};\ )\le 0} f_{\mathbf{x}}(\mathbf{x};\ )\, d\mathbf{x}$$

◎ Part 6: Simulation Methods



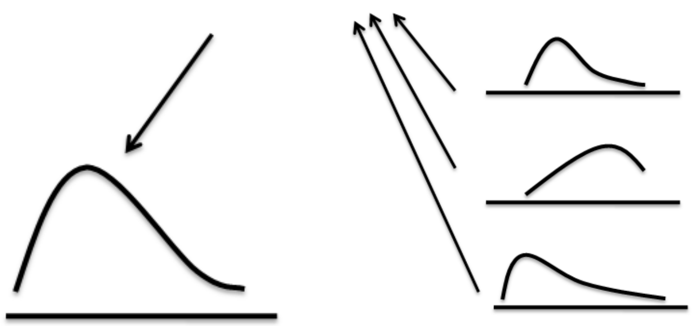
$P_f =$

⇒ Monte Carlo simulations

⇒ Efficient Sampling methods

◎ Part 7: Uncertainty Quantification

$$Y = g(\mathbf{x})$$



input distributions

Find distributions of output

◎ Part 8: Applications

Instructor: Junho Song
junhosong@snu.ac.kr

II. Basic theory of Probability and Statistics

1. Set Theory

Why do we need 'set theory' in uncertainty analysis?

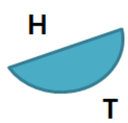
- **Uncertainty:** a (   ) of possible (   )

  e.g.   toss a coin
          roll a dice
          weight of a car

- **Probability:** numerical measure of the (   ) of an event (i.e. a group of outcomes) of interest (   ) the other possible outcomes

  e.g. "unfair coin"

  H:T=   P(H)=

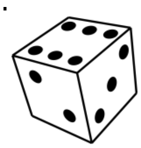


- Uncertainty analysis starts with (   ) the collection of all possible outcomes
- Principles of set theory are essential tool for this task.

2. Definitions

(a) **Sample space** (   ): the set of (   ) possible outcomes
Sample point (   ): an (   ) outcome

e.g.



Criteria	Sample space	Examples
Continuous?	"Discrete": ( ) quantities	# of typhoons at city A in a year S={ }
	"Continuous": ( ) quantities	% of congested traffic in Seoul S={ }
Can count sample points?	"Finite" : ( ) ( ) and ( )	S = { }
	"Infinite" : ( ) ( ) or ( )	S = { } S = { }

Instructor: Junho Song
junhosong@snu.ac.kr

(b) **Event** (   ): any collection of sample (   ) or any (   ) of sample space

e.g. Baseball: outcomes of each "at-bat"

S=

discrete or continuous?

infinite or finite?

"A hitter reaches a base"

E=

(c) Some notable events

- (   ) event: E=

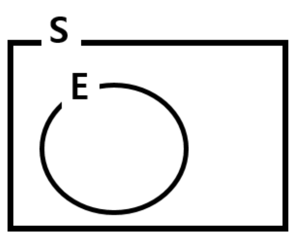
  - Occurs with certainty

- (   ) event: E=

  - cannot occur

- **Complementary** event of E : (   ) or (   )

  - An event that contains (   ) the sample points that are (   ) in E



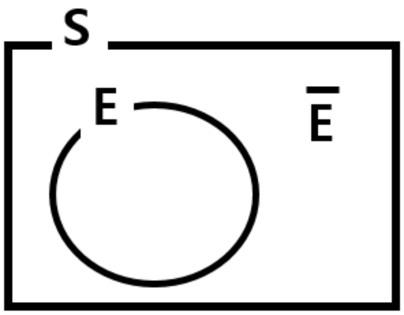
- e.g. "at-bat" outcomes

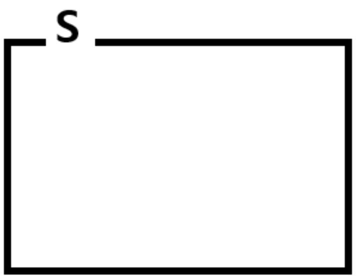
  E: "a hitter reaches a base"

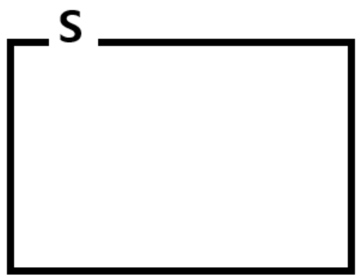
  $\bar{E} =$

- e.g. $\bar{S} =$, $\bar{\phi} =$

(d) **Venn diagram**: (   ) & (   ) representation of the sample space, sample points and events







* GUI-based interactive learning tools for Venn diagrams (and other statistical concepts) are available at http://www.stat.berkeley.edu/~stark/Java/Html/

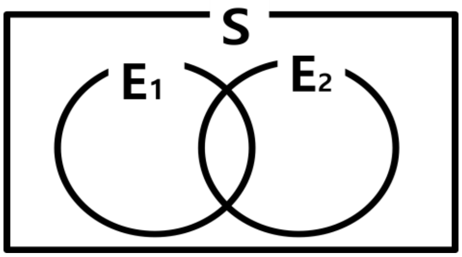
Instructor: Junho Song
junhosong@snu.ac.kr

457.646 Topics in Structural Reliability
In-Class Material: Class 02

◎ Set Operations → useful especially for ( ) reliability analysis

① **"Union" of events:** E_1 E_2

■ An event that contains all the sample points that are in E_1 E_2



e.g., Concrete mixing

- E_1 : shortage of water
- E_2 : shortage of sand
- E_3 : shortage of gravel
- E_4 : shortage of cement

E (concrete can't be produced) =

= E_i

e.g., Wind

- E_1 : blown off due to pressure
- E_2 : missile-like flying objects

$E = E_1$ E_2

e.g., Bridge pier under EQ

- E_1 : reaches displacement capacity
- E_2 : reaches shear capacity

$E =$ E_1 E_2

※ $A \cup S =$

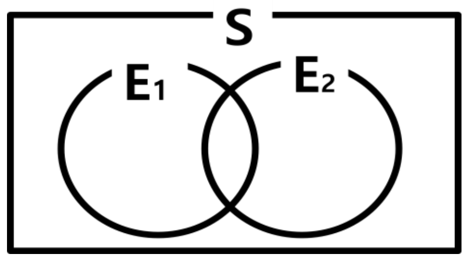
$A \cup \phi =$

$A \cup A =$

If $A \subset B$, then $A \cup B =$

② **"intersection" of events** E_1 E_2 **or**

: an event that contains all the sample points that are both in E_1 E_2



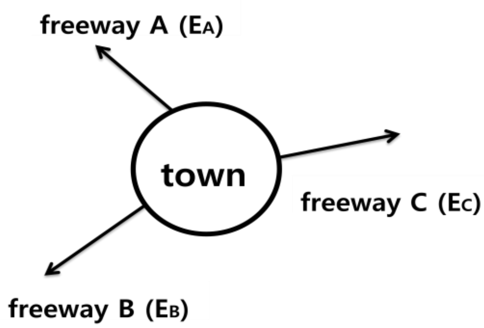
※ $A \cdot S =$

$A \cdot \phi =$

$A \cdot A =$

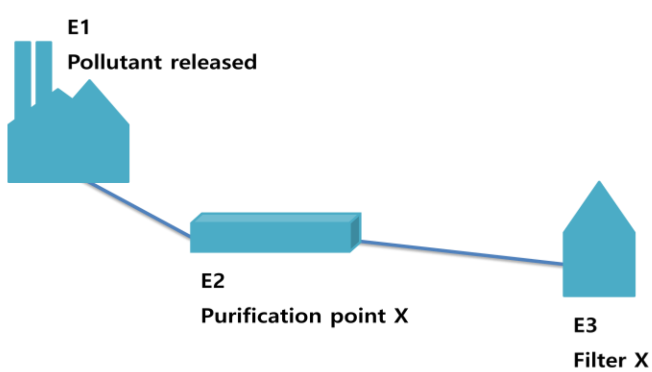
If $A \subset B$, then $A \cdot B =$

e.g.,



No evacuation by freeway $E =$

e.g.,



Exposed to pollutant $E =$

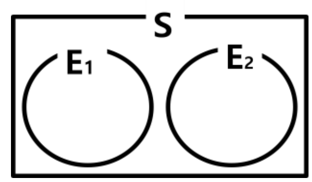
◎ Operation Rules

Commutative Rule	$E_1 \cup E_2 =$ $E_1 E_2 =$
Associative Rule	$(E_1 \cup E_2) \cup E_3 = \qquad =$ $(E_1 E_2) E_3 = \qquad =$
Distributive Rule	$(E_1 \cup E_2) E_3 =$ $(E_1 E_2) \cup E_3 =$
De Morgan's Rule	$\overline{\left(\bigcup_{i=1} E_i\right)} =$ $\overline{\left(\bigcap_{i=1} E_i\right)} =$

◎ Relationship between events

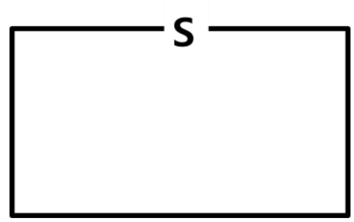
① **Mutually Exclusive events:** $E_1 E_2 =$

- Cannot occur together
- e.g. E_1 and $\overline{E_1}$
- $E_1 \cdots E_n$ and $\overline{E_i}$, $i \in \{1, \cdots, n\}$

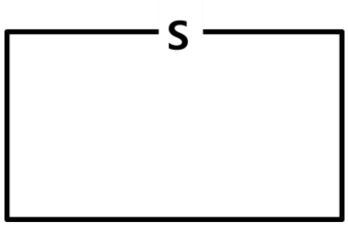


② **Collectively Exhaustive events:** $\bigcup_{i=1}^{n} E_i =$

- The union constitutes the sample space



※ **<u>MECE:</u>**



2. Mathematics of Probability (measure of likelihood of event)

◎ Four approaches for assigning probability of events

Approach	**Description**	Example : Prob. (a "Yut" stick shows the flat side)
Notion of **Relative Frequency**	Relative frequency based on empirical data, Prob. = (# of occurrences) / (# of observations)	
On a **Priori** Basis	Derived based on elementary assumptions on likelihood of events	
On **Subjective** Basis	Expert opinion ("degree of belief")	
Based on **Mixed** Information	Mix the information above to assign probability	

◎ Axioms of Probability

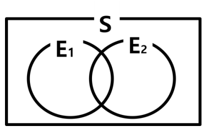
"Axioms": Statements or ideas which people <u>accept</u> as being the foundation of theory

I. $P(E) \quad\quad 0$
II. $P(S) \quad\quad 1$
III. M.E $E_1 \& E_2 : P(E_1 \bigcup E_2) =$

As a result,

① $\le P(E) \le$ $\quad (\because P(S) = P(\quad \bigcup \quad) = \quad + \quad = \quad)$
② $P(\phi) =$ $\quad (\because P(S \cup \phi) = \quad + \quad = \quad)$
③ $P(\bar{E}) =$ $\quad (\because P(E \cup \bar{E}) = \quad)$
④ $P(E_1 \cup E_2) = P(E_1) \quad P(E_2) \quad P(E_1 E_2)$

"Addition Rule"



- Venn Diagram
- Formal Proof

$$P(E_1 \bigcup E_2) = P(E_1 \bigcup \bar{E}_1 E_2) = P\left(E_1\right) + P(\bar{E}_1 E_2)$$
$$P(E_2) = P(E_1 E_2) + P\left(\overline{E_1} E_2\right)$$

"Inclusion-Exclusion Rule"

$$P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i) - \sum\sum P(E_i E_j) + \sum\sum\sum P(E_i E_j E_k) + \cdots + (-1)^{n-1} \times P(E_1 \cdots E_n)$$

* "Inverse version" derived in *Appendix A* of Der Kiureghian et al. (2007)

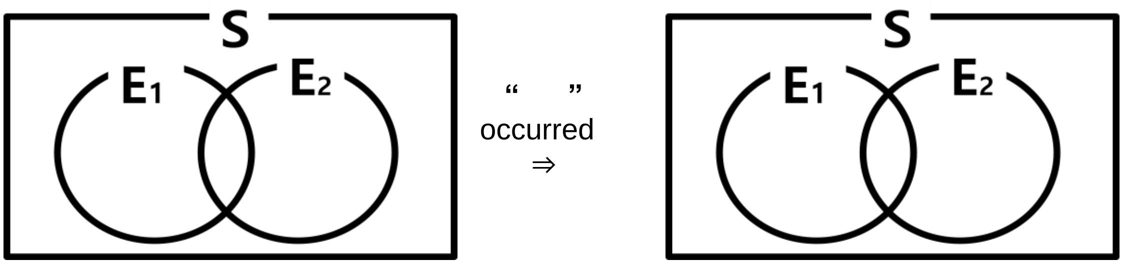
Instructor: Junho Song
junhosong@snu.ac.kr

◎ Conditional Probability & Statistical Independence

① Conditional Probability

■ Conditional probability of given

$P(E_1 | E_2) \equiv$



② $P(E_1|S) =$

③ **"Multiplication Rule":** $P(E_1E_2) =$

($\because$ $P(E_1 | E_2) =$)

- $P(E_1E_2E_3) =$

- $P(E_1 \cdots E_n) =$

④ All the other prob. rules should be applicable to conditional probabilities as long as all the prob. are defined within the same space

- $P(E_1 \cup E_2|E_3) =$
- $P(E_1E_2|E_3) =$
- $P(\overline{E_1}|E_3) =$

⑤ **Statistical Independence:** The occurrence of one event does not affect the likelihood of the other event

- $P(E_1|E_2) =$

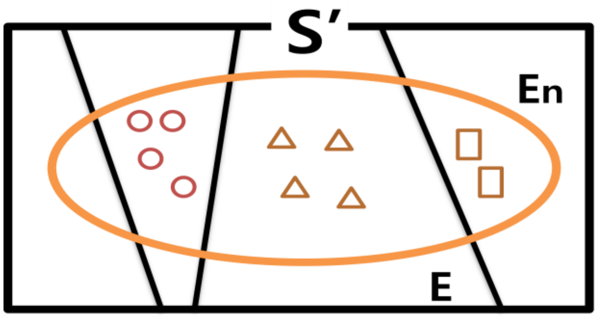
- $P(E_2|E_1) =$

- $P(E_1E_2) =$

cf. Mutually Exclusive $P(E_1E_2) = 0$

◎ Total Prob. Theorem

Setting: $E_1, E_2, ..., E_n$:________ events



$P(E)$ → Not easy to get directly

$P(E\,|E_i)$ → Easier to get

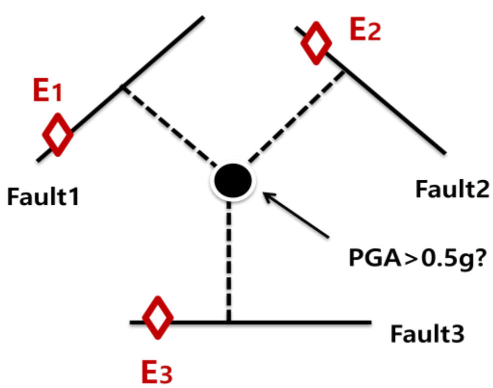
$$P(E) = \sum_{i=1}^{n}$$

Proof:

<u>Examples:</u>

(1) Seismic hazard analysis:

$$P(E) =$$



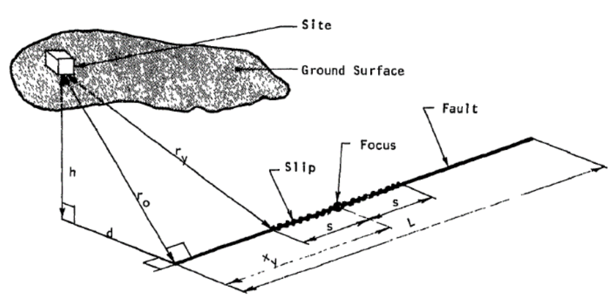
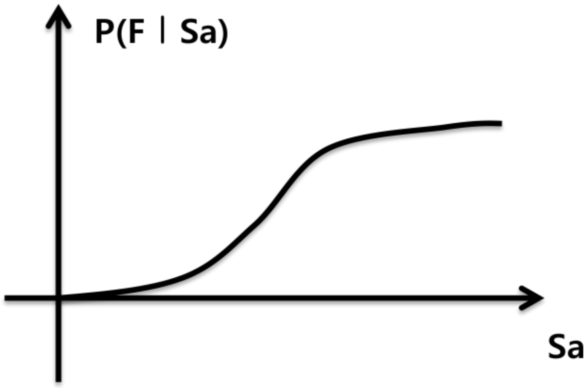


FIG. 3.1 TYPE I SOURCE (BASIC CASE)

Der Kiureghian, A. (1976). *A line source-model for seismic risk analysis*, Ph.D. dissertation, University of Illinois at Urbana-Champaign, Urbana, USA.

Instructor: Junho Song
junhosong@snu.ac.kr

(2) Probability of structural failure under an uncertain input intensity: Fragility



$$P(F) = \sum$$

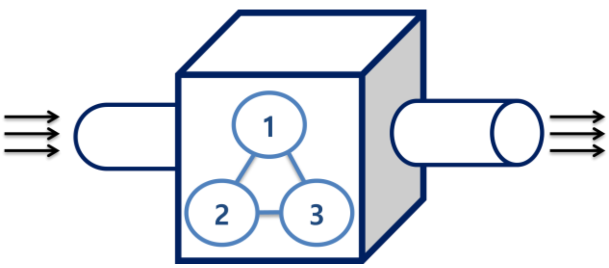
$$= \int$$

◎ **Bayes Theorem**

$$P(E_i | E) = \frac{P(E | E_i)}{}$$

- Decision making
- Parameter estimation
- Inference

Example)



Measure of cleanness, X (0 : contaminated ~ 100 : clean)

	$P(E_i)$	$P(X \leq 20 \| E_i)$
1	0.1	0.9
2	0.3	0.2
3	0.6	0.01

$X \leq 20$ $\Rightarrow$ Which one failed?

$P(E_i | X \leq 20) =$