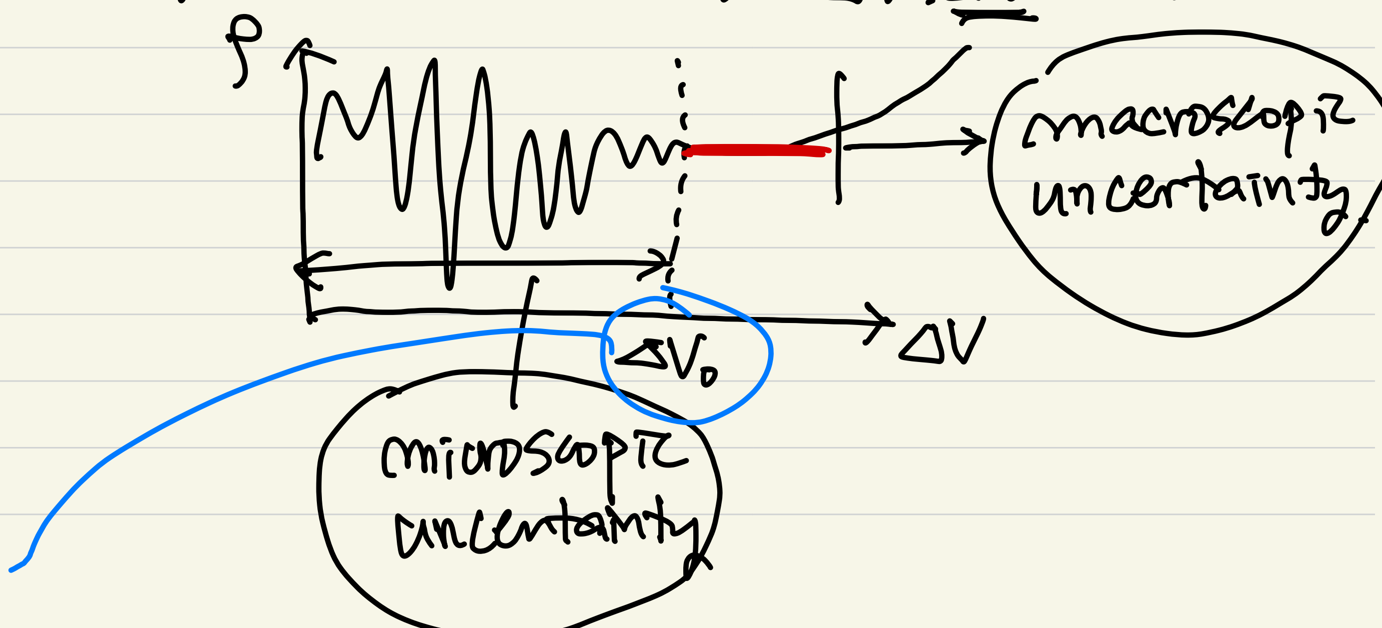
I. BASIC DEFINITIONS & CONCEPTS.

- density and volume fraction
  - density of a continuum: $\rho = \lim_{\Delta V \to \Delta V_0} \dfrac{\Delta M}{\Delta V}$.



one mole of gas : 10^{23} molecules
in 22 liters in the standard condition.

on volume containing 10^{4} molecules ~ $(0.120\mu m)^3$

point measurement
→ continuum assumption is valid.

- number density (# of particles per unit vol.)

$$n = \lim_{\delta V \to \delta V_0} \frac{\delta N}{\delta V}$$

→ to ensure the converged

statistics

- Volume fraction

$$\alpha_c = \lim_{\delta V \to \delta V_0} \frac{\delta V_c}{\delta V}$$ for cont. phase

δV_c → vol. occupied by cont. phase in δV.

δV → total vol.

$$\alpha_d = \lim_{\delta V \to \delta V_0} \frac{\delta V_d}{\delta V}$$ for dispersed phase

$\delta V = \delta V_c + \delta V_d$. $\alpha_c + \alpha_d = 1.0$

for gas : (α_d) : void fraction, gas holdup

porosity, ...

- Bulk (apparent) density

$$\bar{\rho}_d = \lim_{\delta V \to \delta V_0} \frac{\delta M_d}{\delta V}$$

mass of dispersed (cont.) phase.

$$\bar{\rho}_c = \lim_{\delta V \to \delta V_0} \frac{\delta M_c}{\delta V}$$

- Material (actual) density

$$\rho_d = \frac{\delta M_d}{\delta V_d}, \quad \rho_c = \frac{\delta M_c}{\delta V_c}.$$

- Mixture density

$$\bar{\rho}_d = \lim_{\delta V \to \delta V_0} \frac{\rho_d \cdot \delta V_d}{\delta V} = \rho_d \cdot \alpha_d, \quad \bar{\rho}_c = \rho_c \cdot \alpha_c$$

$$\rho_m = \bar{\rho}_d + \bar{\rho}_c = \alpha_d \rho_d + \alpha_c \rho_c$$
$$= \alpha_d \rho_d + (1-\alpha_d)\rho_c .$$

($\mu_m = \alpha_d \mu_d + \alpha_c \mu_c$ for mixture viscosity)

- Mass concentration of dispersed phase.

$$C = \frac{\bar{\rho}_d}{\bar{\rho}_c} = \frac{\alpha_d \rho_d}{\alpha_c \rho_c} \quad \text{(in a mixture)}$$

apparent density

- Loading (loading ratio, mass flow ratio)

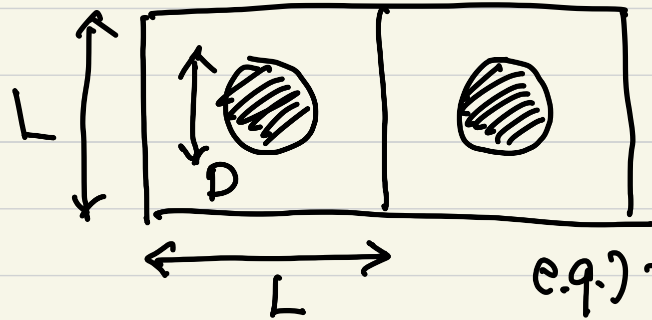
= mass flow of disp. phase / " of cont. phase

- local loading : $z = \bar{\rho}_d v_d / \bar{\rho}_c v_c$ vel. of each phase
- overall (total) loading : $Z_i = \rho_d v_d / \rho_c v_c$.

- Quality (for gas-liq. two-phase flow)

$$= \frac{\text{mass flow rate of gas}}{\text{total mass flow rate}} \sim \frac{\text{mass of gas}}{\text{total mass}}.$$

– spacing between particles (solid, bubbles …)



$$\alpha_d = \left(\frac{\pi}{6}D^3\right) / L^3$$

$$L/D = \left(\pi / 6\alpha_d\right)^{1/3}$$

e.g.) for 1 ppm, $\alpha_d = 1\times 10^{-6}$, $L/D \simeq 80$

if $L/D \lesssim O(1)$, transport phenomena are affected by neighbors.

$$\alpha_d = \frac{\bar{\rho}_d}{\rho_d} = \frac{\bar{\rho}_d}{\bar{\rho}_c}\cdot\frac{\bar{\rho}_c}{\rho_c}\cdot\frac{\rho_c}{\rho_d} = C\cdot\alpha_c\frac{\rho_c}{\rho_d}.$$

$$\kappa = C\frac{\rho_c}{\rho_d}$$

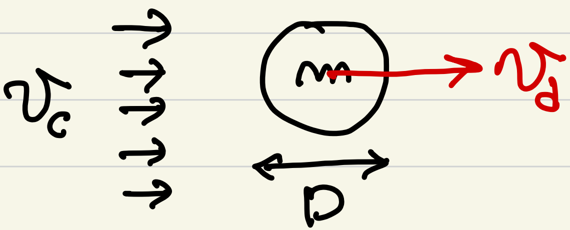
$$\rightarrow\ L/D = \left(\frac{\pi}{6}\cdot\frac{1+\kappa}{\kappa}\right)^{1/3}$$

for most gas-solid, gas-liquid flows,

$\rho_c/\rho_d \simeq O(10^{-3}) \rightarrow L/D \sim O(10)$.

∴ it is safe to assume that the dispersed phase is isolated.

- Mobility of a spherical particle.



· eq. of motion.

$$\boxed{m\frac{dv_d}{dt} = F_D} \leftarrow \text{drag.}$$

if $Re_d = \rho_c D|v_c - v_d|/\mu_c < 1$

· Stokes drag.

$$F_D = 3\pi\mu_c D(v_c - v_d).$$

$$m\frac{dv_d}{dt} = 3\pi\mu_c D(v_c - v_d), \quad B \equiv \frac{1}{3\pi\mu_c D}.$$

$$= \frac{v_c - v_d}{B}$$

$$\cdot\ mB = \frac{m}{3\pi\mu_c D}, \quad m = \rho_d \cdot \overset{\text{volume},}{V_d} = \rho_d \cdot \frac{\pi}{6}D^3.$$

$$= \boxed{\frac{\rho_d D^2}{18\mu_c} \equiv \tau_v} \ (\underline{\text{momentum response time}}).$$

$$\frac{dv_d}{dt} = \frac{v_c - v_d}{\tau_v}, \quad \text{let } \hat{v}_d = v_d/v_c, \ \hat{t} = t/\tau_F$$

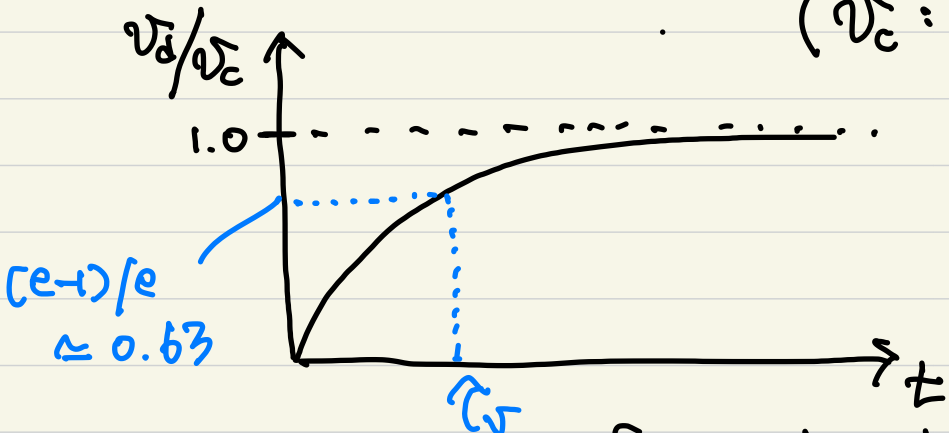
$$\rightarrow \frac{d\hat{v}_d}{d\hat{t}} = \frac{1 - \hat{v}_d}{\boxed{\tau_v/\tau_F}} \quad \tau_F = L_F/v_F \ (\text{flow time scale})$$

$$= St_v \ (\text{Stokes number})$$

flow

$$\therefore \frac{d\hat{v}_d}{d\hat{t}} = \frac{1-\hat{v}_d}{St_v} \Rightarrow \frac{v_d}{v_c} = 1 - e^{-t/\tau_v}$$

(v_c : constant, $v_d(0) = 0$)



if $t = \tau_v$.

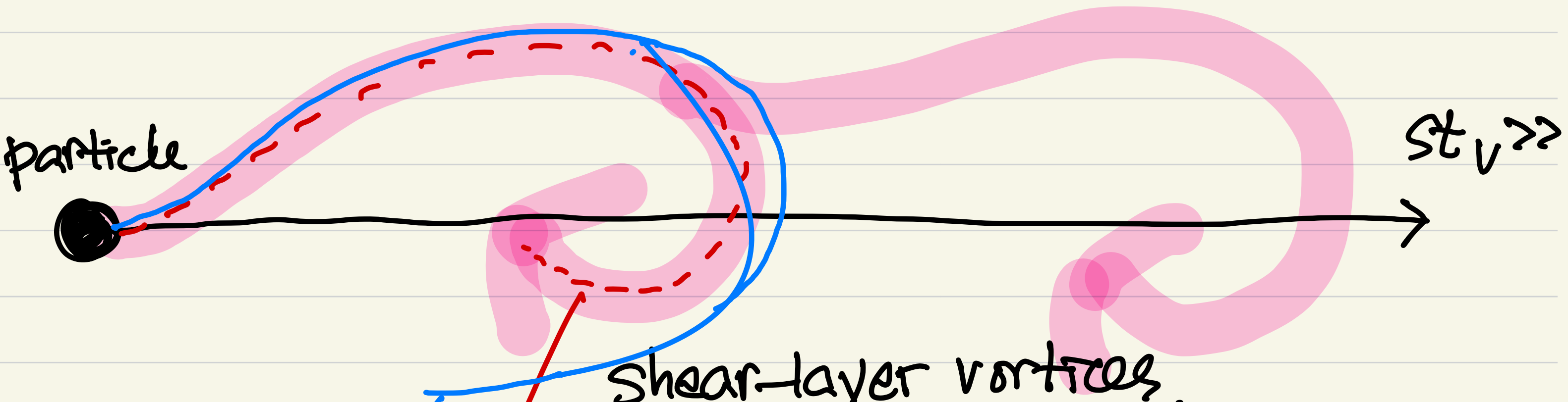
$v_d/v_c = (e-1)/e$.

for water droplet ($\sim 100\,\mu m$)
in air, $\tau_v \simeq 30$ msec.

- Stokes Number.

$St_v = \tau_v/\tau_F \ll 1.0$: dispersed particles and continuous fluid are in near equilibrium.

" $\gg 1.0$: particles are unaffected by the fluid motion.



$St_v \ll 1$. (criteria for seeding particles for PIV).

$St_v \simeq 1.0$

↓

· interaction between particle and eddy

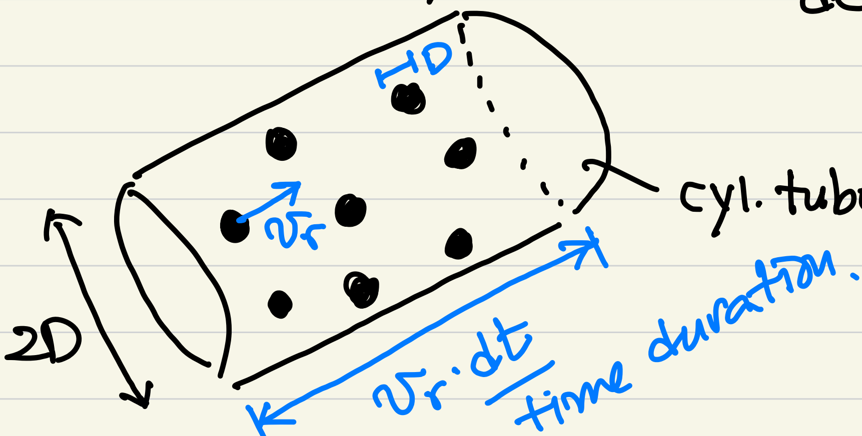
→ preferential concentration.

- Dilute vs. dense flows.

  - mtm response time : τ_v
  - collision time scale : τ_c

→ $\tau_v / \tau_c < 1$: dilute, particle motion is controlled by the fluid forces.

$\tau_v / \tau_c > 1$: dense, 〃 by collisions or contact



⇒ # of particles in the tube

: $\delta N = \underbrace{n}_{\text{\# density of}} \cdot \pi D^2 \cdot v_r \cdot dt$

particles

→ collision frequency : f_c

$$= \frac{\delta N}{\delta t} = n \cdot \pi D^2 \cdot v_r$$

Then, $\tau_c = 1/f_c = \dfrac{1}{n \cdot \pi D^2 \cdot v_r}$.

$$\therefore \tau_v / \tau_c = \frac{n \cdot \pi \rho_d D^4 v_r}{18 \mu_c} = \frac{\bar{\rho}_d D v_r}{3 \mu_c}.$$

$$\left(\bar{\rho}_d = n \cdot m = n \cdot \rho_d \cdot \frac{\pi}{6} D^3 \right)$$

for dilute flow : $D < \dfrac{3\mu_c}{\bar{\rho}_d v_r}$ or $\alpha_d < \dfrac{3\mu_c}{\rho_d D v_r}$

" dense flow : $D > \dfrac{3\mu_c}{\bar{\rho}_d v_r}$ or $\alpha_d > \dfrac{3\mu_c}{\rho_d D v_r}$.