II. PHASE COUPLING (transfer of mass, mtm, and energy)

$$\boxed{\text{carrier fluid}} \;\xleftrightarrow[\text{energy}]{\text{mass, mtm}}\; \boxed{\text{dispersed particles}}$$

Carrier fluid:

- pressure
- temperature
- velocity
- species concentration
- ⋮

Dispersed particles:

- velocity
- temperature
- loading / vol. fraction.
- size
- ⋮

- one way : fluid $\longrightarrow$ particles
- two " : fluid $\longleftrightarrow$ particles.

- mass coupling : phase change
  (boiling, condensation, evaporation ...)
- mtm " : interfacial forces.
- energy " : heat transfer.

- Review of thermodynamics.
  - Heat conduction: flow of heat from high-T to lower-T region toward uniform temp. distribution. (via. direct contact)
  - Fourier's Law

$$\dot{q} = -k_c \nabla T \qquad \left(1D : \dot{q} = -k_c \frac{dT}{dx}\right)$$

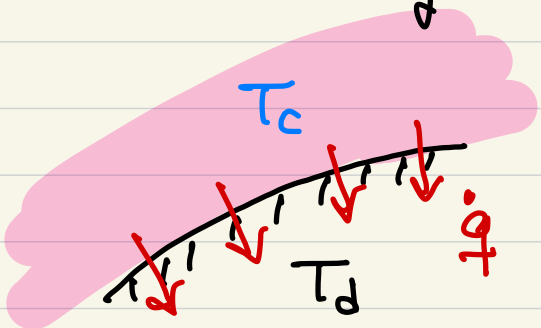
heat flux

thermal conductivity [W/mK]

  - thermal diffusivity : $a \equiv k_c / \rho c_p$ specific heat

- Convective heat transfer:

flow of heat across the boundary (interface) of a body in contact w/ a fluid (flow)



$$\dot{q} = h(T_c - T_d)$$

per unit area and time

heat tr. coefficient $[W/(m^2 K)]$

· Nusselt number (Nu)

① conv.

② cond.

$$\left[\frac{h}{k_c}\right] = \left[\frac{1}{m}\right] \Rightarrow h = Nu \cdot \frac{k_c}{L}.$$

③ char. length

$$\therefore \dot{q} = Nu \cdot \frac{k_c}{L} (T_c - T_d).$$

per unit time,

$$\pi D^2 \cdot \dot{q} = Nu \cdot \pi D^2 \cdot \frac{k_c}{D} (T_c - T_d)$$

analogy

$$m \frac{dv_d}{dt} = 3\pi \mu_c D (v_c - v_d).$$

$$m C_d \frac{dT_d}{dt} = Nu \cdot \pi k_c D (T_c - T_d)$$

specific heat of particle.

for a forced convection,

$Nu = 2 + 0.6 Re^{1/2} \cdot Pr^{1/3}$ (Ranz-Marshall, 1952)

- thermal response time, $\tau_T \equiv \rho_d C_d D^2 / 12 k_c$

$$\frac{dT_d}{dt} = \frac{Nu}{2} \cdot \boxed{\frac{12 k_c}{\rho_d C_d D^2}} (T_c - T_d) \sim \frac{1}{\tau_T}(T_c - T_d).$$

$$\cdot \ \frac{\tau_v}{\tau_T} = \frac{\rho_d D^2}{18 \mu_c} \cdot \frac{12 k_c}{\rho_d C_d D^2} = \frac{2}{3} \cdot \boxed{\frac{k_c}{\mu_c C_c}} \frac{C_c}{C_d} = \frac{2}{3} \frac{C_c}{C_d} \cdot \frac{1}{Pr}$$

- for gas, $Pr \sim O(1) \rightarrow \tau_v / \tau_T \simeq O(1)$.
- for liq. $Pr \sim O(10 - 10^{2})$. $\rightarrow \tau_v / \tau_T \simeq O(10^{-2} - 10^{-1})$.

velocity equilibrium is achieved much rapidly than thermal equilibrium.

- Prandtl number,

· Fourier's Law : $\dot{q} = -k\nabla T$.

· Eq. of heat conduction : $\rho C_p \frac{\partial T}{\partial t} = k\nabla^2 T$

$\partial T/\partial t = (k/\rho C_p)\nabla^2 T$

$= \underbrace{a}_{\text{thermal diffusivity}}\nabla^2 T$.

· Eq. of fluid motion

$\frac{\partial \bar{u}}{\partial t} = \nu \nabla^2 \bar{u}$

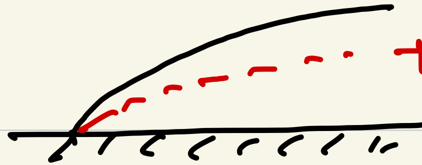
ν : kinematic viscosity (μ/ρ)

(momentum diffusivity)

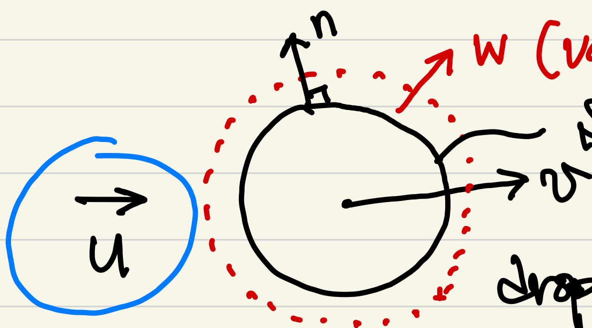
$\rightarrow Pr \equiv \frac{\nu}{a} = \frac{\nu}{k/\rho C_p} = \frac{\mu C_p}{k}$.

$= \frac{\text{viscous diffusion rate}}{\text{thermal} \quad ''}$ → BL vs thermal BL.

$Pr > 1$.



- Mass transfer (phase change)



droplet (m: mass, D: diameter)

droplet continuity eq. states that the rate of change of droplet mass is the negative value of the mass eflux through the droplet surface.

$$\rightarrow \frac{dm}{dt} = -\rho_s w S'$$

vapor density

· Fick's Law: $\dot{M} = \rho_s w = -D_v \dfrac{\partial \rho_A}{\partial n}$

density of species A.

diffusion coefficient $[m^2/s]$

(at the interf.)

mass flux per unit area and time.

$$= -\rho_s D_v \frac{\partial \left(\frac{\rho_A}{\rho_s} \right)}{\partial n}$$

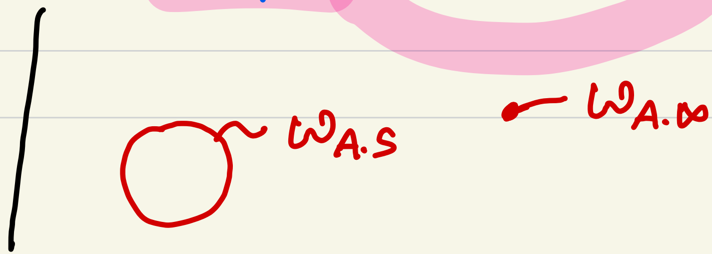
$$= -\rho_s D_v \boxed{\frac{\partial \omega_A}{\partial n}}$$

mass fraction of species 'A' in the mixture.

surf.

at far field

$$\rho_s w \sim \rho_c D_v \frac{\omega_{A.s} - \omega_{A.\infty}}{D}$$



$$\stackrel{\downarrow}{=} \underline{h_D} \cdot \rho_c \left(\omega_{A,s} - \omega_{A,\infty}\right)$$

↳ mass transfer coefficient [m/s].

$$\cdot\ \dot{m} = h_D \cdot \rho_c \left(\omega_{A,s} - \omega_{A,\infty}\right)$$

$$\left[\frac{h_D}{D_v}\right] = \left[\frac{1}{m}\right] \rightarrow h_D = \underline{Sh} \cdot \frac{D_v}{L}.$$

char. length

Sherwood number.

$$\left(\equiv h_D \cdot L / D_v\right)$$

$$\Leftarrow \frac{\text{convective mass tr.}}{\text{diffusion rate.}}$$

$$\dot{m} = Sh \cdot \frac{D_v}{L} \cdot \rho_c \left(\omega_{A,\infty} - \omega_{A,s}\right) \quad \times \pi D^2$$

$L \leftrightarrow D$

$$\pi D^2 \cdot \dot{M} = \frac{dm}{dt} = Sh \cdot \rho_c \cdot D_v \cdot \pi D \left(\omega_{A,\infty} - \omega_{A,s} \right)$$

$$\begin{cases} \omega_{A,\infty} < \omega_{A,s} & : \text{evaporation.} \\ \omega_{A,\infty} > \omega_{A,s} & : \text{condensation.} \end{cases}$$

$$= \frac{M_A}{M_m} \cdot \frac{P_A}{P}$$

$P_A \rightarrow$ saturation pressure corresponding to droplet temp.

$M_A / M_m \rightarrow$ molar weight of species A and mixture ($M_A / M_m \simeq 18/29$ for water vapor in air).